Vibration Analysis of Piezoelectric Composite using Sinc and Discrete Singular Convolution Differential Quadrature Techniques

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Abstract: This research concerns with free vibration analysis of piezoelectric composite materials. Based on the theory of elasticity and piezoelectricity, the governing equations of the problem were derived. Two differential quadrature techniques are employed to reduce the problem to an eigenvalue problem. That is solved for different materials and boundary conditions. The natural frequencies of the composite are obtained. Numerical analysis is introduced to explain influence of computational characteristics of the proposed schemes on convergence, accuracy and efficiency of the obtained results. The obtained results agreed with the previous analytical and numerical ones. Also, the proposed schemes record less execution time than previous ones. Furthermore, a parametric study is introduced to investigate the influence of elastic and geometric characteristics of the composite on the results.

Key words: Vibration, piezoelectric, composite, sinc, discrete singular convolution, execution time

INTRODUCTION

Piezoelectric materials have been frequently arise in many engineering, electro-mechanical problems such as transducers actuators and sensors which have ability of transferring from electrical to mechanical energy and vice-versa (Nechibvute et al., 2012; Hung, 2005). Vibration analysis of such composites can be used to predict the behavior of smart structures.

Due to the complexity of such problems, only limited cases can analytically be solved. A number of approximate theories of the vibration problems are issued by Khdieir (1988), Wu and Chen (1994), Matsunaga (2000) and Cho et al. (1991). Literature on the numerical solution of research subject is sparse. Typical useful numerical methods such as spline finite strip (Pan and Cheung, 1984; Galerkin Chia, 1985) least squares (Zitnan, 1996; meshless (Dorning and Liu, 1998; Rayleigh-RitzYoung, 1950) and finite element (Leung and Chan, 1998) techniques are used to solve such problems. The drawback of these numerical methods is the need to large number of grid points as well as a large computer capacity to attain a considerable accuracy.

More recently, Differential Quadrature Methods (DQM) have many successful applications in engineering fields (Zhang et al., 2006). Earlier approximations depend on lagrange interpolation polynomials. Convergence and stability of the solution are not ensured through that version which is only suitable for rectangular domains. Therefore, this version cannot be individually employed for geometric or material discontinuity problems. This drawback can be overcome by combining DQM with the domain decomposition technique and geometric mapping (Xionghua and Shen, 2004; Zong et al., 2005). Hybrid technique based on State-Space and Differential Quadrature Methods (SSDQM) is used to solve such problems for complex geometry and different boundary conditions (Chen and Liu, 2005; Zhou et al., 2010; Feri et al., 2015). In this approach, DQM is used in two directions and state space method is employed along the thickness direction. Sinc Differential Quadrature Method (SDQM) (Bellomo et al., 2001; El-Gamel et al., 2008; Dockery, 1991; El-Gamel and Layed, 2002; Yin, 1994; Carlson et al., 1987) and Discrete Singular Convolution Differential Quadrature Method (DSCDQM) (Ng et al., 2004; Wei, 1999a; b; 2000a, b; 2001; Wei et al., 2002; Wan et al., 2002; Wei et al., 2001) are more reliable versions than polynomial based DQM.

Up to knowledge of the researchers, SDQM and DSCDQM are not examined for vibration analysis of composite piezoelectric plate materials. Based on these versions, numerical schemes are designed for free vibration of piezoelectric composites. The natural frequencies are obtained and compared with previous analytical and numerical ones. For each scheme the convergence and efficiency is verified. Also, a parametric
MATERIALS AND METHODS

Formulation of the problem: Consider a three-dimensional piezoelectric composite with \((0 < x < a, 0 < y < b, 0 < z < h)\) where, \(a, b\) and \(h\) are length, width and total thickness of the composite. This composite is polarized in \(z\) direction and consists of \(m\) layers with different types of materials as shown in Fig. 1. Based on the theory of elasticity and piezoelectricity, the equations of motion and the charge equation of electrostatic can be written as Feri et al. (2015):

\[
\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \frac{\partial^2 u}{\partial t^2}
\]

\[
\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \frac{\partial^2 v}{\partial t^2}
\]

\[
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} = \frac{\partial^2 w}{\partial t^2}
\]

where, \((\sigma_x, \sigma_y, \sigma_z, \tau_{yx}, \tau_{xy}, \tau_{xz})\) and \((D_x, D_y, D_z)\) are stresses, displacement and induction field in the \(x, y, z\) directions, respectively; \((\tau_{yx}, \tau_{xy}, \tau_{xz})\) are the shear stresses; \(\rho\) is the density of the material. The relation between mechanical and electric material properties is constitutive equations which can be written as:

\[
\begin{pmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yx} \\
\tau_{xy} \\
\tau_{xz}
\end{pmatrix} =
\begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & C_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial u}{\partial y} \\
\frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial v}{\partial z}
\end{pmatrix} +
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & C_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial y} \\
\frac{\partial \phi}{\partial z}
\end{pmatrix}
\]

\[
\begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 & 0 & e_x & 0 \\
0 & 0 & 0 & e_y & 0 & 0 \\
0 & 0 & 0 & 0 & e_z & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial w}{\partial z}
\end{pmatrix} +
\begin{pmatrix}
0 & 0 & 0 & e_y & 0 & 0 \\
0 & 0 & 0 & e_z & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial y} \\
\frac{\partial \phi}{\partial z}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\eta_1 \\
\eta_2 \\
\eta_3
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial w}{\partial z}
\end{pmatrix} +
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial y} \\
\frac{\partial \phi}{\partial z}
\end{pmatrix}
\]

