

## The Task of Optimal Control of the Test Facility “Artificial Lungs”

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**Abstract:** The structure of the predictive control system of the test facility “Artificial lungs” is developed. The statement of the task of optimal control of actuating mechanisms of the test facility “Artificial lungs” is formalized: the equation of object control in the form of differential equations is formulated, vectors states and controls and restrictions on their components are found, optimality criterion in the form of a generalized functional research is determined. An algorithm for the formation of the task and an algorithm for the optimal control over supply and removal of gases from the artificial lungs in order to simulate oxygen consumption are proposed.

**Key words:** Artificial lungs, contained breathing apparatus, optimal control, A.A. Krasovskiy’s functional, predictive algorithm, components

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### INTRODUCTION

Self-Contained Breathing Apparatus (SCBA) working chemical oxygen are used for protection of human respiratory system in a variety of extreme situations: on the earth and under the earth in space and on transport, on the water and under the water.

Since, SCBA tests on humans for many reasons are possible only to a limited extent and in some cases they can be unsafe, these tests are made in specialized test facilities called “Artificial Lungs” (AL).

Currently, different types of automated AL facilities are used for SCBA trials (Kyriazi, 1986; Deno, 1984; Beeckman *et al.*, 1998; Meka and Oostrom, 2004). The main objective of AL facility control systems when simulating human breath is the realization of a given mode of AL functioning adequately reflecting psychophysiological state of a person, characterized by the values of various parameters (pulmonary ventilation, pressure, temperature, humidity, flow and concentration of gases, breathing pneumotachogram, etc.). The existing control systems of AL facilities do not allow to reproduce human breath accurately in particular to simulate oxygen consumption by a human being in a wide range of values of the respiratory rate (ratio of the volume of released carbon dioxide and consumed oxygen) to simulate human respiration pneumotachograms (Kyriazi, 1986; Deno, 1984; Beeckman *et al.*, 1998; Meka and Oostrom, 2004; Gudkov *et al.*, 2009).

The object of the study in this paper is an automated AL test facility where the oxygen consumption is simulated as a release of the calculated volume of Gas-Air Mixture (GAM) into the atmosphere with simultaneous

supply of nitrogen and carbon dioxide in amounts that are removed when you release GAM into the atmosphere (Gudkov *et al.*, 2009). We assume that a three-component mixture of gases  $O_2$ ,  $CO_2$ ,  $N_2$  circulates in the AL. The test facility is equipped with dosing pistons with linear electrical actuators to release the GAM and feed carbon dioxide and nitrogen in the facility. These devices must operate in synchronization with the drive of breathing simulator which reproduces various breathing pneumotachograms (sine, triangular, trapezoidal, etc.) with a predetermined frequency  $n$  and depth  $V_d$ . The release of GAM and supply of gases is carried out in the inhalation phase. Since, SCBA trials in AL facility involve measuring concentrations of inhaled and exhaled gases by gas analyzers which have a certain delay and errors, the AL control system is supposed to predict gas concentrations and volumes of gases released and fed into the breathing simulator at a certain time interval. Furthermore, it is necessary to ensure accurate reproduction of the volumes of released GAM and nitrogen and carbon dioxide fed on each cycle breaths. If inaccurate data on the volumes are reproduced, the accumulated errors lead to inadequate simulation of oxygen consumption and reduce the quality of the SCBA tests.

The purpose of this work is to formulate the problem of optimal control of the AL facility and develop an algorithm of AL actuators control to reduce the number of reproduction errors in the given volumes of gases.

**The structure and working principle of the predictive control system:** Figure 1 shows a simplified block diagram of the control system of the “Artificial Lungs” facility. We consider the principle of operation of the control system.

