Structural Behavior of Highly Strength Reinforced Concrete Beams with Elliptical Arched Bottom

¹Maryam Hameed Naser Al-Mamoori and ²Nameer A. Alwash

¹Department of Civil Engineering, Al-Mustaqbal University College, Hillah, Iraq
maryamhameed02@gmail.com

²Department of Civil Engineering, College of Engineering, University of Babylon, Hillah, Iraq
namer_alwash@yahoo.com

Abstract: This study presents the research results about the testing of four reinforced concrete beams with elliptical arched bottom subjected to static loading. Beams were made and tested under two point load. In each beam have different span of arch (1180, 900, 740 and 600 mm) with the same amount of reinforcing steel and volume of concrete and to find the optimum ratio of the arch length to beam span for the maximum load capacity as well as to validate the numerical results taken from the finite element model. From the obtained results, it can be observed that increase of the arch length to beam span ratio increased the ultimate load capacity to more than 40%. Also, a significant decrease in stiffness of beam were noticed when the length span of arch ratio is less. FEM seem efficient and gives good accuracy through comparison with the experimental results where the maximum difference in the ultimate load value was <9%.

Key words: High Strength Concrete (HSC), elliptical arch beam, non-prismatic beam, Abaqus, arch length, reinforcing

INTRODUCTION

The arch is a structure that extends a space and backings the structure and weight below it. Over the years, arches of different forms have been constructed for bridges, buildings as well as monumental, historical and cultural purposes. These have become architectural masterpieces gradually (Miller *et al.*, 2000).

Numerous specialists introduced experimental and analytical investigations of reinforced concrete arches. These examinations were initiated in 1960 by Jain (1960). Also, the curved beam behavior under static load over various cross-sectional forms and special requirements have been studied by many researchers through many experimental programs such as Al-Thabthabee (2012) and Hamza (2013), most of them focus on the use of Normal Weight Concrete (NWC) but although, there are many studies on High Strength Concrete (HSC) such as those carried out by Hameed (2010) and Annadurai and Ravichandran (2014). Extensive experimental and analytical studies have been performed to investigate the behaviors of non-prismatic beam under different loading methods which are widely used in numerous engineering structures such as Archundia-Aranda *et al.* (2013) by Rojas (2013), Orr *et al.* (2014) and Nabbat (2015).

The above mentioned studies indicate that experimental and theoretical research on behavior of curved and non-prismatic beams remains insufficient. Accordingly, this study attempts to assess the effect span of arch on the behavior of beam with the same size of concrete and amount reinforcing steel and to find the optimal ratio of the arch length to span beam for the maximum load capacity as well as to validate the numerical results taken from the finite element model.

MATERIALS AND METHODS

Experimental program

Description of test specimens: This study includes casting four simply supported beams of elliptical arch at bottom face and were tested under two point load. The width (b) for all beams was 150 mm, the beams have total Length (Lt) of 1500 mm, the effective span (L) was 1350 mm and the shear span (a) was 450 mm. Also, the overall depth at each beam end (h_{max} = 250 mm) and then reduced at the centre according to area and span of the arch as shown in Fig. 1.

Each beam was fabricated and tested under static load condition with the same volume of concrete and

Fig. 1: Details of reinforcement

Fig. 2: Elliptical arch geometry

Table 1: Identification of the specimens

name	Identifications
E1	Beam containing elliptical arch with arch span (Lc = 1.18 m)
E2	Beam containing elliptical arch with arch span (Lc = 0.90 m)
E3	Beam containing elliptical arch with arch span (Lc = 0.74 m)
E4	Beam containing elliptical arch with arch span (Lc = 0.60 m)

amount of steel reinforcement but with different span of arch (1180, 900, 740 and 600 mm). All specimens are identified in Table 1.

