New Application for Generalized Regularized Long Wave (GRLW) Equation, Modified Dispersive Water Wave (MDWW) Equation and Kawahara Equation by Homogeneous Balance Method

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Abstract: This research study examines the homogeneous balance method to obtain an exact solution of travelling wave non-linear equations. The proposed homogeneous balance method is used to obtain new solutions for Generalized Regularized Long Wave (GRLW) equation, Modified Dispersive Water Wave (MDWW) equation and Kawahara equation. Many solitary wave solutions are calculated from the solution by the hyperbolic function when parameters were taken as special values. The obtained results are compared with the F-expansion method solution and (G'/G)-expansion method solution. The comparison reveals that our obtained results are identical to the F-expansion method and (G'/G)-expansion method solutions when certain hypothesis is adopted. Maple Software is used to plot the 3D graph of the obtained exact solution.

Key words: Homogeneous balance method, generalized regularized long wave equation, modified dispersive water wave equation, Kawahara equation and solitary wave solutions, obtained results, hypothesis

INTRODUCTION

There is a wide range of nonlinear phenomena accord in nature and obtaining a solution for such phenomena have a significant impact, especially in physics and applied mathematics. When studying nonlinear physical phenomena, the exact solution of nonlinear evolution equation has a major influence on this types of study (Zayed and Arnow, 2012). There are several algebraic methods to calculate exact travelling wave solution was proposed in the literature. The most important and appropriate method called homogeneous balance method. This method was first proposed by Wang (1995), Wang et al. (1996) then further developed by many researchers (Elwakil et al., 2004; Fan, 2000, 2003; Khalfallah, 2009; Zhao et al., 2006). According to Fan (2000) applied homogeneous balance method was used to calculate solution for Beeklund transformation when an assumption was made in the reduction of the nonlinear partial differential equation. Fan proves that there is a close relationship between the HB method, Clarkson Kruskal (CK) method and Weiss Tabor Carnevale (WTC) method. A new algebraic method was proposed by Fan (2003). The suggested method was used to calculate a new solution for solitary wave of NLPDEs and many other nonlinear evolution equations. The obtained solution has a significant impact in many areas such as applied mathematics and theoretical physics (Antonova and Biswas, 2009; Biswas, 2010; Razborova et al., 2013). The obtained solution can be expressed as a polynomial in an element that satisfies general Riccati equation and Elwakil et al. (2003). When the homogeneous balance method was applied an auto-Becklund transformation for the generalized shallow water wave equation and generalized variable coefficient 2D KdV equation, new exact solitary wave solutions was calculated. This idea of this research study depends on the characteristic of travelling wave solution of nonlinear equations where this solution can be expressed by a polynomial in \( u = \sum_{i=1}^{m} a_i g_i(\xi) \), \( v = u(\xi) \) such that \( \xi = kx+wt \). The degree of this polynomial can be calculated from considering the homogenous balance between the highest order derivatives and the nonlinear term appearing in a giving equation. The coefficient of the consider polynomial can be resulted from solving a set of algebraic equation appeared during the process of using the proposed method (Wang et al., 2008). The significance of the homogeneous balance method is making the calculation of shock or solitary type of solution easy. A general exact solution of the nonlinear equations of the Generalized Long Wave (GRLW), the (MDWW) equation and Kawahara equation was obtained.

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MATERIALS AND METHODS

Mechanism of homogeneous balance method: Consider we have a nonlinear evolution equation in the form of Eq. 1:

\[ F(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0 \]  

(1)

where, \( F \) is a polynomial in \( u = u(x, t) \) and its PDEs in which the highest order derivatives and nonlinear terms are involved. The main steps of applying the homogenous balance method can be illustrated in the following points (Zayed and Arnous, 2012, Zayed and Alurrfi, 2014):

Step 1: Using the wave transformation Eq. 2:

\[ u(x, t) = u(\xi), \xi = kx+wt \]  

(2)

Reduce Eq. 1 to the following ODE in Eq. 3:

\[ P(u, u_t, u_x, \ldots) = 0 \]  

(3)

where, \( P \) is a polynomial in \( u(\xi) \) and its general derivatives while \( k \) and \( w \) are constants.

