Effect of Deep Vibration on Water-Saturated Soil Consolidation

Vladimir Geidt, Larisa Geidt, Svetlana Sheshukova and Andrey Geidt,
Tyumen Industrial University, Volodarskogo Str. 38, 625000 Tyumen, Russia

Abstract: The study considers deep vibration (longitudinal mechanical waves) having an effect on water-saturated soil consolidation. The mechanism is based on the changed physical state of the medium and takes into account its structure. Two soil models have been proposed. The first one-parameter soil model is called a “sandwich (layer-cake)”. The second two-parameter soil model is called a “loose sandwich”. In here, the first model is a special case of the second one. Both parameters have simple physical significance. According to the proposed mechanism, the effect of vibration on soil consolidation is reduced to redistribution of the relationship between the stress in soil skeleton and the pressure of pore water versus the value of “vibration pressure” in such a way that the pore water pressure increases but, nevertheless, cannot exceed the value of the external static load.

Key words: Water-saturated soil, consolidation, pore pressure, deep vibration, harmonic oscillations, stresses

INTRODUCTION

At present, the research devoted to dynamic effects on soil consolidation can be considered to have been formed (Konovalov, 1995; Ivanov, 1991). However, one can state that the effect of vibration on soil consolidation has not been understood yet (Ufukov et al., 2000a, b). From general considerations it is clear that the effect of deep vibration on soil consolidation is due to the following (Ufukov et al., 2000a, b):

- Additional variable pressure
- Changed filtration capacity of the medium
- Changed physical state of the medium

It was actually proved by Ufukov et al. (2000a, b) that the excess pressure in pore water represented both by the harmonic law (Ufukov et al., 2000a, b) and as a plane wave leaves practically no “residual” effects in the medium after switching off the influence. It was shown in that the filtration coefficient does not depend on vibration. Therefore, the second of the above mechanisms is also eliminated. Thus, it is needed to consider the third mechanism of deep vibration having an effect on the consolidation process. This is the subject matter of the study.

Let us suppose that the totally water-saturated soil is subjected to an external static load q directed vertically downwards (gas phase is absent). At a moment of t = 0, a deep vibrator which is a vertically mounted cylinder (of radius r0), functions in this medium. The cylinder “wall” creates harmonically varying perturbations with the frequency ω. It is clear that these perturbations propagate in the medium in the form of mechanical waves.

It is known that mechanical waves can be surface (in terms of the nature of periodic motion of particles in the medium), transverse and longitudinal (Koshkin and Shishkevich, 1980). Surface waves are not discussed in the study, since, the authors are confined to the deep vibrator and the upper soil layer is loaded with sufficient external load. Surface waves appear on the boundary of a liquid and a gas. In regard to transverse waves, they propagate only in solids (Koshkin and Shishkevich, 1980). Transverse waves do not propagate in liquids and bulk materials.

MATERIALS AND METHODS

Let us consider interaction of longitudinal waves (propagating horizontally) with the medium. Taking into account the vibrator geometry which is the source of longitudinal waves in the medium, cylindrical longitudinal waves are considered. For such waves, the dependency of particle’s displacement from the rest position u = u (r, t) is evaluated by the equation (Koshkin and Shishkevich, 1980):

\[ u = \frac{a}{\sqrt{r}} \sin(\alpha r - \omega t) e^{\alpha r} \]  \hspace{1cm} (1)

Where:
\[ a = \text{Value connected to the amplitude of oscillation} \]
\[ \omega_0 = \omega \sqrt{r} \]
\[ \omega = \text{Phase frequency of a wave} (\omega = 2\pi v) \]
\[ v = \text{Oscillation frequency} \]
\[ k = \text{Wave number} (k = \omega / v) \]
\[ V = \text{Wave propagation velocity} \]
\[ r = \text{Distance from the axis of symmetry to the point of observation} \]
$\tau = \text{Vibration time}$

$\alpha = \text{Wave attenuation factor due to absorption}$

In here, the excess pressure created in water by such a wave is evaluated by the equation (Koshkin and Shishkevich, 1980):

$$\tilde{p} = \rho V_0 u = \rho V_0 u_0 \cos(\omega t - kr) e^{-\alpha r}$$  \hspace{1cm} (2)

Where:

$\rho = \text{Water density}$

$\alpha = \text{Wave attenuation factor}$

Intensity of the wave $I(r)$ (the average energy per unit of transverse area) with regard to attenuation is:

$$I(r) = \frac{1}{2} \rho V_0 \frac{\alpha^2 a_0^2}{r} e^{-2\alpha r}$$  \hspace{1cm} (3)

Now let us consider how this wave affects the state of water in the medium. To do this, let us begin with an elementary act of interaction. Let the stress $\sigma$ be established in the soil skeleton and every elementary soil particle be surrounded by a water layer. Thus, if one selects any 2 elementary particles (through which interaction is transferred), then there is a layer of water-b in thickness-between these particles. This pair of particles participates in transferring the stress in the skeleton $\sigma$. The layer of bound water between the particles behaves as a continuation of the skeleton. The forces created by the wave affect this element horizontally and frictional forces counteract.