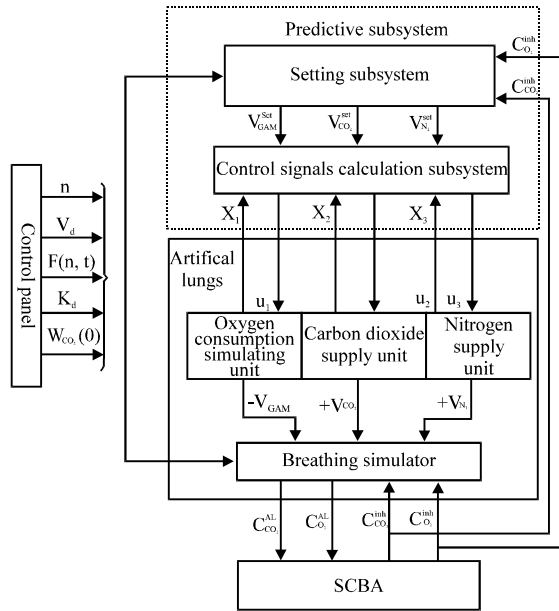


Fig. 1: Block diagram of the control system of artificial lungs facility

The operator sets the SCBA test mode in the AL facility from the control panel and sets the frequency  $n$ , the depth  $V_d$  the breathing factor  $K_d$ , the type of pneumotachogram  $F(n, t)$  and the carbon dioxide flow  $W_{CO_2}(0)$ , imitating  $CO_2$  emissions by a human being, fed into the breathing simulator.

**Setting subsystem:** The process of operation of the system is divided into cycles. At each cycle the necessary volumes of released GAM discharge and fed  $CO_2$  and  $N_2$  are corrected and prediction of the system status for a certain period of time corresponding to the next inhalation is made.

On the basis of a predetermined test mode and measured values of exhaled gas concentrations  $C_{CO_2}^{inh}$  and  $C_{O_2}^{inh}$ , the setting subsystem forms in the time function the preset values of released GAM  $V_{GAM}^{set}(t)$  and fed gases  $V_{CO_2}^{set}(t), V_{N_2}^{set}(t)$ . The predictive model of the setting subsystem is determined by the system of Eq. 1:

$$\left. \begin{aligned} \frac{dV_{GAM}(t)}{dt} &= W_{GAM} F(n, t), V_{GAM}(t_{sc}) = 0 \\ \frac{dV_{CO_2}(t)}{dt} &= (W_{CO_2} + W_{CO_2}(0)) F(n, t), V_{CO_2}(t_{sc}) = 0 \\ \frac{dV_{N_2}(t)}{dt} &= W_{N_2} F(n, t), V_{N_2}(t_{sc}) = 0 \end{aligned} \right\}$$

Where:

- $W_{GAM} = W_{CO_2}(0)/(K_d C_{O_2}^{inh})$  = The flow of released GAM on inhalation
- $W_{CO_2} = C_{CO_2}^{inh} W_{GAM}$  = The flow of additional supply of  $CO_2$  on inhalation from the buffer tank
- $W_{N_2} = (1 - C_{CO_2}^{inh} - C_{CO_2}^{inh}) W_{GAM}$  = The supply flow of  $N_2$  on inhalation from the buffer tank
- $F(n, t)$  = The function, determining the type of breathing pneumotachogram
- $t_{sc}$  = The time of the start of another inhalation-exhalation cycle

Integrating the system Eq. 1 in the interval  $(t_{sc}, t_{sc} + t_{it})$  where  $t_{it}$  is the inhalation time, we obtain in the function of time the preset values of volumes of released and fed gases:  $V_{GAM}^{set}(t), V_{CO_2}^{set}(t), V_{N_2}^{set}(t), t \in [t_{sc}, t_{sc} + t_{it}]$ .

These preset values of volumes of gases and the current state vector  $(X_1, X_2, X_3)^T$  are used by the subsystem to calculate the control signals  $u_1, u_2, u_3$  for direct control of actuators simulating oxygen consumption (GAM release) and supply of carbon dioxide and nitrogen, respectively.

**A control algorithm with a predictive model:** For the construction of control algorithms for actuators that minimize reproduction errors in given volumes of gases, we used the method of analytical design of optimal controllers by the A.A. Krasovskii's criterion of generalized operation (Krasovskii, 1970, 1973, 1987; Krasovskii *et al.*, 1977; Bukov and Krasovskii, 1975; Bukov, 1987; Kabanov, 2005, 1997; Anisimov and Kabanov, 2005; Sizykh, 2005).

On the basis of this method we developed optimal control algorithms for nonlinear dynamic objects that can operate in real time. We used a predictive algorithm on a sliding range of optimization with calculation of sensitivity matrix (Bukov, 1987). Suppose the mathematical model of the control object is described by the differential (Eq. 2):

$$\dot{x} = f(x, y, t), \dot{y} = u, x(0) = x_0, y(0) = y_0 \quad (2)$$

Where:

- $x$  = The state vector of the control object
- $f$  = The differentiable in all arguments nonlinear vector function
- $y$  = The position vector of steering bodies
- $u$  = The rate of change  $y$  (control)

In model Eq. 2 control is performed by changing the rate of influences  $y$ , a  $x = x(y, t)$  is a complex function.

