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Exact Solution of Some Complex Partial Differential Equation

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Abstract: In this study some traveling wave solutions of the Zakharov equation and coupled Higgs field equation are obtained by using reduction method. This method is a direct algebraic method for obtaining exact solutions of complex nonlinear partial differential equations.

Key words: Partial differential equation, Zakharov equation, coupled Higgs field equation, reduction, method, equations

INTRODUCTION

It is well known the nonlinear complex physics phenomena are related to nonlinear partial differential equations which are involved in many fields from Physics to Biology, Chemistry, Mechanics, etc.

Nonlinear evolution equations are widely used to describe nonlinear phenomena in Physics fields like the Fluid Mechanics, Plasma Physics and Optics. In recent years various techniques have been developed to obtain exact solutions of complex nonlinear partial equations such as Zakharov equation and Higgs field equation such that Hirota's bilinear method (Hu *et al*., 2003), modification of truncated expansion method (Mirzazadeh and Khaleghizadeh, 2013), the first integral method (Unal, 2012; Taghizadeh and Mirzazadeh, 2011), the infinite series method (Taghizadeh *et al*., 2010), the tanh method (Abdelkawy *et al*., 2016), the (G′/G)-expansion method (Kumar *et al*., 2012), the Jacobi elliptic function method (Xu, 2014), the ansatz method and finite difference method (Ahmed, 2012).

The aim of this study is to find exact solutions of Zakharov equation and coupled Higgs field equation by the reduction method. The reduction method is powerful solution method for the computation of exact traveling wave solutions.

The traveling wave solution: Consider the nonlinear partial differential equation in the form:

$$H(u, u_x, u_t, u_{xx}, u_{xt}, ...,) = 0 \tag{1}$$

We use the wave variables $\varepsilon = x-2ct$ carries Eq. 1 into the following Ordinary Differential Equation (ODE):

$$G(u, u', u'', u''') = 0 \tag{2}$$

where prime denotes the derivative with respect to the same variable ε. Integrating Eq. 2 once and considering the constants of integration to be zero. Next, by applying reduction of order method (Rainville and Bedient, 1974) by introducing a new independent variable $p = u'$ and $u'' = p\, dp/du$ which changes Eq. 2 to an (ODE) of the form:

$$R\left(u, p, p\frac{dp}{du}\right) = 0 \tag{3}$$

Now, by seeking a solution for Eq. 3 by a desired method that been considered by Zwillinger (1998). Next section devoted to find at an analytical solution for Zakharov equation and Higgs field equation by this method.

Zakharov equations: The Zakharov equation is the coupled nonlinear partial differential equations as follow:

$$iu_t + u_{xx} = uv \tag{4}$$

$$v_{tt} - v_{xx} = (|u|^2)_{xx} \tag{5}$$

Where:

u = The slow variation amplitude of the electric field intensity
v = The perturbed number density of the media or ions in media

The Zakharov equations have various applications in Physics such as theory of deep-water waves, nonlinear

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pulse propagation in optical fibers and interaction of laser plasma (Melrose, 1987). To solving the Zakharov equations. Applying the transformations:

$$u(x, t) = e^{i\theta}f(\varepsilon);\ v(x, t) = g(\varepsilon);\ \theta = cx+t;\ \varepsilon = x-2ct \qquad (6)$$

To the Eq. 4 and 5, we obtain the system of ordinary differential equations:

$$f''(\varepsilon)-(c^2+1)f(\varepsilon) = f(\varepsilon)g(\varepsilon) \qquad (7)$$

$$(4c^2-1)g''(\varepsilon) = \frac{\partial^2 f^2(\varepsilon)}{\partial\varepsilon^2} \qquad (8)$$

Integrating Eq. 8, twice with respect to ε, we have:

$$g(\varepsilon) = \frac{f^2(\varepsilon)}{(4c^2-1)},\ 4c^2 \neq 1 \qquad (9)$$

The first and second integration constant is taken to be zero. Inserting Eq. 9 into Eq. 7, we have:

$$f''(\varepsilon)-(c^2+1)f(\varepsilon)-\frac{1}{4c^2-1}f^3(\varepsilon) = 0 \qquad (10)$$

where, $f''(\varepsilon) = d^2f/d\varepsilon^2$. This is a nonlinear second order ordinary differential equation of ε. The substitution $f'(\varepsilon) = p$ and $f''(\varepsilon) = p\ dp/df$ reduces Eq. 10 to:

$$p\frac{dp}{df}-(c^2+1)f-\frac{1}{4c^2-1}f^3 = 0 \qquad (11)$$

which can be written as:

$$pdp = \left[(c^2+1)f+\frac{1}{4c^2-1}f^3\right]df \qquad (12)$$

The variables are separated then one integration leads to:

$$p^2 = (c^2+1)f^2+\frac{1}{2(4c^2-1)}f^4+c^1 \qquad (13)$$

Let $c_1 = 0$ then:

$$p=f\sqrt{(c^2+1)+\frac{1}{2(4c^2-1)}f^2} \qquad (14)$$

Since, $p = f'(\varepsilon) = df/d\varepsilon$, then:

$$\frac{df}{f\sqrt{(c^2+1)+\frac{1}{2(4c^2-1)}f^2}} = d\varepsilon \qquad (15)$$

by integrating both sides of Eq. 15:

$$\operatorname{csch}^{-1}\left|\frac{f}{\sqrt{(4c^2+1)(2c^2+2)}}\right| = \frac{-\sqrt{(4c^2+1)(2c^2+2)}}{\sqrt{2(4c^2-1)}}\varepsilon \qquad (16)$$

by taking the constant of integration equal zero. The exact solutions of Zakharov equations can be written as:

$$u(x, t) = e^{i\theta}\sqrt{(4c^2+1)(2c^2+2)}\operatorname{csch}\left[\frac{-\sqrt{(4c^2+1)(2c^2+2)}}{\sqrt{2(4c^2-1)}}\varepsilon\right] \qquad (17)$$

$$v(x, t)=\frac{1}{(4c^2-1)}(4c^2+1)(2c^2+2)\operatorname{csch}^2\left[\frac{-\sqrt{(4c^2+1)(2c^2+2)}}{\sqrt{2(4c^2-1)}}\varepsilon\right] \qquad (18)$$

where, θ = cx+t, ε = x-2ct.