Where, E; Elliptical arch, 1-4, refer to span of arch (1180, 900, 740 and 600 mm), respectively. To achieve the same volume of concrete all specimens have the same area of arch:

$$V_{\text{concrete}} = \left[\text{Total area of section} - \text{Area of arch} \right] \times \text{Width of beam}$$

$$V_{\text{concrete}} = [1.5 \times 0.25 - \text{Area of arch}] \times 0.15$$

Let,

$$\text{Area of arch} = 0.06 \text{ m}^2 \text{ (constant)}$$

The arch is having the shape of half an ellipse as shown in Fig. 2:

Table 2: Details of specimen of elliptical arch

Specimens	Lc (m)	yc (m)	Lc/L ratio	A _{arch} (m ²)
E1	1.180	0.065	0.87	0.06
E2	0.900	0.085	0.67	0.06
E3	0.740	0.103	0.55	0.06
E4	0.600	0.127	0.44	0.06

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ (Ellipse equation)} \rightarrow$$

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2} \right)$$

$$y = b \sqrt{1 - \frac{x^2}{a^2}} \text{ but } (b = yc \text{ and } a = Lc/2)$$

$$\therefore y = yc \sqrt{1 - \frac{x^2}{(Lc/2)^2}}$$

$$A_{\text{arch}} = \frac{\pi}{4} \cdot Lc \cdot yc, A_{\text{arch}} = 0.06 \text{ m}^2 \rightarrow yc = \frac{0.24}{\pi \cdot Lc}$$

Where:

Lc = Span of arch

yc = Height of arch

A_{arch} = Area of the arch

L = Beam span

The specimens have half of ellipse shape with different span of arch (1180, 900, 740 and 600 mm) as shown in Table 2 and Fig. 3.

Material properties: Highly strength concrete is obtained by selecting suitable materials, good quality control and proportioning. The material must be conforming to ACI committee 363 R, 1997 requirements. Three different diameter of reinforcement were tested (Ø12, 10 and 6). Bar size of (Ø12, 10 mm) Ukrainian organize were used as

Fig. 3: Specimens of elliptical arch: a) E1; b) E2; c) E3 and d) E4

Fig. 4: Casting and curing process

longitudinal reinforcement and bars of size ($\varnothing 6$ mm) ukrainian organize were used as lateral reinforcement (closed stirrups). Ordinary portland cement known as KAR cement was used in the present study. Natural sand from Al-Akhaidur Region in Iraq with maximum size of (4.75 mm) and fineness modulus of (2.94) was used. Natural crushed gravel of maximum size 19 mm was used in this research. It was selected from Al-Nebai Region. Also, silica fume and superplasticizers (SikaViscocrete-5930) are used to achieve the high workability required to produce High-Strength Concrete (HSC).

Mixing, casting, curing and concrete testing of specimens: The drum laboratory mixer with a capacity of 0.04 m^3 is used to mixing of concrete. Therefore, for each specimen and corresponding cubes, cylinders and prisms,

two batches were used. High strength concrete was used to cast all specimens. The mix was by weight at 1:1.25:2.03:0.31 for cementitious material, sand, gravel and water ratios, respectively. Moulds for flexural strength testing were prepared with internal dimensions of 150 mm width, 1500 mm length and variable height along length of the beam. Four beam samples were cast and cured under laboratory conditions at the Civil Engineering Department in the University of Babylon. Also, three standard cubes ($150 \times 150 \times 150$ mm) for compressive strength test, three standard cylinders (100×200 mm) for splitting tensile strength test and three prisms ($100 \times 100 \times 400$ mm) for modulus of rupture test were cast from the concrete of each beam specimens. After the water curing period, the beam models were painted by white emulsion to ensure clear appearance of crack growth during the test as described in Fig. 4.