Step 2: We suppose that Eq. 3 has a solution shown in Eq. 4:

\[ u(\xi) = \sum_{i=0}^{N} q_i \psi^i \]  

(4)

where, \( q_i (i = 0, 1, \ldots, N) \) are constants to be calculated, such that \( q_0 \neq 0 \) and \( \psi(\xi) \) is the solution of Eq. 5:

\[ \psi'(\xi) = \psi''(\xi)-\psi(\xi) \]  

(5)

Equation 5 has the solution:

\[ \psi(\xi) = \frac{1}{1+e^{-\xi}} \]  

(6)

Step 3: We calculate all the derivatives of \( u', u'', \ldots \), of the polynomial \( u(\xi) \) which results:

\[
\begin{align*}
  u &= q_0 + q_1 \psi + q_2 \psi^2 + \\
  u' &= (\psi-1)\psi[q_1 + 2q_2 \psi^2 + \ldots] \\
  u'' &= (\psi-1)\psi((-1+2\psi)q_1 + 2(\psi-3)q_2 + q_3 + \ldots) \\
  u''' &= (\psi-1)\psi((6\psi+6\psi^2)q_1 + 2(4-15\psi + 12\psi^3)q_2 + q_3 + \ldots)
  \\
  u^{(4)} &= (\psi-1)\psi((-1+14\psi-36\psi^2+24\psi^3)q_1 + 2(8+57\psi-108\psi^2+60\psi^3)q_2 + q_3 + \ldots) \\
  u^{(5)} &= (\psi-1)\psi((-1+30\psi+150\psi^2-240\psi^3+120\psi^4)q_1 + 2(16-165\psi+660\psi^2-840\psi^3+360\psi^4)q_2 + \ldots)
\end{align*}
\]

Step 4: We determine the positive integer \( N \) in Eq. 4 by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. 3.

Step 5: Substitute Eq. 4 into Eq. 3, we calculate all the necessary derivatives \( u', u'', \ldots \), of the equation \( u(\xi) \). As a result of this substitution, we obtain a polynomial of \( \psi \)

\[ (\psi(1) = 0, 1, 2, \ldots, N) \]. In this polynomial, we gather all terms of same powers and equating them to zero, we obtain a system of algebraic equations which can be solved by the Maple to get the unknown parameters \( q(i = 0, 1, \ldots, N), k \) and \( w \) consequently, we get the exact solutions of Eq. 1.

Application of homogeneous balance method

Generalized Regularized Long Wave (GRLW) equation:

In this study, our proposed method is applied to obtain a new and more general exact solution of the Generalized Regularized Long Wave (GRLW) equation read (Abazari, 2010):

\[ u_x + u_x + a(u^3)_x - bu_{xx} = 0 \]  

(7)

where, \( a \) and \( b \) are positive constants. Equation 7 was first proposed as a model for small amplitude long waves on the surface of the water in a channel by Peregrine (1996) and then by Benjamin et al. (1972). In physics phenomena such as unidirectional waves propagation in a water channel, long-crested waves in near shore zones and many other, the Generalized Regularized Long Wave (GRLW) equation works as an alternative model to the KdV equations. Suppose the following Eq. 8:

\[ u(x, t) = u(\xi), \xi = kx+wt \]  

(8)

we obtain:

\[ (w+k)u' + ak(u^3)_x - bku'' = 0 \]  

(9)

Integrating Eq. 9 once with respect to \( \xi \) and the setting the integration constant as zero yields:

\[ (w+k)u + aku^2 - bku^3 = 0 \]  

(10)

Now, we are balancing between \( u' \) and \( u'' \) then we obtain \( N+2 = 2N \), then, \( N = 2 \), suppose that Eq. 10 has the following formal solution:

\[ u(\xi) = q_1 + q_2 \psi + q_3 \psi^2 \]  

(11)
If \( \lambda = 1, c_1 = 1, c_2 = 0 \) then, Eq. (34a) becomes

\[
\begin{align*}
\text{Proposed homogeneous balance solution} & \\
\text{The solution of (G'/G)-expansion method (Abazari, 2010)} & \\
\text{If } \beta > 0, \alpha > 0 \text{ then, Eq. (12) becomes} & \\
\end{align*}
\]

\[
\begin{align*}
\frac{3bk^2}{2a(bk^2-1)} \sec h^2 \left( \frac{kx}{2} + \frac{k}{2(bk^2-1)} \right) \\
\end{align*}
\]