It is necessary to find the control that minimizes the functional of the generalized operation on a sliding range  $(t_u, t_u+T)$ :

$$I = \int_{t_u}^{t_u+T} Q(x, y, \tau) d\tau + \frac{1}{2} \int_{t_u}^{t_u+T} (u^T K^{-1} u + u_{opt}^T K^{-1} u_{opt}) d\tau \quad (3)$$

Where:

$Q$  = The scalar function differentiable by its arguments and characterizing the quality of the control process on the interval  $(t_u, t_u+T)$

$K$  = The diagonal matrix of coefficients of control effectiveness

$u_{opt}$  = The optimal rate of change  $y$  (optimal control)

$t_u$  = The moment determining control influences

$T$  = The prediction interval

According to Kyriazi (1986) and Deno (1984), control  $u_{opt}$  is calculated as follows:

$$u_{opt}(t_u) = -K \frac{\partial S(t_u)}{\partial y(t_u)} \quad (4)$$

where  $S(t)$  is the solution of the Lyapunov Eq. 5:

$$\frac{\partial S}{\partial t} + \frac{\partial S}{\partial x} f(x_f, t) = -Q(x_f, t), \quad S(t_u + T) = 0 \quad (5)$$

at free motion of the object Eq. 2 on the interval  $(t_u, t_u+T)$ :

$$\dot{x}_f = f(x_f, y, t), \quad \dot{y} = 0 \quad (6)$$

On the free movement Eq. 6 the left side Eq. 5 is transformed into a full derivative by t:

$$\dot{S} = -Q(x_f, t), \quad S(t+T) = 0 \quad (7)$$

Differentiating Eq. 7 with respect to  $y$  according to the rule of differentiating a complex function, we obtain (Beeckman *et al.*, 1998):

$$\frac{d}{dt} \left( \frac{\partial S(t_u)}{\partial y(t_u)} \right) = - \frac{\partial Q(t)}{\partial x_f} \cdot \frac{\partial x_f(t)}{\partial y(t_u)} - \frac{\partial Q(t)}{\partial y(t_u)} \quad (8)$$

where,  $\frac{\partial x_f(t)}{\partial y(t_u)} = Z(t)$  is a sensitivity matrix of the solution of the Eq. 6 for the parameter vector  $y$ , satisfying the sensitivity (Eq. 9):

$$\dot{Z}(t) = \frac{\partial f(x_f, y, t)}{\partial x_f(t)} Z(t) + \frac{\partial f(x_f, y, t)}{\partial y(t_u)} \quad (9)$$

with the initial condition:

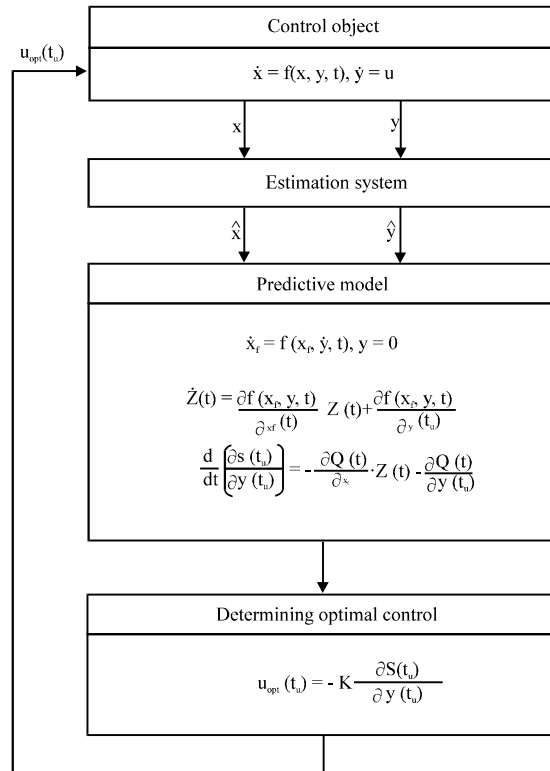


Fig. 2: A block diagram of the control algorithm

$$Z(t_u) = 0 \quad (10)$$

where Jacobi matrices  $\frac{\partial f(x_f, y, t)}{\partial x_f(t)}$ ,  $\frac{\partial f(x_f, y, t)}{\partial y(t_u)}$  are calculated for  $x_f(t)$ . A block diagram of the control algorithm is shown in Fig. 2.