Let us introduce the tangential stress $\tau$ produced by the wave in the water layer. It is clear that if $\tau$ does not exceed the tangential stress created by the frictional forces $\tau_{fr}$, then the water layer will form a single whole with the soil particles. However, if $\tau$ exceeds $\tau_{fr}$, this layer of water will move in relation to the skeleton and interaction between these two particles will stop and the water in this layer can be considered as free. These two particles will hang in the water and the pressure $\sigma$ acting on the upper particle will not be transmitted to the lower one; it will be transferred to the water. Thus, the upper particle taking the vertical pressure $\sigma$ will interact with the water, i.e., the pore water pressure will increase by $\sigma$. Taking into account the negligibly small compressibility of water, destruction of water structure between the particles occurs without changing the porosity coefficient of the medium $e$. Now let us estimate the quantitative parameters of $\tau$ and $\tau_{fr}$. It is well known that the maximum value of $\tau_{fr}$ is evaluated by the Coulomb formula in a macro volume (Ivanov, 1991; Abelev, 1983; Zareckij, 1970):

$$\tau_{fr} = \tau_{Coul} = (\sigma + \sigma_t) \tan \varphi = \sigma \tan \varphi + C$$  \hspace{1cm} (4)

Where:

$\sigma = \text{Vertical load}$

$\sigma_t = \text{Structural strength}$

$C = \text{Consolidation}$

$\varphi = \text{Angle of internal friction}$

Taking into account the principle of superposition, when the macro properties are determined by particles’ behavior at the micro level, one can assume that $\tau_{fr}$ equals to $2\tau_{Coul}$. The multiplier is “2”, since, both the upper particle and the lower one participate in friction.

Now let us estimate the shear stress $\tau = F/S$ created by a plane wave located between the particles in the given water layer (Fig. 1). Thus, in the case of a plane wave:

![Fig. 1: Effect of a mechanical plane wave on water layer enclosed between the isolated particles (“shells”) of soil. Here, $\sigma$-stresses in the soil skeleton; $b$-distance between the soil particles; $p$ -excess wave pressure at a given point at a moment of time $t$; $S$-area of the soil element](image-url)
Fig. 2: Effect of a cylindrical wave on water layer enclosed between soil particles. The source of cylindrical waves is shown in the center. Here, $\phi$ the sector angle; $r$-distance from the wave source to the layer under consideration; $\Delta r$-width of the element

$$\tau = \frac{F}{S} = \frac{\tilde{F} \Delta z \Delta x}{\Delta r \Delta x} = \frac{\tilde{F} \Delta z}{\Delta r}$$  \hspace{1cm} (5)

Where:

$\tilde{F} = \frac{\partial}{\partial z}$

$\Delta z, \Delta x$ = Horizontal dimensions of the isolated soil

$b$ = Average distance between the particles

The $\sim$ sign above the letter $F$ means the excess wave pressure in contrast to the excess pore water Pressure $P$. In the case of a cylindrical wave (Fig. 2):

$$\tau = \frac{F}{S} = \frac{b \frac{\partial}{\partial r} \left[ r P(r,t) \right]}{\frac{\partial}{\partial r} \left[ r P(r,t) \right]}$$  \hspace{1cm} (6)

Thus, the following is the restriction of total soil liquefaction:

$$\tau \geq \tau_{res} = \sigma \phi_{qf}$$  \hspace{1cm} (7)

where, $\tau$-tangential stress created by the wave in the water layer, it is evaluated by the Eq. 5 and 6. In this case (complete liquefaction) the pore pressure $P$ reaches its maximum value of $q$.

RESULTS AND DISCUSSION

Now let us consider the case when $\tau < \tau_{res} = \sigma \phi_{qf}$. The transfer of stress $\sigma$ from the overlying soil layers to the underlying ones is formed in such a way that some particles are strongly consolidated and some are weakly consolidated. When such a medium is affected by longitudinal waves, there will always exist some particles for which the effect of the wave will be sufficient to destroy the structural bonds. This means that the structural bonds will not be violated in the whole of particles but only in some of them in here, $\sigma$ decreases by $\Delta \sigma$ equal to:

$$\Delta \sigma = \frac{\tau}{2\phi_{qf}}$$  \hspace{1cm} (8)

In other words, the element $\Delta \sigma \phi_{qf}$ is assumed to be equal to $\tau$ in the inequality $\tau < \tau_{res} = \sigma \phi_{qf} = (+\Delta \sigma \phi_{qf})$. The stress $\sigma$ on the skeleton decreases by $\Delta \sigma$ and becomes equal to $\delta$ under the influence of the wave. Thus, the stress $\sigma$ with no wave is related to the residual stress $\delta$ with the wave, i.e., $\sigma = \delta - \Delta \sigma$.