Substituting Eq. (11) and the derivatives in step 3 and collecting all terms with the same order of \( (c^2) \), we have the following:

\[
\begin{align*}
(\psi)^2 & : a_k q_{\psi}^2 + kq_\psi + aq_{\psi} = 0 \\
(\psi)^3 & : -bk^2q_\psi + 2akq_\psi q_2 + 2bkq_\psi + aq_\psi = 0 \\
(\psi)^2 & : 3bk^2q_\psi w + 4bkq_\psi w + 2akq_\psi q_2 + aq_\psi + kq_\psi \psi_{\psi} w_\psi = 0 \\
(\psi)^3 & : -2bk^2q_\psi w + 10bkq_\psi + 2akq_\psi q_2 = 0 \\
(\psi)^4 & : -6bkq_\psi w + aq_\psi w = 0
\end{align*}
\]

And solving the system of equation by using to maple, we obtain Table 1.

**Status 1:**

\[
\begin{align*}
q_\psi &= 0, q_\psi = \frac{-6bk^2}{a(bk^2-1)} q_\psi = \frac{-6bk^2}{a(bk^2-1)} w = \frac{k}{bk^2-1} \\
u(x, t) &= \frac{-3bk^2}{2a(bk^2-1)} \sec h \left( \frac{kx}{2} + \frac{k}{2(bk^2-1)} \right) \\
\end{align*}
\]

**Status 2:**

\[
\begin{align*}
q_\psi &= \frac{-bk^2}{a(bk^2+1)} + \frac{6bk^2}{a(bk^2+1)} q_\psi = \frac{-6bk^2}{a(bk^2+1)} \\
\psi &= \frac{-k}{bk^2+1} \\
u(x, t) &= \frac{-bk^2}{a(bk^2+1)} + \frac{3bk^2}{a(bk^2+1)} \sec h \left( \frac{kx-k}{2} \right) \right)
\end{align*}
\]

The **MDWW equation:** The MDWW equations will be studied in this study (Alzaidy, 2013; Neyrame et al., 2010) by the form of the following Eq. 13 and 14:

\[
\begin{align*}
u_x & = \frac{1}{4} \nu_x + \frac{1}{2} (uv)_x \\
v_t & = -u_x - 2u a_x + \frac{3}{2} v v_x
\end{align*}
\]

The traveling wave variables as:

\[
u(x, t) = \nu(\xi), \psi(x, t) = \psi(\xi), \xi = x - ct
\]
\[ \psi^0: c_a + \frac{1}{2} a_1 b_1 = 0 \]
\[ \psi^1 : c_a + \frac{1}{2} b_1 + a_1 b_1 = 0 \]
\[ \psi^2: b_1 + a_1 b_1 = 0 \]
\[ \psi^3: \frac{3}{4} b_1 - a_1^2 + c b_1 = 0 \]
\[ \psi^4: a_1^2 + \frac{3}{4} b_1 - c b_1 = 0 \]

Solving the above algebraic equations from \( \psi^0 \) into \( \psi^4 \) by using the maple, yields.

**Status 1:**
\[ a_0 = 0, \ a_1 = 1/2, \ b_0 = 0, \ b_1 = 1, \ c = -1/2 \]
\[ u(x,t) = \frac{1}{4} \tanh\left(\frac{x + t}{2}\right) \]  
\[ v(x,t) = \frac{1}{2} \tanh\left(\frac{x + t}{2}\right) \]  
(25)

**Status 2:**
\[ a_0 = 0, \ a_1 = 1/2, \ b_0 = 0, \ b_1 = -1, \ c = 1/2 \]
\[ u(x, t) = \frac{1}{4} \tanh\left(\frac{x + t}{2}\right) \]  
\[ v(x, t) = \frac{1}{2} \tanh\left(\frac{x + t}{2}\right) \]  
(27)

**Status 3:**
\[ a_0 = 1/2, \ a_1 = -1, \ b_0 = 1, \ b_1 = 0, \ c = -1/2 \]
\[ u(x, t) = \frac{1}{2} \tanh\left(\frac{x + t}{2}\right) \]  
(29)