Table 3: Mechanical properties of mixing concrete for all beams

Samples name	Compressive strength (f_c') MPa $f_c' = f_{cu}/1.15$	Tensile strength (MPa)		Modulus of Elasticity* (E_c) GPa
		f_{sp}	f_t	
E1	66.38	5.41	5.93	33.95
E2	67.45	5.55	6.07	34.17
E3	68.21	4.76	5.88	34.32
E4	66.23	5.22	6.20	33.92

*ACI 363R-92 ($E_c = 3320 \sqrt{f_c'} + 6900$)

Table 4: Experimental results for each specimens

Samples name	First cracking load			P_u (kN)	P_s (kN) (0.7 P_u)	Δ_s (mm)
	P_{cr} (kN)	Location from mid span (cm)	Δ_{cr} (mm)			
E1	28	11.5	1.14	95.05	66.54	5.61
E2	25	0.6	0.84	84.18	58.93	6.51
E3	23	9.5	1.20	80.05	56.04	6.79
E4	19	8.5	0.86	55.66	38.96	3.45

Fig. 5: Recording the development of cracks

RESULTS AND DISCUSSION

Mechanical properties: Table 3 explains the mechanical properties of concrete. Each value in this Table 3 is the average of three samples.

Cracking behavior and ultimate load: During testing, flexural, flexural-shear (diagonal) and splitting cracks were formed as shown in Fig. 5. Flexural cracks were observed firstly while diagonal cracks and splitting cracks developed as loading continued.

The first flexural cracks in all the beams was between two point load region occurred randomly on the tension face of the beam. As the load increased, cracks formed along arch at bottom face. Then, the longitudinal cracks

formed along the arch toward mid-span. The cracks pattern of each of these specimens is described in Table 4 and Fig. 6.

Load-deflection curves: Three dial gages were put one at mid span one at the point load and one at the start point of the arch. The recorded ultimate load, type of failure and deflection at mid span, under point load and at start of arch are presented in Table 5 for each specimens.

Figure 7 represent the load deflection curves for the tested specimens with elliptical arched bottom. It can be concluded that decreasing the length of span arch reduced the ultimate load capacity (compared with E1) by about 11.44% for E2, 15.78% for E3 and 41.44% for E4. Also, a significant decrease in stiffness of beam were noticed when the length span of arch is smaller.

Fig. 6: a-d) Cracks pattern for beams with elliptical arched bottom at failure

Fig. 7: Load deflection curves of beams with elliptical arch: a) Beam E1; b) Beam E2; c) Beam E3 and d) Beam E4

Table 5: Deflections at ultimate load and type of failure

Samples name	Ultimate load			Deflection Δu (mm)		
	P_u (kN)	Decrease in ultimate load (%)	Failure mode	At mid span	Under point load	Start of arch
E1	95.05	----	Crushing of concrete and flexural failure	16.87	14.57	4.60
E2	84.18	11.44	Crushing of concrete and flexural failure	16.39	13.33	8.88
E3	80.05	15.78	Crushing of concrete and flexural failure	15.52	12.82	9.50
E4	55.66	41.44	Crushing of concrete and splitting failure	11.40	9.39	8.71

Table 6: Comparison between experimental and FEM at first cracking load, ultimate load and deflections at mid Span

Samples name	First cracking load			Ultimate load (kN)			Deflection Δu (mm)		
	P_{cr}	P_{cr}	P_{cr}	P_u	P_u	P_u	Δu	Δu	Δu
	Exp	FEM	Exp/FEM	Exp	FEM	Exp/FEM	Exp	FEM	Exp/FEM
E1	28	26.775	0.96	95.05	98.48	1.04	16.87	15.99	0.95
E2	25	22.575	0.90	84.18	86.90	1.03	16.39	15.30	0.93
E3	23	23.850	1.04	80.05	83.87	1.05	15.52	14.24	0.92
E4	19	18.150	0.96	55.66	56.99	1.02	11.40	11.00	0.96

Fig. 8: Comparison of load-mid span deflection curves for beams; at mid span

Fig. 9: Effect of the arch length/beam span ratio on the ultimate load; beam with elliptical arched bottom

For Fig. 8, it is observed that beam (E1) has more stiffness and deflection compared with other beams due to the distribution of stresses on larger length where leads to less intensity of stress. The relation between the ultimate load and arch length/beam span ratio is described in Fig. 9 (Table 6).