Fig. 1: Piezoelectric composite (sensor is PZT-4 and actuator is Ba2Na Nb5O15)
where, C, e and η are the components of the effective elastic, piezoelectric and dielectric constants of the same piezoelectric material, respectively. Also, φ is the electrical potential. For harmonic behavior, one can assume that:

\[
\begin{align*}
\psi(x, z, t) &= U e^{i\omega t}, \quad V(x, z, t) = V e^{i\omega t}, \quad w(x, z, t) = W e^{i\omega t}, \quad \phi(x, z, t) = \Phi e^{i\omega t}
\end{align*}
\]

Where:

\[
\begin{align*}
\omega &= \text{The natural frequency of the plate} \\
W &= \text{The amplitudes for } u, w \text{ and } \phi, \text{ respectively}
\end{align*}
\]

The elastic material constants can be determined as follows using the reciprocal theorem (Zhou et al., 2010):

\[
C_{11} = \frac{E_t \left(1 - v_{23}^2 \frac{E_t}{E_2} \right)}{1 - v_{23}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)}
\]

\[
C_{12} = \frac{C_{21}}{1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)} \left(1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)\right)
\]

\[
C_{13} = \frac{C_{31}}{1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)} \left(1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)\right)
\]

\[
C_{22} = \frac{E_2 \left(1 - v_{13}^2 \frac{E_3}{E_2} \right)}{1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)}
\]

\[
C_{23} = \frac{E_3 \left(1 - v_{13}^2 \frac{E_3}{E_2} \right)}{1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)}
\]

\[
C_{33} = \frac{E_3 \left(1 - v_{13}^2 \frac{E_3}{E_2} \right)}{1 - v_{12}^2 \frac{E_2}{E_1} - v_{13}^2 \frac{E_3}{E_2} - v_{13}^2 \frac{E_3}{E_1} \left(1 - v_{12}^2 \frac{E_2}{E_3} \right)}
\]

\[
C_{44} = C_{23}, \quad C_{55} = C_{13}, \quad C_{66} = C_{12}
\]

Substituting from Eq. 5-14 into 1-4 the problem can be reduced to a quasi-static one as:

\[
\begin{align*}
&\frac{\partial^2 U}{\partial x^2} + C_{66} \frac{\partial^2 U}{\partial y^2} + C_{11} \frac{\partial^2 V}{\partial x^2} + C_{11} \frac{\partial^2 V}{\partial y^2} + C_{13} \frac{\partial^2 W}{\partial z^2} + C_{33} \frac{\partial^2 \Phi}{\partial z^2} = -\rho \omega^2 U \\
&\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\rho \omega^2 V \\
&\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\rho \omega^2 W
\end{align*}
\]

The boundary conditions can be described as:

\[
\begin{align*}
w &= v = 0, \quad \text{at } x = 0, a \\
w &= u = \sigma_z = 0, \quad \text{at } y = 0, b
\end{align*}
\]

For simply Supported edge (S):

\[
\begin{align*}
w &= v = u = 0, \quad \text{at } x = 0, a \quad y = 0, b
\end{align*}
\]

For Clamped edge (C):

\[
\begin{align*}
\sigma_x = \tau_{xz} = \tau_{xy} &= 0, \quad \text{at } x = 0, a \\
\sigma_y = \tau_{yz} = \tau_{xy} &= 0, \quad \text{at } y = 0, b
\end{align*}
\]

For Free edge (F) Mechanical and electrical boundary conditions at lower and upper surfaces of the composite are:

\[
\begin{align*}
\sigma_x = \tau_{xz} = \tau_{xy} &= D_z = 0, \quad \text{at } z = 0 \\
\sigma_z = \tau_{xz} = \tau_{xy} &= \Phi = 0, \quad \text{at } z = h
\end{align*}
\]

To ensure the continuity between electric and elastic layers, the following conditions can be considered:

\[
\begin{align*}
U(x, y, h_i) &= U(x, y, h_i) \\
V(x, y, h_i) &= V(x, y, h_i) = W(x, y, h_i) = W(x, y, h_i) \\
\Phi(x, y, h_i) &= \Phi(x, y, h_i)
\end{align*}
\]
U \left( x, y, (h, h_n^+) \right) = U \left( x, y, (h, h_n^-) \right),
V \left( x, y, (h, h_n^+) \right) = V \left( x, y, (h, h_n^-) \right),
W \left( x, y, (h, h_n^+) \right) = W \left( x, y, (h, h_n^-) \right),
\Phi \left( x, y, (h, h_n^+) \right) = \Phi \left( x, y, (h, h_n^-) \right) \tag{24}

Also, the continuity conditions between different elastic materials are:

\begin{align*}
U \left( x, y, h_n^+ \right) &= U \left( x, y, h_n^- \right), \quad V \left( x, y, h_n^+ \right) = V \left( x, y, h_n^- \right), \\
W \left( x, y, h_n^+ \right) &= W \left( x, y, h_n^- \right), \quad \Phi \left( x, y, h_n^+ \right) = \Phi \left( x, y, h_n^- \right) \tag{25}
\end{align*}

Method of solution: Two differential quadrature techniques are employed to reduce the governing equations into an eigenvalue problem as follows:

**Sine Differential Quadrature Method (SDQM):** Cardinal sine function is used as a shape function such that the unknown \( \psi \) and its derivatives can be approximated as a weighted linear sum of nodal values, \( \psi_i \), \( i = \{-N, N\} \) as follows (Korkmaz and Dag, 2011; Bellomo et al., 2001; El-Gamel et al., 2003; Dockery, 1991; El-Gamel and Zayed, 2002; Yin, 1994; Carlson et al., 1997):

\begin{align*}
\psi(x_i) &= \sum_{j=\pm N}^{N} \frac{\sin \left[ \pi \left( x_i - x_j \right) h_x \right]}{\pi \left( x_i - x_j \right) h_x} \psi(x_j), \quad (i = \{-N, N\}, h_x > 0) \tag{26}
\end{align*}

\begin{align*}
\left. \frac{\partial \psi}{\partial x} \right|_{x = x_i} &= \sum_{j=\pm N}^{N} A_{ij} \psi(x_j), \quad (i = \{-N, N\}) \tag{27}
\end{align*}

\begin{align*}
\left. \frac{\partial^2 \psi}{\partial x^2} \right|_{x = x_i} &= \sum_{j=\pm N}^{N} B_{ij} \psi(x_j), \quad (i = \{-N, N\}) \tag{28}
\end{align*}

