

Stagnation-Point Flow of Casson Fluid in Darcy-Forchheimer Porous Media with Newtonian Heating Effect

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Abstract: Flow of Casson fluid in Darcy-Forchheimer porous medium near a stagnation point over a stretching sheet with Newtonian heating effect is investigated in this study. The boundary-layer equations are solved numerically for $f'(0)$ and $\theta(0)$ using the Keller-box method for some values of Casson fluid parameter (β), porous media parameter (κ_1), inertia-coefficient parameter (κ_2), Newtonian heating parameter (γ), Prandtl number (Pr) and stretching parameter (c).

Key words: Newtonian heating, stretching sheet, Darcy-Forchheimer, medium, boundary-layer, parameter

INTRODUCTION

The flow behaviour near stagnation-point environs has many applications, particularly in engineering and industry fields. The needs of such study attracted researchers to explore such a situation incorporating various cases, i.e., Newtonian or non-Newtonian fluid, steady or unsteady, two or three-dimensional, etc. Furthermore, the effects of stretching and heat transfer are common aspects in determining the final product. Chiam (1994) discussed stagnation-point flow of inviscid fluid over a linear stretching wall and Mahapatra and Gupta (2002) examined the heat transfer rate in stagnation-point flows towards a stretching sheet. Stagnation-point flow of a micropolar fluid towards a stretching sheet was discussed by Nazar *et al.* (2004) which was later extended by Attia (2006) taking into account heat generation/absorption and Ishak and Nazar (2009) in the case of permeable sheet. Ahmad *et al.* (2011) investigated boundary layer flows and heat transfer of micropolar fluids near stagnation point on a stretching vertical surface with prescribed skin friction. Mustafa *et al.* (2012) solved stagnation-point flow and heat transfer of Casson fluid towards a stretching sheet along with viscous dissipation effects using the HAM method. Shateyi and Marewo (2014) investigated the hydro magnetic boundary layer flow, heat and mass transfer of Casson fluid near a stagnation point over a stretching sheet in the presence of thermal radiation, viscous dissipation and chemical reaction and recently, Medikare *et al.* (2016) analyzed the effects of viscous

dissipation on MHD stagnation-point flow of Casson fluid over a nonlinearly stretching sheet, to name a few. All the above mentioned studies were dedicated to prescribed surface temperature.

According to Merkin (1994), there are four common heating processes, i.e., constant or prescribed wall temperature, constant or prescribed surface heat flux, conjugate condition and Newtonian heating or conjugate convective flow. In modelling convective boundary layer flow, much attention was focused on and only recently has Newtonian heating condition gained considerable attention. Some of the related works can be found in Lesnic *et al.* (2004), Salleh *et al.* (2009), Mohamed *et al.* (2012) and Ahmad *et al.* (2017).

The study of convective heat transfer in porous media plays an important role in many applications such as thermal insulation, extraction of crude oil, chemical reactors, etc. Many studies have been done on Darcy porous medium which is valid only for slow flows with low permeability. At higher flow rates, there is a departure from linear law and inertial effects. The Darcy-Forchheimer model describes the effect of inertia and viscous forces in porous media (Rawat *et al.*, 2007). Pal and Chatterjee (2012) analyzed two-dimensional stagnation-point flow of an electrically-conducting micropolar fluid over a stretching sheet embedded in non-Darcian porous medium with uniform magnetic and electric fields. Arifin *et al.* (2016) solved forced convection boundary layer stagnation point slip flow in Darcy-Forchheimer porous medium over a shrinking sheet by shooting method and recently,

Mishra *et al.* (2017) investigated boundary layer magnetohydrodynamic stagnation-point flow past a stretching sheet through non-Darcy porous media in the presence of a heat source/sink. Meanwhile, studies of Casson fluid flow in non-Darcian porous medium were discussed by Prasad *et al.* (2013), Kumar *et al.* (2016) and Vijaya *et al.* (2017). However, to the best knowledge of the researchers, no attempts have been made for stagnation-point Casson fluid flow in non-Darcian porous medium. Hence, motivated by the above mentioned works, this study is aimed at considering Casson fluid flow near a stagnation point in Darcy-Forchheimer porous medium with Newtonian heating effects.