Taking into account that the stresses in the skeleton both with and without the wave ($\delta$ and $\sigma$) are related to the pore water pressure, i.e., $q = \sigma + P$, one can obtain that $\tau < \tau_{res}$ for the case under consideration. Pressure in the pore water does not reach the maximum value of $q$ in the presence of the wave $\tilde{P}$. Such a state of the medium is called partial liquefaction, i.e., when the pressure in the pore water increases by $\Delta(\tilde{P} = \phi_{qf} - \phi) q$ but does not reach its maximum possible value of $q$.

Thus, under partial liquefaction ($\tau < \tau_{res}$), the effect of the wave on the medium means that the pressure in the pore water $P$ increases by $\Delta \sigma = \tau/2\phi_{qf}$ as compared to $\tilde{P}$ where, $\tau$ is estimated as (Eq. 5) for the plane wave and (Eq. 6) for the cylindrical wave.

The evaluation formulas are obtained for $\tau$ with regard to the cylindrical wave when the pressure in the pore water changes by the following law:

$$P(r,t) = \rho \omega v a \sqrt{\frac{\pi}{r}} \cos(\omega t - kz)e^{\gamma t}$$

Then:

$$\tau = \frac{\rho \omega v a \sqrt{\pi}}{r} \cos(\omega t - kz)e^{\gamma t} \left[ \frac{1}{2} \Phi(\phi) \right]$$

where, $\Phi(\phi)$ value which depends on the wave phase $\phi = \omega t - kr$. When the oscillation period $T = 1/\omega = 2\pi/\omega$ is much shorter than the relaxation time, $\Phi(\phi)$ can be replaced by the maximum value of $\Phi_{max} = \max \Phi(\phi)$ ($0 \leq \phi \leq 2\pi$), since, if the frictional forces between the particles are overcome in the water layer at some value of $\phi$. 

1723
the phase (at some moment of time), then the structural bonds will not be capable to recover during oscillations in this water layer. Thus:

$$\Phi_m = \frac{1}{2} \sqrt{1 - 2\alpha^2 + (2k\alpha)^2} = \frac{1}{2} \sqrt{1 - 4\alpha^2 + 4\alpha^2(\alpha^2 + k^2)}$$ (10)

Then the maximum value $\tau_m(\tau) = \max_{\tau \in \mathbb{R}} \tau(r, t)$ is as follows:

$$\tau_m = \frac{b \rho V}{\pi} \left( \frac{a}{r} \right) \frac{1}{\sqrt{r}} 2 \sqrt{1 - 4\alpha^2 + 4\alpha^2(\alpha^2 + k^2)}$$ (11)

$\alpha$-attenuation factor. Connection between the interporous gap $b$ and the porosity factor must be revealed. As this connection depends upon soil structure, the value of $b$ will depend on the accepted ideas about soil structure. If one takes the "sandwich" soil model ("layer cake") composed of the dense shells, $\delta$-in thickness and $b$-in distance, the following is obtained:

$$b = e \delta$$ (12)

If one takes the "loose sandwich" soil model with the porosity factor $e_{\text{shl}}$, then:

$$b = \delta \frac{e - e_{\text{shl}}}{1 + e_{\text{shl}}}$$ (13)

It can be seen that if one takes $e_{\text{shl}} = 0$ in the Eq. 13, then Eq. 13 passes into Eq. 12. Thus, the "shell" soil model Eq. 12 is a special case of the "loose shell sandwich" model Eq. 13. Let us consider the model Eq. 13, although, it is not enough in order to cover various soil types.

CONCLUSION

Proposed is the model of interaction of longitudinal mechanical waves with water-saturated soil. The study has revealed that: when the condition Eq. 7 is fulfilled, a liquefaction state occurs (when the stress in the skeleton decreases to zero, i.e., $v = 0$ and hence, pressure in the pore water increases up to the maximum possible value $= q$). If $v_m < v_{\text{res}}$, the stress in the skeleton decreases up to the residual stress $= \sigma - \Delta \sigma$ but not to zero; $\Delta \sigma$ can be evaluated by the Eq. 8 taking into account $\tau - v_m$ where, $v_m$ is evaluated by the Eq. 6 for cylindrical waves and the parameter $b$ is evaluated by the Eq. 13 for the loose shell soil model. In the given case when $v_m < v_{\text{res}}$ (partial liquefaction), pressure in the pore water increases by $\Delta \sigma$ and does not reach the limit value of $q$.

REFERENCES

Abelev, M.I., 1983. [Construction of Industrial and Residential Buildings on Weak Water-Saturated Soils]. Stroyizdat Publisher, Moscow, Russia, Pages: 248 (In Russian).