Where:

\( \psi \) = Denotes to U, V, W and \( \Phi \)

\( N \) = The number of grid points \( hx \) is grid size

The weighting coefficients \( A_{ij} \), \( B_{ij} \) can be determined by differentiating (Eq. 26) as:

\begin{align*}
A_{ij} &= \begin{cases} 
(1)^{ij} h_x \left( ij \right) & \text{if } i \neq j \\ 0 & \text{if } i = j 
\end{cases} \\
B_{ij} &= \begin{cases} 
\frac{\left( 2 \left( 1 \right)^{ij} \right) - \pi^2}{3 h_x^2} & \text{if } i = j \\
-\frac{\left( 2 \left( 1 \right)^{ij} \right) - \pi^2}{3 h_x^2} & \text{if } i \neq j
\end{cases} \tag{29}
\end{align*}

**Discrete Singular Convolution Differential Quadrature Method (DSCDQM):** A singular convolution can be defined as Korkmaz and Dag (2011), Bellomo et al. (2001), El-Gamel et al. (2003), Dockery (1991); El-Gamel and Zayed (2002), Yin (1994) and Carlson et al. (1997):

\begin{align*}
\psi_i(t) &= \left( T \ast \eta \right)(t) = \int_{-\infty}^{\infty} T(t-x) \eta(x) dx \tag{30}
\end{align*}

where, \( T(t-x) \) is a singular kernel. The DSC algorithm can be applied using many types of kernels. These kernels are applied as shape functions such that the unknown \( \psi \) and its derivatives are approximated as a weighted linear sum of \( \psi_i \), \( i = \{-N, N\} \), over a narrow bandwidth (\( x-x_{y_0} \), \( x+x_{y_0} \)) (Ng et al., 2004; Wei, 1999a, b; Wei, 2002a, b; 2001, Wei et al., 2000; Wan et al., 2002; Wei et al., 2001). Two kernels of DSC will be employed as follows: Delta Lagrange Kernel (DLK) can be used as a shape function such that the unknown \( \psi \) and its derivatives can be approximated as a weighted linear sum of nodal values, \( \psi_i \), \( i = \{-N, N\} \) as follows:

\begin{align*}
\psi(x_i) &= \sum_{j=M}^{\infty} \prod_{k=1}^{M} \left( x_i - x_k \right) \\
\prod_{j=M}^{\infty} \left( x_i - x_k \right) \psi(x_j), \quad (i = \{-N, N\}, M \geq 1) \tag{31}
\end{align*}

\begin{align*}
\left. \frac{\partial \psi}{\partial x} \right|_{x = x_i} &= \sum_{j=M}^{\infty} A_{ij} \psi(x_j), \\
\left. \frac{\partial^2 \psi}{\partial x^2} \right|_{x = x_i} &= \sum_{j=M}^{\infty} B_{ij} \psi(x_j), \quad (i = \{-N, N\}) \tag{32}
\end{align*}

where, \( 2M+1 \) is the effective computational band width. \( A_{ij}, B_{ij} \) are defined as:

\begin{align*}
A_{ij} &= \begin{cases} 
1 & \text{if } i = j \\
\prod_{k=-M}^{M} \left( x_i - x_k \right) & \text{if } i \neq j
\end{cases} \\
B_{ij} &= \begin{cases} 
\frac{\left( 2 \left( 1 \right)^{ij} \right) - \pi^2}{3 h_x^2} & \text{if } i = j \\
-\frac{\left( 2 \left( 1 \right)^{ij} \right) - \pi^2}{3 h_x^2} & \text{if } i \neq j
\end{cases} \tag{33}
\end{align*}

Regularized Shannon Kernel (RSK) can also be used as a shape function such that the unknown \( \psi \) and its derivatives can be approximated as a weighted linear sum of nodal values, \( \psi_i \), \( i = \{-N, N\} \) as follows:
\[
\psi(x_i) = \sum_{j=M}^{M} \left[ \sin \left( \frac{\pi(x_i-y_j)}{d_i} \right) \right] e^{-x_j^2/\sigma^2} \psi(x_j)
\]

\[i = -N_i, N_i, \sigma = (r/h_i) > 0\]  

\[\frac{\partial \psi}{\partial x} |_{x=x_i} = \sum_{j=M}^{M} A_{ij} \psi(x_j), \frac{\partial^2 \psi}{\partial x^2} |_{x=x_i} = \sum_{j=M}^{M} B_{ij} \psi(x_j), (i = -N_i, N_i)\]  

Where:
\( \sigma = \) Regularization parameter  
\( r = \) A computational parameter

The weighting coefficients \( A_{ij}, B_{ij} \) are defined as (Ng et al., 2004; Wei, 1999a, b; Wei, 2000a, b; Wei et al., 2001; Wan et al., 2002; Wei et al., 2002):

\[A_{ij} = \begin{cases} (-1)^{i+j} \left( \frac{i+j}{2\sigma} \right)^{2} e^{-x_j^2/\sigma^2}, & i \neq j \\ 0, & i = j \end{cases}\]

\[B_{ij} = \begin{cases} 2(-1)^{i+j+1} \left( \frac{i+j}{2\sigma} \right)^{2} e^{-x_j^2/\sigma^2}, & i \neq j \\ -\frac{1}{2\sigma^2} x_j^2, & i = j \end{cases}\]  