Functioning of the algorithm is as follows. At each correction cycle, the information on the state variables of the control object (the signals from the meters) is entered into the estimationsystem which estimates the vector of the system status (noise filtering and coordinate restoration of vector which cannot be measured). Estimates of the state vector  $\hat{x}, \hat{y}$  are entered into the predictive model.

Control signals for the object Eq. 2 calculated by Eq. 4 at time points  $t_u$  at intervals  $\Delta t_c$  where  $\Delta t_c$  is a correction control cycle. Optimal equation  $u_{opt}(t_u)$  is calculated at each cycle by integration on the interval  $(t_u, t_u+T)$  predictive model Eq. 6, 8 and 9.

**Statement of the problem and the implementation of the algorithm of optimal control of actuators:** Consider statement of the optimal control problem and the implementation of the algorithm for calculating the subsystem of control signals  $u_1-u_3$ . We assume that the estimation system (it is not considered in this example)

provides an accurate estimate of the state vector  $(X_1, X_2, X_3)^T$ , noise of sensors (Fig. 1) are absent. We form the the equation, realizing the proposed algorithm. Suppose the mathematical model of control object in accordance with Eq. 2 has the form:

$$\left. \begin{aligned} \frac{dV_i}{dt} &= S_{pi} \vartheta_i, V_i(t_u) = 0, 0 \leq V_i \leq V_i^{max} \\ \frac{d\vartheta_i}{dt} &= \frac{1}{T_{di}} (-\vartheta_i + k_{di} \mu_{icon}), \vartheta_i(t_u) = 0, |\vartheta_i| \leq \vartheta_i^{max} \\ \frac{d\mu_{icon}}{dt} &= u_i, i = 1, 2, 3 \end{aligned} \right\} \quad (11)$$

Where:

- $V_1 = V_{GAM}(t)$  = The GAM volume removed from the system in the inhalation phase
- $v_2 = v_{CO_2}(t), v_3 = v_{N_2}(t)$  = The volumes of carbon dioxide and nitrogen fed to the system at the inhalation phase
- $\vartheta_1, \vartheta_2, \vartheta_3$  = The linear velocities of drive rods, respectively the units simulating oxygen consumption and supply of carbon dioxide and nitrogen
- $\mu_{1con}, \mu_{2con}, \mu_{3con}$  = Control signals in the respective channels
- $u_1, u_2, u_3$  = Rates of change of control signals (control)
- $S_{p1}, S_{p2}, S_{p3}$  = Areas of pistons of reciprocating feeders of GAM, CO<sub>22</sub> and N<sub>2</sub>, respectively
- $T_{di}, k_{id} (i = 1, 2, 3)$  = Parameters of drive models

The quality control criteria to be minimized for the object Eq. 11 will take the form:

$$I = \frac{1}{2} \int_{t_u}^{t_u+T} \sum_{i=1}^3 [\alpha_i (V_i - V_i^{set})^2 + \beta_i (\dot{V}_i)^2 + Q_i^f] dt + \frac{1}{2} \int_{t_u}^{t_u+T} \sum_{i=1}^3 (k_i^{-1} u_i^2 + k_i^{-1} u_{iopt}^2) dt \quad (12)$$

Where:

- $\alpha_i, \beta_i$  = Weighting factors (positive) which can be varied to achieve the predetermined quality of the transition process
- $Q_i^f = Q_{ii}^f + Q_{ii}^f$  = A penalty function

$$Q_{ii}^f = \begin{cases} 0 & \text{if } V_i \leq V_i^{max} \\ \gamma_i (V_i - V_i^{max})^2 & \text{if } V_i > V_i^{max} \end{cases}$$

$$Q_{2i}^f = \begin{cases} 0 & \text{if } |\vartheta_i| \leq \vartheta_i^{max} \\ \eta_i (\vartheta_i - \vartheta_i^{max})^2 & \text{if } |\vartheta_i| > \vartheta_i^{max} \end{cases}$$

$\gamma_i, \eta_i$  are parameters (positive coefficients) of the penalty function defining “rigor” of the predetermined boundary. In accordance with the notation of 2.2 we have:  $x = (X1, X2, X3)^T = [V_{GAM}, \vartheta_1, v_{CO_2}, \vartheta_2, v_{N_2}, \vartheta_3]^T$  is the state vector of control object.  $y = (\mu_{1con}, \mu_{2con}, \mu_{3con})^T$  is the position vector of steering bodies.  $u = (u_1, u_2, u_3)^T$  control vector (rate of change of y):

$$Q(x, y, t) = \sum_{i=1}^3 [\alpha_i (V_i - V_i^{set})^2 + \beta_i (\dot{V}_i)^2 + Q_i^f]$$

Function characterizing the desired quality of the control process on the interval  $(t_u, t_u+T)$ ;  $K = \text{diag}[k_i]$  matrix of coefficients of control efficiency.