Problem formulation: Consider steady two-dimensional Casson fluid flow near a stagnation point over a stretching sheet immersed in a porous medium. The sheet is stretched with velocity $u_w(x) = ax$ and the external velocity of the fluid flow is $u_e(x) = bx$ where a and b are constants. It is assumed that the sheet is heated due to Newtonian heating and the ambient fluid temperature is T_∞ . Under the above conditions, the boundary layer equations that govern the fluid flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{v \epsilon}{\kappa} (u - u_e) - \frac{F \epsilon^2}{\sqrt{\kappa}} (u^2 - u_e^2) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where:

- u and v = The velocity components along the x and y directions, respectively
- ν = The kinematic viscosity
- β = The non-Newtonian (Casson) fluid parameter
- ϵ = The dimensionless stretching parameter
- κ = The permeability of the porous medium
- T = The Temperature
- α = The thermal diffusivity

It should be pointed out that the last two terms in Eq. 2 represents the Darcy-Forchheimer porous medium. The appropriate boundary conditions for the problem are:

$$\begin{aligned} u = u_w, \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_f T \text{ (NH)} \quad \text{at } y = 0 \\ u \rightarrow u_e, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \tag{4}$$

with h being the heat transfer coefficient. Equation 1-3 subject to Eq. 4 can be transformed to ordinary differential equations by the following similarity transformation (Mohamed *et al.*, 2013):

$$\Psi = \sqrt{u_e \nu x} f(\eta), \quad \eta = \sqrt{\frac{u_e}{\nu x}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty} \text{ (NH)} \tag{5}$$

Where:

- Ψ = The stream function defined in usual fashion, i.e., $u = \partial \Psi / \partial y$ and $v = \partial \Psi / \partial x$
- η = The similarity variable

$f(\eta)$ and $\theta(\eta)$ are the dimensionless stream function and dimensionless temperature, respectively and prime denotes differentiation with respect to η . Making use of Eq. 5 into Eq. 1-3, Eq. 1 is automatically satisfied and Eq. 2-3 reduced to:

$$\left(1 + \frac{1}{\beta} \right) f''' + f f'' - f'^2 - \kappa_1 (f' - 1) - \kappa_2 (f'^2 - 1) + 1 = 0 \tag{6}$$

$$\theta'' + Pr f \theta' = 0 \tag{7}$$

Similarly, the boundary conditions Eq. 4 after implementing Eq. 5 is reduced to:

$$\begin{aligned} f(\eta) = 0, \quad f'(\eta) = c, \quad \theta'(\eta) = -\gamma [1 + \theta(0)] \quad \text{at } \eta = 0 \\ f'(\eta) \rightarrow 1, \quad \theta(\eta) \rightarrow 0 \quad \text{as } \eta \rightarrow \infty, \end{aligned} \tag{8}$$

where, prime denotes differentiation with respect to η , $\kappa_1 = \nu \epsilon / \kappa b$ is the porous media parameter, $\kappa_2 = F \epsilon^2 x / \sqrt{\kappa}$ is the inertia-coefficient parameter, $Pr = \mu c_p / k$ is the Prandtl number, $c = a/b (\geq 0)$ is the stretching parameter and $\gamma = h_f \sqrt{\nu} / b$ is the conjugate parameter for the Newtonian heating.

