ON $\alpha$-g-closed Sets with Respect to an Ideal

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Abstract: This research informs new types of weakly $\alpha$-open sets via. ideals which generalize a usual notion for $\alpha$-g-open sets which is finer than $\alpha$-open set. The relationship between this notion and previously defined concepts was studied. Some properties are studied like compactness by using this notion. Some examples have been shown that the opposite is not true for many propositions and various types of ideals were used in these examples. The relationship between different types of compactness has been clarified in a flowchart. As a result, we obtained many propositions and remarks of knowledge sets, including that the existence of two different ideals on the same set such that one of them subset of the other does not necessarily indicate that there is a relationships between the two sets defined on these ideals. It is also by using different types of topologies and ideals to know the form of new knowledge sets.

Key words: $\alpha$-g-O set, $\alpha$-g-C set, $\alpha$-g-compact, $\alpha$-g-compact, ideals, $\alpha$-g-open

INTRODUCTION

Throughout this research T.S. indicates to Topological space and I.T.S. indicate to Ideal Topological Space. If(A) and C(A) will indicate the interior and closure of A, respectively. The nonempty family IcP(X) known as ideal on X, if it hold the heredity and finite additive property (Kuratowski, 1969; Nasef et al., 2015). For any T.S. (X, T), the aggregation $\{I\in T: I\subseteq I\}$ is a base for $T'$ where $T'$ is a topology, finer than T, constructed by Vaidyanathaswamy (1945). Each element in $T'$ is called $T'$-open sets ($T'$-O). $T=\{\varnothing\}$ implies $T=T'$ (Donchev et al., 1999; Jarkovski and Hamlet, 1990). Any set A of a T.S. (X, T) is $\alpha$-open ($\alpha$-O) (Njastad, 1965), if $A\subseteq C(I(A))$ and then $X\setminus A$ is $\alpha$-closed ($\alpha$-C) (Nasef et al., 2015). $I_{g}(A)$ and $C_{g}(A)$ will indicate the interior and closure of A in $(X, T_{g})$, resp.

A subset A of a T.S. $(X, T)$ is said to be e-compact (Viglion, 1969) where each closed subset F and each open cover of F, there is a finite subfamily W that hold $\{C(V): V\in W\}$ covers F. Every compact space is e-compact (Esmaeel, 2009; Viglion, 1969).

MATERIALS AND METHODS

$\alpha$-g-closed sets via. ideal
Definition 2.1: Any set A of a I.T.S. $(X, T, I)$ is $\alpha$-g-closed set via. $I$ ($\alpha$-g-C) iff $C_{g}(A)/5cI$, whenever $A\setminus 5cI$ and 5 is an $\alpha$-O. So, $X\setminus A$ is $\alpha$-g-open set ($\alpha$-g-C). The collection of all $\alpha$-g-C sets is indicated by $\alpha$-g-C $(X)$.

Remark 2.2: For any I.T.S. $(X, T, I)$
- $\alpha$-g-C$(X) = P(X)$ where $I = P(X)$
- If $FeI$ for each $\alpha$-C set F, then $\alpha$-g-C$(X) = P(X)$
- Every $\alpha$-C set is $\alpha$-g-C
- All closed sets are $\alpha$-g-C

The converse of the implication in 2.2, need not be true.

Example 2.3: For the I.T.S. $(X, T, I)$ when $X = \{e_1, e_2, e_3\}$, $T = P(X)$ and $I = \{\varnothing\}$ it’s clear that $\alpha$-g-C$(X) = P(X)$.

Example 2.4: For the I.T.S. $(X, T, I)$ when $X = \{e_1, e_2, e_3\}$, $I = \{\varnothing, \{e_3\}\}$ and $T = \{\varnothing, \{e_1\}\}$. A subset $A = \{e_1, e_2\}$ of $X$ is $\alpha$-g-C which isn’t closed (resp., $\alpha$-C).