Similarly, one can approximate \( \psi_j, \psi_{ij}, \psi_{nm}, \psi_{mn} \) and calculated \( A^x_{ij}, A^y_{ij}, B^x_{ij}, B^y_{ij} \). On suitable substitution from equations of weighting coefficients (Eq. 26-36) into (Eq. 15-18), the problem can be reduced to the following Eigen-value problem:

\[C_{11} \sum_{i=1}^{N_i} B^x_{ij} U_{ij} + C_{66} \sum_{m=-N_i}^{N_i} B^y_{jm} U_{nm} + C_{55} \sum_{n=-N_i}^{N_i} B^x_{kn} U_{nj} + \]

\[(C_{12} + C_{66}) \sum_{i=1}^{N_i} A^x_{ij} \sum_{m=-N_i}^{N_i} A^y_{jm} U_{nm} + C_{55} \sum_{n=-N_i}^{N_i} B^x_{kn} U_{nj} + \]

\[(C_{13} + C_{55}) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} W_{nj} + \]

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 U\]  

\[(C_{11} \sum_{m=-N_i}^{N_i} B^x_{jm} V_{mj} + C_{44} \sum_{n=-N_i}^{N_i} B^x_{kn} V_{jn} + \]

\[(C_{13} + C_{44}) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} W_{mn} + \]

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 V\]  

\[(C_{55} + C_{13}) \sum_{i=1}^{N_i} A^x_{ij} \sum_{m=-N_i}^{N_i} A^x_{kn} U_{jn} + \]

\[(C_{13} + C_{55}) \sum_{m=-N_i}^{N_i} B^x_{jm} W_{nj} + C_{44} \sum_{m=-N_i}^{N_i} B^x_{kn} W_{mn} + \]

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} B^y_{jm} \Phi_{mn} + e_3 \sum_{n=-N_i}^{N_i} B^x_{kn} \Phi_{ij} = -\rho v^2 W\]  

\[(e_1 + e_3) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{jn} = -\rho v^2 U\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 V\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} B^y_{jm} \Phi_{mn} + e_3 \sum_{n=-N_i}^{N_i} B^x_{kn} \Phi_{ij} = -\rho v^2 W\]  

\[(e_1 + e_3) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{jn} = -\rho v^2 U\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 V\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} B^y_{jm} \Phi_{mn} + e_3 \sum_{n=-N_i}^{N_i} B^x_{kn} \Phi_{ij} = -\rho v^2 W\]  

\[(e_1 + e_3) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{jn} = -\rho v^2 U\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 V\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} B^y_{jm} \Phi_{mn} + e_3 \sum_{n=-N_i}^{N_i} B^x_{kn} \Phi_{ij} = -\rho v^2 W\]  

\[(e_1 + e_3) \sum_{i=1}^{N_i} A^x_{ij} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{jn} = -\rho v^2 U\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} A^y_{jm} \sum_{n=-N_i}^{N_i} A^x_{kn} \Phi_{mn} = -\rho v^2 V\]  

\[(e_2 + e_4) \sum_{m=-N_i}^{N_i} B^y_{jm} \Phi_{mn} + e_3 \sum_{n=-N_i}^{N_i} B^x_{kn} \Phi_{ij} = -\rho v^2 W\]  

The boundary conditions Eq. 19-25 can also be approximated using two DQMs as. Simply Supported (S):
\[
W_{ijk} = V_{ijk} = C_{11} \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + C_{12} \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} + C_{13} \sum_{n=-N_z}^{N_z} A_{kn} W_{ijn} + e_1 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = 0, \text{ at } x = 0, a
\]

\[
W_{ijk} = U_{ijk} = C_{12} \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + C_{11} \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} + C_{13} \sum_{n=-N_z}^{N_z} A_{kn} W_{ijn} + e_2 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = 0, \text{ at } y = 0, b
\]

\[
W_{ijk} = V_{ijk} = U_{ijk} = 0, \text{ at } x = 0, a \quad y = 0, b \quad (42)
\]

Clamped (C). Free surface (F):

\[
C_{11} \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + C_{12} \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} + C_{13} \sum_{n=-N_z}^{N_z} A_{kn} W_{ijn} + e_1 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = 0 \quad (43)
\]

\[
C_{55} \left( \sum_{l=-N_x}^{N_x} A_{ll} W_{ijk} \sum_{n=-N_z}^{N_z} A_{kn} U_{ijn} \right) + e_2 \sum_{l=-N_x}^{N_x} A_{ll} \Phi_{ijk} = C_{66} \left( \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} \right) = 0, \text{ at } x = 0, a
\]

\[
C_{12} \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + C_{11} \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} + C_{13} \sum_{n=-N_z}^{N_z} A_{kn} W_{ijn} + e_2 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = 0
\]

\[
C_{44} \left( \sum_{m=-N_y}^{N_y} A_{jm} W_{imk} + \sum_{n=-N_z}^{N_z} A_{kn} V_{ijn} \right) + e_4 \sum_{m=-N_y}^{N_y} A_{jm} \Phi_{imk} = C_{66} \left( \sum_{l=-N_x}^{N_x} A_{ll} V_{ijk} + \sum_{m=-N_y}^{N_y} A_{jm} U_{imk} \right) = 0, \text{ at } y = 0, b
\]