**Status 4:**
\[ a_0 = 1/2, \ a_1 = -1, \ b_0 = -1, \ b_1 = 0, \ c = -1/2 \]
\[ u(x, t) = \frac{1}{2} \tanh\left(\frac{x + t}{2}\right) \]  
(30)

**Status 5:**
\[ a_0 = 1/2, \ a_1 = -1, \ b_0 = -1, \ b_1 = 0, \ c = -1/2 \]
\[ u(x,t) = \frac{1}{4} \tanh\left(\frac{x + t}{2}\right) \]  
(31)

**Status 6:**
\[ a_0 = -1/2, \ a_1 = 1/2, \ b_0 = -1, \ b_1 = 1, \ c = 1/2 \]
\[ u(x,t) = \frac{1}{4} \tanh\left(\frac{x + t}{2}\right) \]  
(33)

**Kawahara equation:** Let us consider the so-called Kawahara equation (Ozis and Aslan, 2010; Wazwaz, 2006):
\[ u_t + \alpha u u_x + \beta u_x u_x - \gamma u_{xxx} = 0 \]  
(35)

where, \( a - c \) is not equal zero arbitrary constants. Equation 36, proposed first by Kawahara (1972) and this equation is a model equation for plasma waves, capillary-gravity water waves (Bridges and Derks, 2002). Moreover, this equation characterized the water waves with surface tension (Benjamin et al., 1972) (Table 2).

Let us now solve Eq. 35 by applying the homogeneous balance method. To this end, we apply the wave transformation Eq. 2 to reduce Eq. 36 to the following ODE:
\[ w u'' + \alpha k^2 u u'' + \beta k^2 u u'' - \gamma k^4 u^{(4)} = 0 \]  
(36)

And integrating the resulting ordinary differential equation once, result:
\[ w u'' + \alpha k^2 \left(\frac{u'}{2}\right)^2 + \beta k^2 u u'' - \gamma k^4 u^{(4)} = 0 \]  
(37)

Balancing \( u'' \) with \( u^{(4)} \) yields \( N = 4 \). Consequently, Eq. 37 has the formal solution:
\[ u = q_x + q_1 \psi + q_2 \psi^2 + q_3 \psi^3 + q_4 \psi^4 \]  
(38)
Table 2: Comparison of proposed solution with solution of (G'/G)-expansion method (Nayrame et al., 2010)

<table>
<thead>
<tr>
<th>The solution of (G'/G)-expansion method</th>
<th>Proposed homogeneous balance solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $c_1 = 1$, $c_2 = 0$, $\lambda = 1$, $e = 1/2$ then Eq. 12 becomes</td>
<td>If $e &gt; 0$ then, Eq. 33 and Eq. 34 becomes</td>
</tr>
<tr>
<td>$u(x, t) = \frac{1}{4} \frac{1}{4} x^\frac{-1}{2} \tanh(-\frac{x}{2})$</td>
<td>$u(x, t) = \frac{1}{4} \frac{1}{4} x^\frac{-1}{2} \tanh(-\frac{x}{2})$</td>
</tr>
<tr>
<td>$v(x, t) = \frac{1}{2} \frac{1}{2} x^\frac{-1}{2} \tanh(-\frac{x}{2})$</td>
<td>$v(x, t) = \frac{1}{2} \frac{1}{2} x^\frac{-1}{2} \tanh(-\frac{x}{2})$</td>
</tr>
</tbody>
</table>

where, $q_0, q_1, q_2$ are constants such that $q_0 \neq 0$, now determine the derivative of Eq. 38 and we get the following system:

\[
\begin{align*}
(\psi)^1 :& \frac{1}{2} q_1^2 + cq_0 = 0 \\
(\psi)^2 :& \alpha q_0 q_1 + \beta q_1 - \alpha q_1 - \gamma q_1 = 0 \\
(\psi)^3 :& -c_1 q_0 + \frac{1}{2} q_1^2 + 4 \beta q_1 - 3 \beta q_1 + 15 q_1 - 16 \gamma q_1 = 0 \\
(\psi)^4 :& q_0 q_2 + c_2 q_1 = 0 \\
(\psi)^5 :& \alpha q_0 q_3 + \alpha q_1 q_2 + 2 \beta q_1 + 10 \beta q_1 - 9 \beta q_1 - c_2 - 50 q_1 + 130 q_2 - 81 \gamma q_1 = 0 \\
(\psi)^6 :& \frac{1}{2} q_1^2 + 6 \beta q_1 - 21 q_1 + 525 q_1 - 250 q_1 - 330 q_1 + 60 q_1 + 2 q_1 + 36 \beta q_1 - 2 q_1 + 336 q_1 - 1164 q_1 + 1476 \gamma q_1 = 0 \\
(\psi)^7 :& \frac{1}{2} q_1^2 + 120 q_1 + 1080 q_1 - 3020 \gamma q_1 + 20 \beta q_1 + 2 q_1 + 8 q_1 = 0 \\
(\psi)^8 :& q_0 q_4 + c_4 q_1 = 0 \\
(\psi)^9 :& q_0 q_4 + c_4 q_1 = 0 \\
(\psi)^10 :& \frac{1}{2} q_1^2 - 840 q_1 = 0
\end{align*}
\]

Solving the system of equations from $(\psi)^6$ to $(\psi)^9$ by using the Maple, we get:

**Status 1:**

$q_0 = 0$, $q_1 = 0$, $q_2 = 1680 \gamma / \alpha$, $q_3 = -3360 \gamma / \alpha$, $q_4 = 1680 \gamma / \alpha$, $\beta = 13 \gamma$, $e = -36 \gamma$

\[
\begin{align*}
 u(x, t) = & \frac{105 \gamma}{\alpha} \frac{210 \gamma}{\alpha} \tanh^2 \left( \frac{x - 36 \gamma t}{2} \right) \\
 & \frac{105 \gamma}{\alpha} \tanh^2 \left( \frac{x - 36 \gamma t}{2} \right) + (39)
\end{align*}
\]

**Status 2 A:**

$\beta = 13 \gamma$, $q_0 = 0$, $q_1 = $ \frac{840}{\alpha} \left( \frac{1}{2} \gamma + \frac{1}{62} \gamma \sqrt{31} \right)$, $q_2 = \frac{840}{\alpha} \left( \frac{5}{2} \gamma + \frac{1}{62} \gamma \sqrt{31} \right)$, $q_3 = \frac{-3360 \gamma}{\alpha}$, $q_4 = 1680 \gamma / \alpha$,

\[
\begin{align*}
 C = & \frac{-13}{2} \gamma \frac{39}{62} \gamma \sqrt{31}
\end{align*}
\]

\[
\begin{align*}
 u(x, t) = & \frac{105 \gamma}{\alpha} \frac{210 \gamma}{\alpha} \tanh^2 \left( \frac{x - 36 \gamma t}{2} \right) \\
 & \frac{105 \gamma}{\alpha} \tanh^2 \left( \frac{x - 36 \gamma t}{2} \right) + (40)
\end{align*}
\]

**Status 3:**

$\beta = 13 \gamma$, $q_0 = $ \frac{-72 \gamma}{\alpha}$, $q_1 = 0$, $q_2 = 1680 \gamma / \alpha$, $q_3 = \frac{-3360 \gamma}{\alpha}$, $q_4 = 1680 \gamma / \alpha$, $C = -36 \gamma$

\[
\begin{align*}
 u(x, t) = & \frac{33 \gamma}{\alpha} \frac{210 \gamma}{\alpha} \tanh^2 \left( \frac{x + 36 \gamma t}{2} \right) \\
 & \frac{105 \gamma}{\alpha} \tanh^2 \left( \frac{x + 36 \gamma t}{2} \right) + (42)
\end{align*}
\]
Status 4 A:
\[
\begin{align*}
\beta &= \frac{-13}{2} \gamma + \frac{39}{62} \gamma \sqrt{31}, \\
q_1 &= \frac{6}{\alpha} \left( \frac{5}{2} \gamma + \frac{13}{62} \gamma \sqrt{31} \right), \\
q_2 &= \frac{840}{\alpha} \left( \frac{5}{2} \gamma - \frac{1}{62} \gamma \sqrt{31} \right), \\
q_3 &= \frac{3360 \gamma}{\alpha}, \\
q_4 &= \frac{1680 \gamma}{\alpha}, \\
q_5 &= \frac{15}{2} \gamma + \frac{39}{62} \gamma \sqrt{31}.
\end{align*}
\]
\[
\begin{align*}
u(x,t) &= \frac{105 \gamma}{\sqrt{31} \alpha} \tanh^{i} \left( x - \frac{-15}{2} \gamma - \frac{39}{62} \gamma \sqrt{31} \right)_t \\
&+ \frac{105 \gamma}{\alpha} \tanh^{i} \left( x - \frac{-15}{2} \gamma + \frac{39}{62} \gamma \sqrt{31} \right)_t \\
&+ \frac{15 \gamma}{\alpha} \frac{66 \gamma}{\sqrt{31} \alpha} = 0
\end{align*}
\]