Proposition 2.5: Let $(X, T, I)$ be a I.T.S., A and B any two sets in X, $A\cup B$ is $\alpha$-g-C, whenever both A and B are $\alpha$-g-C sets.

Proof: Let $(A\cup B)/5cI$ whenever 5 is $\alpha$-O then we get that $A5cI$ and $B5cI$. Because of, A and B are both $\alpha$-g-C sets, that’s lead $C_{g}(A)/5cI$ and $C_{g}(B)/5cI$. So $(C_{g}(A)/5cI)\cup (C_{g}(B)/5cI)$ implies $(C_{g}(A)\cup C_{g}(B))/5cI$, so, $C_{g}(A\cup B)/5cI$. Therefore, $Q\cup B$ is $\alpha$-g-C.

Corollary 2.6: Let $(X, T, I)$ be a I.T.S., A and B any 2 sets in X. If A and B are both $\alpha$-g-O sets then, so is their intersection.

Remark 2.7: The arbitrary union of $\alpha$-g-C sets in an I.T.S. $(X, T, I)$ may be not $\alpha$-g-C in general.
Example 2.8: Consider the I.T.S. $(N, T_{ov}, I)$, where $T_{ov} = \{5cN; (N-S) \text{ is finite set or } S = \emptyset \}$ and $I = P(E)$ is an ideal on $X$ “where $E$ is set of all even natural numbers”. \{e\} is $\alpha_{g}$-C set for all $e \in E$ but $\cup \{e\}: e \in E = E$ is not.

Proposition 2.9: A subset $A$ of a I.T.S. $(X, T, I)$ is $\alpha_{g}$-O set iff $F(A) \in I$ where $F(A) \in E$ and $F$ is an $\alpha$-C set.

Proof: For a $\alpha_{g}$-O set $A$ and $F(A)\in I$ when $F$ is an $\alpha$-C set, $F(A) = (X \cap A)(X(F))$ and $(X(A) \cap (X(F)))$ is a $\alpha_{g}$-C set, so, $C(A \cap (X(A) \cap (X(F)))) = (X \cap A)(X(F)) \in E$. Hence, $F(A) \in I$. Sufficient by the same idea we can show that if $F(A) \in I$, when $F(A) \in E$ with $F$ is an $\alpha$-C set then $A$ is an $\alpha_{g}$-O set.

Proposition 2.10: Let $(X, T, I)$ be an I.T.S. such that $T = \{X, \emptyset, \{x\}\}$ for any $x \in X$, if $I = \{\leq X; \{X\} \in I\}$ then $\alpha_{g}$-C$(X) = P(X)$. If $I = \{\emptyset\}$ then, $\alpha_{g}$-C$(X) = I(X)$.

Proposition 2.11: Let $(X, T)$ be a T.S. if $I = I_{c} = \{A \subseteq X; I(C(A)) = \emptyset\}$ then $\alpha_{g}$-C$(X) = P(X)$.

Proof: For any $A \in X$, if $A \subseteq E$, then $\leq T_{ov}$ implies $I(C(A) \cap (X(S))) = \emptyset$. Now, $I(C(A) \cap (X(S))) = \emptyset$ implies $I(C(A)) \cap (X(S)) = \emptyset$. Hence, $C(A) \cap (X(S)) = \emptyset$. We claim that, $I(C(A)) \cap (X(S)) = \emptyset$. Let $x \in C(A) \cap (X(S))$, implies that there is $v \in T$ such that $x \in C(A) \cap (X(S))$. So, $\forall v \in C(A) \cap (X(S)) = \forall v \in (X(A) \cap (X(S))) = X(A) \cap (X(S)) = X(A) \cap (X(S)) = \emptyset$.

That is, $\forall x \in C(A) \cap (X(S)) = \emptyset$ which a contradiction. Hence, $I(C(A)) \cap (X(S)) = \emptyset$. Therefore, $C(A) \subseteq I_{c}$ and $A$ is an $\alpha_{g}$-C set.