Mechanical and electrical boundary conditions at lower and upper surfaces of the composite are:

\[
C_{13} \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + C_{12} \sum_{m=-N_y}^{N_y} A_{jm} V_{imk} + C_{33} \sum_{n=-N_z}^{N_z} A_{kn} W_{ijn} + e_3 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = C_{55} \left( \sum_{l=-N_x}^{N_x} A_{ll} W_{ijk} + \sum_{n=-N_z}^{N_z} A_{kn} U_{ijn} \right)
\]

\[
e_3 \sum_{l=-N_x}^{N_x} A_{ll} \Phi_{ijk} = 0 \quad C_{44} \left( \sum_{m=-N_y}^{N_y} A_{jm} W_{imk} + \sum_{n=-N_z}^{N_z} A_{kn} V_{ijn} \right) + e_4 \sum_{m=-N_y}^{N_y} A_{jm} \Phi_{imk} = e_1 \sum_{l=-N_x}^{N_x} A_{ll} U_{ijk} + e_2 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn}
\]

\[
e_3 \sum_{m=-N_y}^{N_y} A_{jm} \Phi_{imk} = C_{55} \left( \sum_{l=-N_x}^{N_x} A_{ll} W_{ijk} + \sum_{n=-N_z}^{N_z} A_{kn} U_{ijn} \right) + e_3 \sum_{l=-N_x}^{N_x} A_{ll} \Phi_{ijk} = 0 \quad C_{44} \left( \sum_{m=-N_y}^{N_y} A_{jm} W_{imk} + \sum_{n=-N_z}^{N_z} A_{kn} V_{ijn} \right)
\]

\[
e_3 \sum_{n=-N_z}^{N_z} A_{kn} \Phi_{ijn} = \Phi_{ijk} = 0, \text{ at } z = h \quad (44)
\]

The continuity conditions between the interfaces of layers can be assumed as:

\[
W_{ijk}^{1} = W_{ijk}^{2}, \quad V_{ijk}^{1} = V_{ijk}^{2}, \quad U_{ijk}^{1} = U_{ijk}^{2}, \quad \Phi_{ijk}^{1} = \Phi_{ijk}^{2} \quad (45)
\]

We have solved the generalized eigen value problem (Vel et al., 2004; Zhang et al., 2006):

\[
KX = \omega^2 MX \quad (46)
\]

Where:

- \( K \) = The coefficient matrix of previous system
- \( M \) = The mass matrix can be diagonal with zero diagonal elements and
- \( \omega \) = Free vibration frequencies squared. Rewriting the equation in the form
\[
\begin{align*}
\begin{pmatrix}
[K_{aa}] & [K_{ac}] \\
[K_{ca}] & [K_{cc}]
\end{pmatrix}
\begin{pmatrix}
\phi_a \\
\phi_c
\end{pmatrix}
&= \omega^2
\begin{pmatrix}
[M] & [0] \\
[0] & [0]
\end{pmatrix}
\begin{pmatrix}
\phi_a \\
\phi_c
\end{pmatrix}

\text{where}

K_{aa} = \begin{pmatrix}
[K_{11}] & [K_{12}] & [K_{13}] \\
[K_{21}] & [K_{22}] & [K_{23}] \\
[K_{31}] & [K_{32}] & [K_{33}]
\end{pmatrix},
K_{ac} = \begin{pmatrix}
[K_{14}] \\
[K_{24}] \\
[K_{34}]
\end{pmatrix}
\end{align*}
\]

\[
K_{ca} = \begin{pmatrix}
[K_{41}] \\
[K_{42}] \\
[K_{43}]
\end{pmatrix},
K_{cc} = \begin{pmatrix}
[K_{44}]
\end{pmatrix}
\]

\[
\phi_a = \begin{pmatrix}
[U] \\
[V] \\
[W]
\end{pmatrix},
\phi_c = \begin{pmatrix}
([\Phi])
\end{pmatrix},
M = -\rho l
\]

\[
\Omega = \frac{c_0 h \sqrt{\frac{E_2}{\rho}}}{\omega} = \Omega * \left( \frac{\alpha}{h} \right)^2
\]

where, \(\rho\) and \(E\) are the density and the Young’s modulus of the bottom layer, respectively. For the present results, material parameters for the composite are listed in Table 1.

For sine DQ scheme, the problem is solved over a regular grids ranging from \(3*5*5-11*5*5\). Table 2 shows convergence of the obtained results. They agreed with exact ones (Khdeir, 1988; Wu and Chen, 1994; Matsunaga, 2000; Zhou et al., 2010; Korkmaz and Dag, 2011) over grid size \(\geq 7*5*5\).

For DSCDQ scheme based on delta Lagrange kernel, the problem is also solved over a uniform grids ranging from \(3*5*5-11*5*5\). The bandwidth \(2M+1\) ranges from 3-11. Table 3 shows convergence of the obtained fundamental frequency which agreed with exact ones (Khdeir, 1988; Wu and Chen, 1994; Matsunaga, 2000; Zhou et al., 2010; Korkmaz and Dag, 2011) over grid size \(\geq 7*5*5\) and bandwidth \(\geq 5\). Table 4 shows that the obtained results are more accurate than that were obtained using state space DQM (Zhou et al., 2010; Korkmaz and Dag, 2011). This table also shows that execution time of DSCDQM-DLK is less than that of sine DQM.

For DSCDQ scheme based on Regularized Shannon Kernel (RSK), the problem is also solved over a uniform grids ranging from \(3*5*5-9*5*5\). The bandwidth \(2M+1\) ranges from 3-11 and the regularization parameter \(\sigma = \Delta x \text{ ranges from 1.0-2.5} \text{ h} \) where \(\Delta x = 1/N-1\). Table 5 shows convergence of the obtained fundamental frequency to the exact ones (Khdeir, 1988; Wu and Chen, 1994; Matsunaga, 2000; Zhou et al., 2010; Korkmaz and Dag, 2011) over grid size \(\geq 5*5*5\), bandwidth \(\geq 3\) and regularization parameter \(\sigma = 2\Delta x\).

Table 6-8 insist that the obtained results from DQ schemes are more accurate than that of state space DQM (Zhou et al., 2010; Korkmaz and Dag, 2011; Zhang et al., 2006). Further, execution time of this scheme is the least.