Description of specimens in finite element: This research includes the analysis of beams tested by using a powerful nonlinear finite element method package Abaqus that

Fig. 10: Details modeling of beam models

provides a relatively acceptable numerical procedure for investigating the behavior of beam containing arch at bottom face using high strength concrete, assuming full bond between concrete and steel reinforced. In the modeling of the concrete, plates support and shaft a 3-Dimensional 8-node element (C3D8) was used and for reinforcement a 3 Dimensional 2 node Truss element (T3D2) was used as shown in Fig. 10.

Results of finite element analysis: The validity and accuracy of the adopted finite element models by using Abaqus computer program are studied and checked by analyzing all beams (E1-E4) that have been studied experimentally in this research. The numerical load-deflection curves for the all beams compared with the experimental results showed that the FE results showed rather stiffer structure as shown in Fig. 11. From the load-deflection behavior of all beams can be noticed that results of experimental work and numerical analysis have three regions, elastic-uncracked, elastic-cracked and elasto-plastic, the first region was terminated when the cracks happen. The deflected shape at ultimate load for E1-E4 is shown in Fig. 12. Table 6 shows that the maximum difference of the deflection in ultimate load was <5%. Also, it was found that the different between the numerical and experimental ultimate load for all beam was not more than 9%.

Fig. 11: Load-deflection curve for beams with elliptical arched bottom; a) Beam E1; b) Beam E2; c) Beam E3 and d) Beam E4

Fig. 12: Deformed shape of for the tested beams at ultimate load; a) E1; b) E2; c) E3 and d) E4

CONCLUSION

Depending on the results collected from the experimental work and numerical analysis, the following conclusions can be stated within the scope of this study. It can be concluded that the increase in the arch

length/beam span ratio increased the ultimate load capacity where it was found when beams with elliptical arch the increase in the ultimate load may reach to more than 40%.

Generally, the failure often occurred due to the top chord concrete crushing due to compression stresses.

The first cracking load of numerical data showed results lower than the experimental data recorded with ratio P_{cr}/P_{cr}^{exp} from (0.9-1.04). The FEM Model appears efficient and gives good accuracy through comparison with the experimental results where the maximum difference in the deflection at ultimate load was lower than 5%. Also, it was found that the ultimate numerical load for all beams was about 4% higher than the ultimate experimental load.

REFERENCES

- Al-Thabthawee, D.W., 2012. Nonlinear analysis for behavior of R.C. arch beams with opening. Msc Thesis, University of Babylon, Hillah, Iraq.
- Annadurai, A. and A. Ravichandran, 2014. Development of mix design for high strength concrete with admixtures. IOSR. J. Mech. Civil Eng., 10: 22-27.
- Archundia-Aranda, H.I., A. Tena-Colunga and A. Grande-Vega, 2013. Behavior of reinforced concrete haunched beams subjected to cyclic shear loading. Eng. Struct., 49: 27-42.
- Hameed, D.H., 2010. Mix design for high strength concrete with silica fume. MSc Thesis, University of Babylon, Hillah, Iraq.
- Hamza, B.H., 2013. Behavior of RC curved beams with openings and strengthened by CFRP laminates. Ph.D Thesis, University of Basrah, Basrah, Iraq.
- Jain, O.P., 1960. Ultimate strength of reinforced concrete arches. ACI. J., 77: 697-714.
- Miller, A.B., K.M. Clark and M.C. Grimes, 2000. A survey of masonry and concrete arch bridges in Virginia (No. VTRC 00-R11). Master Thesis, Virginia Transportation Research Council, Virginia.
- Nabbat, R.A., 2015. Flexural and shear behavior of non-prismatic reinforced high strength concrete beams with openings and strengthened with CFPR products. Msc Thesis, University of Babylon, Iraq.
- Orr, J.J., T.J. Ibell, A.P. Darby and M. Evernden, 2014. Shear behaviour of non-prismatic steel reinforced concrete beams. Eng. Struct., 71: 48-59.
- Rojas, A.L., 2013. Mechanical elements of rectangular nonprismatic members for symmetrical parabolic haunches subjected to a uniformly distributed load. J. Archit. Eng. Technol., 2: 1-8.