Proposition 2.12: Let $I$ and $J$ are ideals in T.S. $(X, T)$ then, the condition $I \subseteq J$ is not sufficient to get every $\alpha_{g}$-C to be $\alpha_{g}$-C set and not conversely.

Example 2.13: Let $(X, T)$ be a T.S. where $X = \{e_{1}, e_{2}, e_{3}\}$, $T = \{X, \emptyset, \{e_{1}\}, \{e_{2}\}, \{e_{3}\}, \{e_{1}, e_{2}\}\}$ and $A = \{e_{1}\}$. Clear that $A$ is $\alpha_{g}$-C which isn’t $\alpha_{g}$-C since, $\{e_{1}\} \subseteq I_{c}$ but $C_{\emptyset}(\{e_{1}\}) = \{e_{1}\} \subseteq I_{c}$.

Example 2.14: Let $(X, T)$ be a T.S. where $X = \{e_{1}, e_{2}, e_{3}\}$, $T = \{X, \emptyset, \{e_{1}\}, \{e_{2}\}, \{e_{1}, e_{2}\}, \{e_{1}, e_{3}\}, \{e_{2}, e_{3}\}\}$ and $A = \{e_{1}, e_{2}\}$. Clear that A is $\alpha_{g}$-C which isn’t $\alpha_{g}$-C since, $\{e_{1}, e_{2}\} \subseteq I_{c}$ but $C_{\emptyset}(\{e_{1}, e_{2}\}) = \{e_{1}, e_{2}\} \subseteq I_{c}$.

RESULTS AND DISCUSSION

$\alpha_{g}$-compactness

Definition 3.1: An I.T.S. $(X, T, I)$ known as $\alpha_{g}$-compact if any $\alpha_{g}$-O cover has a finite subcover.

Definition 3.2: An I.T.S. $(X, T, I)$ is known as $\alpha_{g}$-go-compact if for each $\alpha_{g}$-C set $A \subseteq X$ each family of $\alpha_{g}$-O subset of $X$ which covers $A$ has a finite subfamily whose $\alpha$-closures in $X$ covers $A$.

Proposition 3.3: For an I.T.S. $(X, T, I)$

- Every $\alpha_{g}$-g-cover is compact
- Every $\alpha_{g}$-go-cover is $\alpha_{g}$-go-compact
- Every $\alpha_{g}$-go-cover is $c$-compact

The relations among different types of compactness in 3.3, explain in following flowchart.

The converses of flowchart may be not hold.

Example 3.4: For a I.T.S. $(R, T, I)$ where $T = \{ R, \emptyset \}$ with $I = P(X), (R, T, I)$ is compact and $\alpha_{g}$-go-compact but the cover $\{r\} : r \in R$ shows $(R, T, I)$ isn’t $\alpha_{g}$-go-compact.

Example 3.5: Consider the I.T.S. $(N, T_{ov}, I_{c})$ where $T_{ov} = \{5cN; (N-S) \text{ is finite set or } S = \emptyset \}$ and $I_{c} = \{5cN, 5 = \{e_{1}, e_{2}, e_{3}, ..., e_{n}\}\}$. Clear that $(N, T_{ov}, I_{c})$ is a $c$-compact but the cover $\{x\} : x \in N$ shows $(N, T_{ov}, I_{c})$ isn’t $\alpha_{g}$-go-compact.

CONCLUSION

This research informs new types of weakly $\alpha$-open sets via ideals namely $\alpha_{g}$-go-closed set via $I$ ($\alpha_{g}$-C) we obtained many remarks and propositions of knowledge sets, some examples have been shown that the opposite is not true for many propositions and some properties are studied like $\alpha_{g}$-C-compactness. The relationship between some types of compactness and $\alpha_{g}$-go-compactness has been clarified in a flowchart.

REFERENCES