Therefore, DSCDQM-RSK scheme is the best choice among the examined quadrature schemes for vibration analysis of piezoelectric composite materials. Also, Table 7 shows the convergence of normalized frequencies at total thickness \(h_c = 0.04\). As well as for different boundary conditions Table 8 ensures that DSCDQM-RSK scheme is the best choice for free vibration analysis of piezoelectric composite materials.

Furthermore, a parametric study is introduced to investigate the influence of elastic, geometric characteristics of the composite and type of material on
Table 1: Material property for composite piezoelectric plate (Feri et al., 2015)

<table>
<thead>
<tr>
<th>Material property</th>
<th>Young’s modulus (GPa)</th>
<th>Shear modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective elastic (GPa)</td>
<td>E2</td>
<td>E1</td>
<td>E3</td>
<td>G12</td>
</tr>
<tr>
<td>Sensor PZT-4</td>
<td>139</td>
<td>78</td>
<td>74</td>
<td>139</td>
</tr>
<tr>
<td>Sensor BaTiO3</td>
<td>166</td>
<td>77</td>
<td>78</td>
<td>166</td>
</tr>
<tr>
<td>Actuator Ba2NaNb5O15</td>
<td>129</td>
<td>104</td>
<td>5</td>
<td>247</td>
</tr>
<tr>
<td>Actuator PZT-5A</td>
<td>121</td>
<td>77</td>
<td>77</td>
<td>121</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material property</th>
<th>Dielectric constants (ε/m²)</th>
<th>Dielectric constants *10^9</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor PZT-4</td>
<td>-5.2</td>
<td>-5.2</td>
<td>15.1</td>
</tr>
<tr>
<td>Sensor BaTiO3</td>
<td>-4.4</td>
<td>-4.4</td>
<td>18.6</td>
</tr>
<tr>
<td>Actuator Ba2NaNb5O15</td>
<td>-5.4</td>
<td>-5.4</td>
<td>15.8</td>
</tr>
</tbody>
</table>

Table 2: Comparison between the normalized fundamental frequencies by using SincDQM, grid size N and the previous exact and numerical ones for simply supported square plate (a/h = 5, h/b = 25)

<table>
<thead>
<tr>
<th>Results/Normalized frequencies (N)</th>
<th>α1</th>
<th>α2</th>
<th>α3</th>
<th>α4</th>
<th>α5</th>
<th>α6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinc DQM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.682829</td>
<td>11.85211</td>
<td>17.16022</td>
<td>34.21303</td>
<td>68.96996</td>
<td>78.75195</td>
</tr>
<tr>
<td>5</td>
<td>10.682829</td>
<td>11.85856</td>
<td>17.16215</td>
<td>34.21560</td>
<td>68.94715</td>
<td>78.76118</td>
</tr>
<tr>
<td>7</td>
<td>10.68214</td>
<td>11.80986</td>
<td>17.16001</td>
<td>34.22266</td>
<td>68.98614</td>
<td>78.78893</td>
</tr>
<tr>
<td>9</td>
<td>10.68214</td>
<td>11.80866</td>
<td>17.16001</td>
<td>34.22266</td>
<td>68.98614</td>
<td>78.78893</td>
</tr>
<tr>
<td>11</td>
<td>10.68214</td>
<td>11.80866</td>
<td>17.16001</td>
<td>34.22266</td>
<td>68.98614</td>
<td>78.78893</td>
</tr>
<tr>
<td>SSDQM (Zhou et al., 2010) N = 7</td>
<td>10.68218</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSDQM (Korkmaz and Dag, 2011) N = 11</td>
<td>10.68221</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Individual layer plate theory (Cho et al., 1993)</td>
<td>10.673</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two dimensional local (Wu and Chen, 1994)</td>
<td>10.682</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global higher order theory (Matsuura, 2000)</td>
<td>10.6876</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Execution time (sec)</td>
<td>5.691851 over N = 7**5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Comparison between the normalized fundamental frequency by using DSCDQM-DLK, band width (2M+1) and grid size N for simply supported square plate (a/h = 25, a/h = 25)

<table>
<thead>
<tr>
<th>Fundamental frequency (α)</th>
<th>DSCDQM-DLK</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10.87465</td>
</tr>
<tr>
<td>5</td>
<td>10.65705</td>
</tr>
<tr>
<td>7</td>
<td>10.65705</td>
</tr>
<tr>
<td>9</td>
<td>10.65705</td>
</tr>
</tbody>
</table>

Table 4: Comparison between the normalized frequencies by using DSCDQM-DLK, grid sizes N and the previous exact and numerical ones for simply supported square plate (2M+1= 5, a/h = 25, a/h = 25)

<table>
<thead>
<tr>
<th>Normalized frequencies/</th>
<th>DSCDQM-DLK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results/N</td>
<td>α1</td>
</tr>
<tr>
<td>Sinc DQM</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.91236</td>
</tr>
<tr>
<td>5</td>
<td>10.52187</td>
</tr>
<tr>
<td>7</td>
<td>10.68214</td>
</tr>
<tr>
<td>9</td>
<td>10.68214</td>
</tr>
<tr>
<td>Analytical (Korkmaz and Dag, 2011)</td>
<td>10.68214</td>
</tr>
<tr>
<td>SSDQM (Zhou et al., 2010) N = 7</td>
<td>10.68218</td>
</tr>
<tr>
<td>SSDQM (Korkmaz and Dag, 2011)</td>
<td>10.68221</td>
</tr>
<tr>
<td>N = 11</td>
<td></td>
</tr>
<tr>
<td>Individual layer plate theory (Cho et al., 1993)</td>
<td>10.67300</td>
</tr>
<tr>
<td>Two dimensional local (Wu and Chen, 1994)</td>
<td>10.68200</td>
</tr>
<tr>
<td>Global higher order theory (Matsuura, 2000)</td>
<td>10.87600</td>
</tr>
<tr>
<td>Execution time (sec)</td>
<td>5.592470 over N = 7**5</td>
</tr>
</tbody>
</table>
Table 5: Comparison between the normalized fundamental frequency by using DSCDQM-RSK, band width (2M+1), regularization parameter \( \sigma \) and grid size \( N \) for simply supported square plate \((h/h_p = 25, a/h = 5)\)