Status 4 B:
\[
\begin{align*}
\beta &= \frac{-13}{2} \gamma + \frac{39}{62} \gamma \sqrt{31}, \\
q_1 &= \frac{6}{\alpha} \left( \frac{5}{2} \gamma - \frac{13}{62} \gamma \sqrt{31} \right), \\
q_2 &= \frac{840}{\alpha} \left( \frac{5}{2} \gamma + \frac{1}{62} \gamma \sqrt{31} \right), \\
q_3 &= \frac{3360 \gamma}{\alpha}, \\
q_4 &= \frac{1680 \gamma}{\alpha}, \\
q_5 &= \frac{15}{2} \gamma - \frac{39}{62} \gamma \sqrt{31}.
\end{align*}
\]
\[
\begin{align*}
u(x,t) &= -\frac{105 \gamma}{\sqrt{31} \alpha} \tanh^{i} \left( x - \frac{-15}{2} \gamma - \frac{39}{62} \gamma \sqrt{31} \right)_t \\
&+ \frac{105 \gamma}{\alpha} \tanh^{i} \left( x - \frac{-15}{2} \gamma + \frac{39}{62} \gamma \sqrt{31} \right)_t \\
&+ \frac{15 \gamma}{\alpha} \frac{66 \gamma}{\sqrt{31} \alpha} = 0
\end{align*}
\]

RESULTS AND DISCUSSION

Graphical representations of the solutions: Maple software is used to plot the obtain result. Figure 1-3 show the graphical representation of the solution (Table 3).

Fig. 1: The solitary wave 3D graphics of Eq. 13 for \(a = 2, b = 2, k = 2, x = -10, ..., 10\) and \(t = 0, ..., 10\)

Fig. 2: a) The solitary wave 3D graphics of Eq. 26 for \(x = -10, ..., 10\) and \(t = 0, ..., 10\) and b) The solitary wave 3D graphics of Eq. 27 for \(x = -10, ..., 10\) and \(t = 0, ..., 10\)

Fig. 3: The solitary wave 3D graphics of Eq. 39 for \(a = 3, \gamma = 3, x = -10, ..., 10\) and \(t = 0, ..., 10\)
Table 3: Comparison of proposed solutions with solution of (G'/G)-expansion method (Wazwaz, 2006)  

<table>
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<tr>
<th>Solution of (G'/G)-expansion method</th>
<th>Proposed homogeneous balance solution</th>
</tr>
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<tbody>
<tr>
<td>If $\beta = 4$, $\gamma = -1/13$ then Eq. 33 becomes $u(x, t) = \frac{5283360}{13\alpha} \tanh^{2}(x + 576/t)$</td>
<td>If $\gamma = 16/13$ then, Eq. 42 $u(x, t) = \frac{5283360}{13\alpha} \tanh^{2}(x + 576/t)$</td>
</tr>
</tbody>
</table>

CONCLUSION

In this research study, the homogeneous balance method is proposed to find the an exact solution for nonlinear equations such as the Generalized Regularized Long Wave (GRLW) equation, the MDWW equation and the kawahara equation. The proposed method has high efficiency and practicality in finding exact solutions. The obtained result was compared with other proposed methods in the literature under special condition. The finding reveals that our proposed method is less complex and more efficient than the methods used in the literature. Additionally, the proposed method is capable to be applied to wide range of different selected equation and the result would be useful in different applied situation.

ACKNOWLEDGEMENT

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