RESULTS AND DISCUSSION

Equation 6 and 7 subjected to Eq. 8 were solved using the method described by Cebeci and Bradshaw (1988) for arbitrary values of Casson fluid parameter (β), porous media parameter (κ_1), inertia-coefficient parameter (κ_2), Newtonian heating parameter (γ), stretching parameter (c) and Prandtl number (Pr). As far as engineering interest is concerned, there are two physical quantities of interest, i.e., the skin friction coefficient and the local Nusselt number which can be found once the values of $f'(0)$ and $\theta'(0)$ are secured. Retained that the value of $\theta'(0)$ can be acquired once $\theta(0)$ is known as given in Eq. 8.

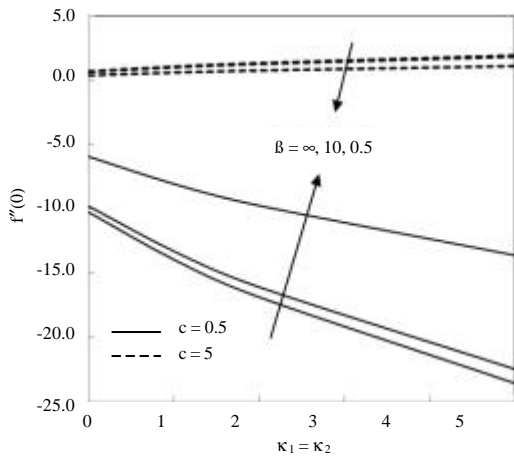


Fig. 1: Variations of $f''(0)$ for various values of κ_1 , κ_2 and β when $c = 0.5$ and 5 , respectively

Table 1: Values of $f''(0)$ when $\kappa_1 = \kappa_2 = 0$ and $\beta = 8$

c	Present results	Mohamed <i>et al.</i> (2013)	Wang (2008)
0	1.2315306	1.2325877	1.232588
0.5	0.7121119	0.7132949	0.71330
1	0	0	0
2	-1.8859019	-1.8873066	-1.88731

Table 2: Values of $f''(0)$ for various values β , κ_1 , κ_2 and c

$\kappa_1 = \kappa_2/c$	$\beta = 0.5$	$\beta = 10$
0		
0	0.7126263	1.1741682
0.5	0.4106638	0.6789219
1	0	0
2	-1.0882539	-1.7980719
0.1		
0	0.7630589	1.2607018
0.5	0.4365975	0.7217265
1	0	0
2	-1.1420484	-1.8868910
0.5		
0	0.9444179	1.5602419
0.5	0.5281054	0.8727675
1	0	0
2	-1.3368236	-2.2084359

Values obtained for $f''(0)$ when $\kappa_1 = \kappa_2 = 0$ and $\beta = \infty$ in our study are compared with Wang (2008) and Mohamed *et al.* (2013) as tabulated in Table 1 and were found to be satisfactory. It should be pointed out that Eq. 6 and 7 are decoupled. As such, $f''(0)$ is independent of Pr number and γ . The effects of β , κ_1 and κ_2 on $f''(0)$ are given in Table 2 for various values of c . It is seen that at fixed κ_1 and κ_2 and as β increases, $f''(0)$ increases for $c < 1$ and contrary impact is seen for $c > 1$. These results are further supported by the graph depicted in Fig. 1. Increments of κ_1 and κ_2 are found to increase and decrease $f''(0)$ for $c < 1$ and $c > 1$, respectively. However, $c = 1$ impacted to nil $f''(0)$ throughout the boundary layer for any values of κ_1 , κ_2 and β .

The effect of c does not only influence the $f''(0)$ but also the surface temperature $\theta(0)$. The accession of