<table>
<thead>
<tr>
<th>Regularization parameter (2M+1)</th>
<th>( \sigma = 1^{st}hx )</th>
<th>( \sigma = 1.5^{st}hx )</th>
<th>( \sigma = 2^{nd}hx )</th>
<th>( \sigma = 2.5^{nd}hx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10.01106</td>
<td>10.27275</td>
<td>10.31357</td>
<td>10.16323</td>
</tr>
<tr>
<td>5</td>
<td>10.32780</td>
<td>10.69052</td>
<td>10.45065</td>
<td>10.51694</td>
</tr>
<tr>
<td>7</td>
<td>10.33925</td>
<td>10.48258</td>
<td>10.54917</td>
<td>10.54542</td>
</tr>
<tr>
<td>9</td>
<td>10.33436</td>
<td>10.52011</td>
<td>10.64838</td>
<td>10.68571</td>
</tr>
<tr>
<td>11</td>
<td>10.33436</td>
<td>10.52011</td>
<td>10.63909</td>
<td>10.68571</td>
</tr>
<tr>
<td>3</td>
<td>10.00594</td>
<td>10.07620</td>
<td>10.68214</td>
<td>10.68214</td>
</tr>
<tr>
<td>5</td>
<td>10.05511</td>
<td>10.66973</td>
<td>10.68214</td>
<td>10.68214</td>
</tr>
<tr>
<td>7</td>
<td>10.12542</td>
<td>10.29284</td>
<td>10.68214</td>
<td>10.68214</td>
</tr>
<tr>
<td>9</td>
<td>10.32286</td>
<td>10.59687</td>
<td>10.68214</td>
<td>10.68214</td>
</tr>
<tr>
<td>11</td>
<td>10.32286</td>
<td>10.59687</td>
<td>10.68214</td>
<td>10.68214</td>
</tr>
</tbody>
</table>

Table 6: Comparison between the normalized frequencies by using DSCDQM-RSK, grid sizes \( N \) and the previous exact and numerical ones for simply supported square plate \((2M+1 = 3, \sigma = 2^{nd}hx, h/h_p = 205, a/h = 5)\)

<table>
<thead>
<tr>
<th>Normalized frequencies</th>
<th>( N ) = 5</th>
<th>( N ) = 5</th>
<th>( N ) = 5</th>
<th>( N ) = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSCDQM-RSK</td>
<td>( \tilde{\alpha} )</td>
<td>( \tilde{\alpha} )</td>
<td>( \tilde{\alpha} )</td>
<td>( \tilde{\alpha} )</td>
</tr>
<tr>
<td>3</td>
<td>10.31357</td>
<td>11.48038</td>
<td>17.46617</td>
<td>34.57366</td>
</tr>
<tr>
<td>5</td>
<td>10.68214</td>
<td>11.89086</td>
<td>17.16601</td>
<td>34.22266</td>
</tr>
<tr>
<td>7</td>
<td>10.68214</td>
<td>11.89086</td>
<td>17.16601</td>
<td>34.22266</td>
</tr>
<tr>
<td>9</td>
<td>10.68214</td>
<td>11.89086</td>
<td>17.16601</td>
<td>34.22266</td>
</tr>
<tr>
<td>Analytical Korkmaz and</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td></td>
<td>10.68214</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>Dag (2011) SSDQM (Zhou et al., 2010) ( N = 7 )</td>
<td>10.68218</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>SSDQM (Korkmaz and</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>Dag, 2011 ( N = 11 )  Individual plate theory</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>Cho et al. (1991) Two dimensional local</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>(Wu and Chen, 1994) Global higher order theory</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
<tr>
<td>(Ma et al., 2009)</td>
<td>10.6876</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
<td>( \bar{\alpha} )</td>
</tr>
</tbody>
</table>

Execution time (sec) 1.915793 over \( N = 5^5 \times 5 \)

the values of natural frequencies. Table 9 and Fig. 2-8 show that the natural frequencies increase with increasing side to thickness ratio \((a/h)\), Young's modulus gradation ratio, \((E1/E2)\), shear modulus gradation ratio \((G13/G12)\) and number of layers.

Figure 4 shows the natural frequencies decrease with decreasing the piezoelectric layer thickness \((h/h_p)\). Further, Fig. 6 show the natural frequencies decrease with increasing the aspect ratio \((a/b)\) at different values of \((a/h)\) (Fig. 7-9).
Table 7: Comparison between the normalized frequencies, thickness (h) and the previous exact and numerical ones for simply supported rectangular plate. (a/h = 4, a/b = 0.005)

<table>
<thead>
<tr>
<th>Result</th>
<th>a1</th>
<th>a2</th>
<th>a3</th>
<th>a4</th>
<th>a5</th>
<th>a6</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSCDM-Q-RSK</td>
<td>N = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>2.2428</td>
<td>10.5956</td>
<td>24.1111</td>
<td>42.5443</td>
<td>49.8596</td>
<td>60.4815</td>
</tr>
<tr>
<td>0.02</td>
<td>2.2453</td>
<td>10.5135</td>
<td>24.1187</td>
<td>41.5900</td>
<td>49.5002</td>
<td>60.3542</td>
</tr>
<tr>
<td>0.03</td>
<td>2.2604</td>
<td>10.1289</td>
<td>24.2830</td>
<td>41.4257</td>
<td>49.5221</td>
<td>59.8660</td>
</tr>
<tr>
<td>0.05</td>
<td>2.2604</td>
<td>10.0485</td>
<td>24.0968</td>
<td>41.2223</td>
<td>49.5218</td>
<td>59.8449</td>
</tr>
<tr>
<td>0.04</td>
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<td>10.0421</td>
<td>24.0968</td>
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<td>59.8447</td>
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<td>0.05</td>
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<td>41.1223</td>
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<td>59.8447</td>
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<tr>
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<td>10.0877</td>
<td>24.0888</td>
<td>41.6633</td>
<td>49.5111</td>
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<tr>
<td>Liew (Zhang et al., 2006)</td>
<td>2.2630</td>
<td>10.0896</td>
<td>23.7761</td>
<td>40.4813</td>
<td>48.5155</td>
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<td>h = 0.05, N = 7</td>
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<tr>
<td>Liew (Zhang et al., 2006)</td>
<td>2.2630</td>
<td>10.0896</td>
<td>23.7761</td>
<td>40.4813</td>
<td>48.5155</td>
<td>—— ——</td>
</tr>
</tbody>
</table>