Then, following the procedure of formation of the optimal control algorithm, we obtain the implementation of this algorithm by actuators. Optimal controls of the type (Eq. 4) are determined by the expression:

$$u_{iopt}(t_u) = -k_i \frac{\partial S(t_u)}{\partial \mu_{icon}(t_u)} \quad i = 1, 2, 3 \quad (13)$$

The predictive model consists of equations of the form of free movement of the object (Eq. 6):

$$\left. \begin{aligned} \frac{dV_i}{dt} &= S_{pi} \vartheta_i; \\ \frac{d\vartheta_i}{dt} &= \frac{1}{T_{di}} (-\vartheta_i + k_{di} \mu_{icon}); \\ \frac{d\mu_{icon}}{dt} &= 0, \quad i = 1, 2, 3 \end{aligned} \right\} \quad (14)$$

of the equations of sensitivity of the type (Eq. 9):

$$\left. \begin{aligned} \frac{dZ_{i1}}{dt} &= S_{pi} Z_{i2}; \\ \frac{dZ_{i2}}{dt} &= -\frac{1}{T_{di}} Z_{i2} + \frac{k_{di}}{T_{di}}, \quad i = 1, 2, 3 \end{aligned} \right\} \quad (15)$$

where,  $Z_{i1} = \partial V_i / \partial \mu_{icon}$ ,  $Z_{i2} = \partial \vartheta_i / \partial \mu_{icon}$  are sensitivity functions of the equations for the partial derivative of the Lyapunov function of the form (Eq. 8):

$$\left\{ \begin{aligned} \frac{d}{dt} \left( \frac{\partial S}{\partial \mu_{icon}} \right) &= -[\alpha_i (V_i - V_i^{set}) + \gamma_i (V_i - V_i^{max})] Z_{i1} - \\ &[\beta_i \dot{V}_i S_{pi} + \eta_i (\vartheta_i - \vartheta_i^{max})] Z_{i2}, \quad \gamma_i = 0 \quad \text{if } V_i \leq V_i^{max} \\ \eta_i &= 0 \quad \text{if } |\vartheta_i| \leq \vartheta_i^{max}, \quad i = 1, 2, 3 \end{aligned} \right. \quad (16)$$

Equation 15 and 16 are integrated on the interval  $(t_u, t_u+T)$  under zero initial conditions and the system (Eq. 14) is integrated with the initial conditions corresponding to the current state of the object (Eq. 11) at the end of the previous cycle  $\Delta t_c$  of control correction.

### CONCLUSION

The structure of the predictive control system for the "Artificial Lungs" facility which consists of setting and control subsystems. We proposed an algorithm for the operation of the setting subsystem. We formalized the task of optimal control for AL facility and proposed the implementation of an optimal control algorithm for the subsystem of control signals by actuators simulating oxygen consumption, supply of carbon dioxide and nitrogen, respectively.

Functioning of the control system is as follows. At each inhalation cycle in the interval  $[t_{sc}, t_{sc}+t_{il}]$  the system of Eq. 1 is integrated and functions  $V_{GAM}^{set}(t)$ ,  $V_{CO_2}^{set}(t)$ ,  $V_{N_2}^{set}(t)$  are calculated. These functions are entered into the subsystem for calculation of control signals. Then we used the algorithm of optimal control. Next comes the algorithm of optimal control. At each cycle of correction control in the interval  $[t_u, t_u+T]$  ( $t_u$  is current time of controls calculation,  $T$  is prediction interval) at intervals  $\Delta t_c$  equations of predictive model (Eq. 14-16) are integrated and optimal controls by actuators using Eq. 13 are calculated. By appropriate selection of controls correction cycle  $\Delta t_c$ , prediction interval  $T$ , weighting factors  $\alpha_i$ ,  $\beta_i$ , parameters,  $\gamma_i$ ,  $\eta_i$ , one can ensure the desired quality of transient processes in the system.

Thus, the proposed algorithm can be used to control the gas volume in the AL facility with a given accuracy and minimal energy control costs.

### ACKNOWLEDGEMENT

This study was supported by the Russian Science Foundation (Agreement No. 15-19-10028).

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