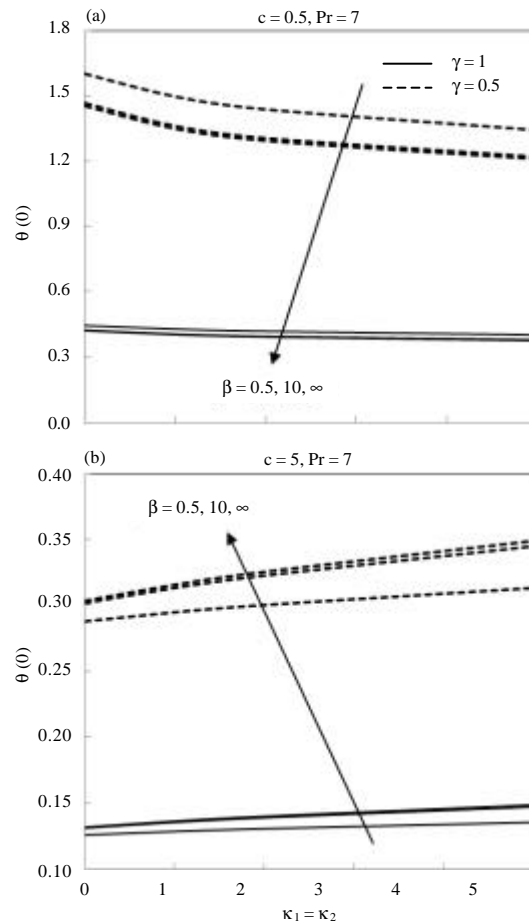


Fig. 2: Variations of $\theta(0)$ for various values of κ_1 , κ_2 , β and γ when $Pr = 7$ and $c = 0.5$ and 5 , respectively

stretching parameter c results in low surface temperature $\theta(0)$ at fixed κ_1 , κ_2 and γ . However, as κ_1 and κ_2 increase, $\theta(0)$ also experience the same contrary behavior for $c < 1$ and $c > 1$ as depicted in Fig. 2, i.e., $\theta(0)$ is found to decrease as κ_1 and κ_2 increase for $c < 1$ ($c = 0.5$) and reverse features for $c > 1$ ($c = 5$). The property behaviour of the fluid itself from Newtonian ($\beta = \infty$) to non-Newtonian fluid ($\beta = 0.5$) is seen to give higher surface temperature $\theta(0)$ for $c < 1$ and an opposite trend occurs for $c > 1$.

On top of that, the increment of κ_1 and κ_2 had an outstanding impact on $\theta(0)$ for $\gamma = 1$ as compared to $\gamma = 0.5$, i.e., the impact of κ_1 and κ_2 are not very critical. As the sheet was being heated due to Newtonian heating γ , the surface temperature $\theta(0)$ for $\gamma = 1$ was higher than $\gamma = 0.5$ which was expected to happen.

Pr number is very much significant in determining the heat transfer rate at the surface. The effect of this parameter can be seen in Fig. 3. It is very clear that increments of the Pr number results in the decrement of surface temperature $\theta(0)$. This is because as Pr increases,

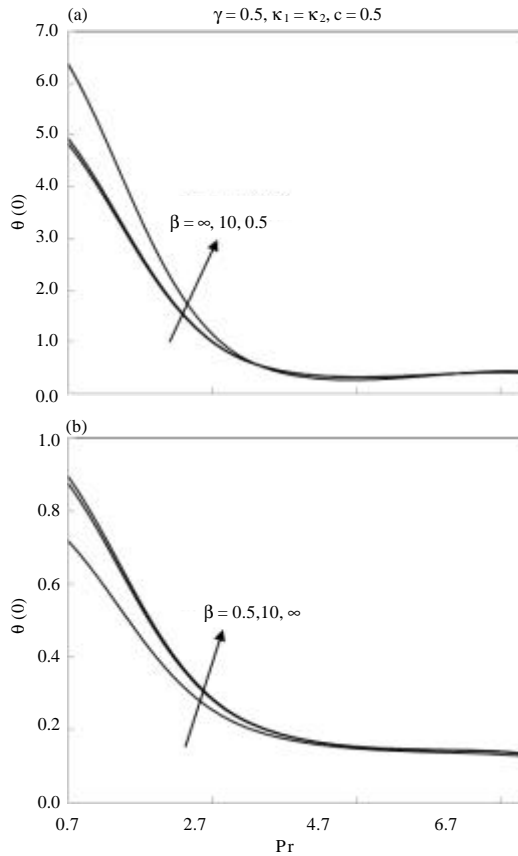


Fig. 3: Variations of $\theta(0)$ for various values of β and Pr when $\gamma = 0.5$, $\kappa_1 = \kappa_2 = 0.5$ and $c = 0.5$ and 5 , respectively