Table 8: Comparison between the natural frequencies, different boundary conditions and the previous numerical ones. (h/h = 25, a/b = 2, a/h = 10)

<table>
<thead>
<tr>
<th>BCS/Results</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
<th>Q6</th>
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<tr>
<td>SSSS</td>
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<td>0.0672</td>
<td>0.0753</td>
<td>0.0962</td>
<td>0.1045</td>
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<td>DSCDM (Korkmaz and Dag, 2011)</td>
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<td>0.0533</td>
<td>0.0678</td>
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<tr>
<td>CCCCC</td>
<td>0.0577</td>
<td>0.0816</td>
<td>0.113</td>
<td>0.2538</td>
<td>0.3528</td>
<td>0.4523</td>
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<td>0.0377</td>
<td>0.0802</td>
<td>0.110</td>
<td>——</td>
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<td>SCSC</td>
<td>0.0296</td>
<td>0.0534</td>
<td>0.0807</td>
<td>0.1253</td>
<td>0.1795</td>
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<td>0.0878</td>
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<tr>
<td>CCF</td>
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<td>0.0193</td>
<td>0.0256</td>
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Fig. 2: Variation of the fundamental frequency with Young's modulus(E1/E2), different boundary conditions and different materials for square plate (h/h = 45, a/h = 10); a) Actuator is Ba2Na Nb5O15 and b) Actuator is PZT-5A

Fig. 3: Variation of fundamental frequency with shear modulus(G13/G12), different boundary conditions and different materials of square plate (h/h = 45, a/h = 10); a) Sensor is PZT-4 actuator is Ba2Na Nb5O15 and b) Sensor is BaTiO3 actuator is PZT-5A
Fig. 4: Variation of fundamental frequency with composite layer thickness to piezoelectric thickness ratio $E_1/E_2 = 25$; a) $a/h = 5$ and b) $a/h = 10$

Fig. 5: Variation of fundamental frequency with side to thickness ratio $(a/h)$, different boundary conditions and different materials for square plate ($E/E_1 = 25$) a) Sensor is BaTiO$_3$, actuator is Ba$_2$Na$_5$Nb$_{2}O_{15}$ b) sensor is BaTiO$_3$, actuator is PZT-5A

Fig. 6: Variation of fundamental frequency with aspect ratio $(a/b)$, different values of $a/h$ and different boundary conditions for square plate $(h/h_1 = 25, E_1/E_2 = 25)$; a) SSSS, b) CSCS, c) SCSC and d) CCCF
Fig. 7: Variation of fundamental frequency with number of layers, different boundary conditions and different materials of square plate (h/h = 45, a/h = 5): a) Sensor is PZT-4, actuator is Ba2NaNb5O15 and b) Sensor is BaTiO3, actuator is PZT-5A

Fig. 8: a, b) Variation of fundamental frequency with Young's modulus(E1/E2), shear modulus(G13/G12) and different number of layers for different materials (sensor is BaTiO3, and actuator is PZT-5A) of square plate (h/h = 45, a/h = 10)

Fig. 9: Variation of normalized mode shape W with time for first three modes at different boundary conditions for square plate with total thickness 0.04 (h/h = 25, E1/E2 = 25, a/h = 10): a) SSS; b) CCC; c) SCSC and d) CCCF
Moreover, the natural frequencies remain constant when \( h/h_p > 20 \) in all different edge conditions. Figure 2-3 and 5 show the effect of material type on the natural frequency. It is seen that, the sensor is more significant than actuators. The natural frequency is almost unchanged when \( a/h > 40 \). Also, the influence of BaTiO₃ material is more affected on the natural frequencies than PZT-4 material. Furthermore, Fig. 9 shows the first three mode shapes of normalized transverse displacement \( \omega_t \). From previous figure, it is seen that, the normalized transverse displacement is maximum for the CCCF plate and is minimum for the CCCC plate.

**CONCLUSION**

Two different quadrature schemes have been successfully applied for free vibration analysis of piezoelectric composite materials. A MATLAB program is designed for each one such that the maximum error (comparing with the previous exact results) is \( \leq 10^{-8} \). Also, execution time for each scheme is determined. It is concluded that discrete singular convolution differential quadrature method based on regularized Shannon kernel (DSCDQM-RSK) with grid size \( 5 \times 5 \times 5 \), bandwidth \( 2M+1 \times 3 \) and regularization parameter \( \sigma = 2h \) leads to best accurate efficient results for the concerned problem (three layered piezoelectric composite with total thickness \( = 0.04 \)). Based on this scheme, a parametric study is introduced to investigate the influence of elastic, geometric characteristics of the composite and type of material of the vibrated plate, on results. The thinner composite has larger frequencies more than thick one. Composite plate with CCCC conditions has minimum normalized transverse displacement where as it has maximum value for CCCF conditions. Further, it is aimed that these results may be useful for piezoelectric dampers as a part of smart structures system for buildings.

**REFERENCES**