Coupled Higgs field equation: The Higgs field equation (Tang and Xia, 2011; Tajiri, 1983):

$$u_{tt}-u_{xx}-\alpha u+\beta|u|^2u-2uv = 0 \qquad (19)$$

$$v_{tt}+v_{xx}-\beta\left(|u|^2\right)_{xx} = 0 \qquad (20)$$

Describes a system of conserved scalar nucleons interacting with a neutral scalar meson. Here, real constant v represents a complex scalar nucleon field and u a real scalar meson field. To find exact solutions of coupled Higgs field Eq. 19 and 20, first we make the transformation:

$$u(x, t) = e^{i\theta}f(\varepsilon);\ v(x, t) = g(\varepsilon) \qquad (21)$$

where, θ = kx+ωt; ε = x+ct, we have a relation k = ωc and reduce Eq. 19 and 20 to the following system of ordinary differential equations:

$$\left[\omega^2(c^2-1)-\alpha\right]f(\varepsilon)+(c^2-1)f''(\varepsilon)+\beta f^3(\varepsilon)-2f(\varepsilon)g(\varepsilon) = 0 \qquad (22)$$

$$(c^2+1)g''(\varepsilon)-\beta(f^2(\varepsilon))'' = 0 \qquad (23)$$

Integrating Eq. 23 twice with respect to ε and taking the first and second integration constant to be zero, we have:

$$g(\varepsilon) = \frac{\beta f^2(\varepsilon)}{(c^2+1)} \quad (24)$$

Inserting Eq. 24 into Eq. 22 yields:

$$(c^2-1)f''(\varepsilon)+\left(\beta-\frac{2\beta}{c^2+1}\right)f^3(\varepsilon)+\left[\omega^2(c^2-1)-\alpha\right]f(\varepsilon) = 0 \quad (25)$$

Rewrite this second-order ordinary differential equation as follows:

$$f''(\varepsilon)+\frac{\beta}{c^2+1}f^3(\varepsilon)+\frac{\left[\omega^2(c^2-1)1-\alpha\right]}{c^2-1}f(\varepsilon) = 0 \quad (26)$$

where, $f''(\varepsilon) = d^2f/d\varepsilon^2$. This is a nonlinear second order ordinary differential equation of ε. The substitution $f'(\varepsilon) = p$ and $f''(\varepsilon) = p\, dp/df$ reduces Eq. 26 to:

$$p\frac{dp}{df}+\frac{\beta}{c^2+1}f^3(\varepsilon)+\left[\frac{\omega^2(c^2-1)-\alpha}{(c^2-1)}\right]f(\varepsilon) = 0 \quad (27)$$

Equation 27 can be written as:

$$pdp = \left[\frac{\omega^2(c^2-1)-\alpha}{(1-c^2)}f(\varepsilon)-\frac{\beta}{(c^2+1)}f^3(\varepsilon)\right]df \quad (28)$$

The variables are separated, then one integration leads to:

$$p^2 = \left[\frac{\omega^2(c^2-1)-\alpha}{(1-c^2)}\right]f^2(\varepsilon)-\frac{\beta}{2(c^2+1)}f^4(\varepsilon) \quad (29)$$

where the integration constant is to be zero then:

$$p=f\sqrt{\frac{\omega^2(c^2-1)-\alpha}{1-c^2}-\frac{\beta}{2c^2+2}f^2} \quad (30)$$

Since, $p = f'(\varepsilon) = df/d\varepsilon$ then by integrating both sides of Eq. 30:

$$\operatorname{sech}^{-1}\left(\frac{\sqrt{\beta}f(\varepsilon)}{\sqrt{2\omega^2(1-c^2+\alpha)}}\right) = -\frac{\sqrt{\beta}}{\sqrt{2\omega^2(1-c^2)+\alpha}}\varepsilon \quad (31)$$

by taking the constant of integration to be zero. The exact solutions of the Higgs field equation can be written as:

$$u(x, t) = e^{i\theta}\left(\frac{\sqrt{2\omega^2(1-c^2)+\alpha}}{\sqrt{\beta}}\right)\operatorname{sech}\left(-\frac{\sqrt{\beta}}{\sqrt{2\omega^2(1-c^2)+\alpha}}\varepsilon\right) \quad (32)$$

$$v(x, t) = \left[\frac{2\omega^2(1-c^2)+\alpha}{\beta}\right]\operatorname{sech}^2\left(-\frac{\sqrt{\beta}}{\sqrt{2\omega^2(1-c^2)+\alpha}}\varepsilon\right) \quad (33)$$

CONCLUSION

In this study, the reduction method has been used to construct exact traveling wave solutions of complex nonlinear partial differential equations, the Zakharov equation and the coupled Higgs field equation. The performance of this method is found to be reliable and effective and it gives more solutions. The method has the advantages of being direct and concise.

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