the thermal diffusivity decreases which in turn decreases the thermal energy. Additionally, the effect of stretching parameter c is found to decrease the surface temperature. Flow of Casson fluid is very much profound for small values of Pr number ($Pr \ll 3$) where Casson fluid β is seen to offer variations in the surface temperature, unlike for $Pr \gg 3$. It is observed that the non-Newtonian fluid caused higher $\theta(0)$ for $c < 1$ and a contrary trend was observed for $c > 1$. This scenario was only valid for $Pr \ll 3$.

The velocity and temperature profiles for $\beta = \gamma = 0.5$ and $Pr = 0.7$ are depicted in Fig. 4. The velocity profile is seen to have inverted boundary layer structured when $c > 1$ and no momentum boundary layer was formed when $c = 1$. This is in compliance with the results obtained in Table 1 and 2. As κ_1 and κ_2 increase, the velocity decreases within the boundary layer for $c > 1$ which results in the decrement of $f''(0)$ as demonstrated in Fig. 1. It should be noted that the opposite situation occurs for $c < 1$. Nevertheless, the temperature distribution within the boundary layer can also be seen on the right of Fig. 4.

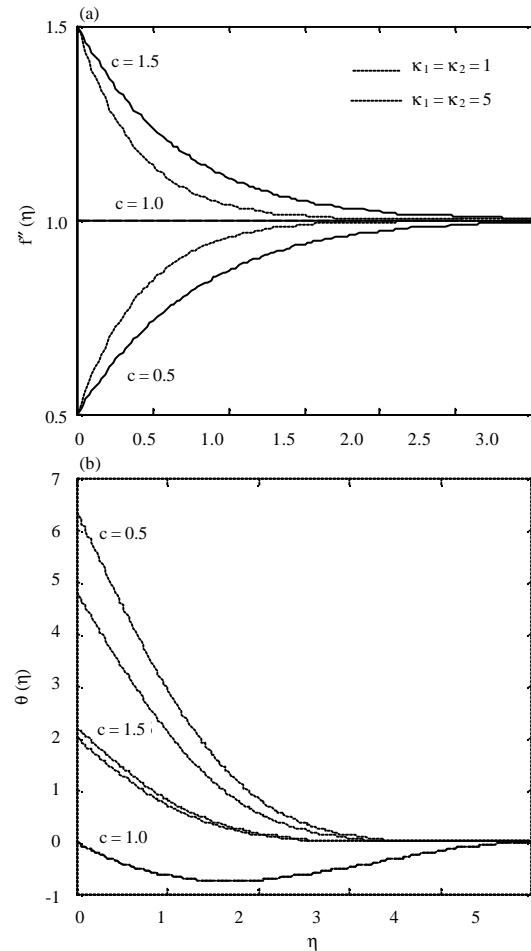


Fig. 4: Velocity and temperature profiles for $\beta = \gamma = 0.5$ and $Pr = 0.7$

CONCLUSION

In this study, we discussed the effect of stretching sheet and Newtonian heating for flow of Casson fluid embedded in Darcy-Forchheimer porous medium near a stagnation point. It can be concluded that c plays a major role in determining $f''(0)$ which later affects surface temperature $\theta(0)$. The repercussion of the stretching parameter c on $f''(0)$ and $\theta(0)$ for various values of Casson fluid parameter β , porous media parameter κ_1 , inertia-coefficient parameter κ_2 , Prandtl number Pr and Newtonian heating parameter γ very much depends on whether $c > 1$ or $c < 1$.

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