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Representation of Half Wing of Butterfly and Hamiltonian Circuit for Complete Graph using Starter Set Method

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Abstract: Generating distinct circuits from complete graphs have been an active study due to vast applications particularly in computer science. Several interesting and challenging methods have used to formulate distinct circuits but this research is motivated through the process of wings movement in butterfly to develop distinct circuits from complete graphs. The beauty of method is using the concept of mirror image of the wings movement and this method in called Half Butterfly Method (HBM). The advantage of HBM is a visualisation of half wing of butterfly. Inspired by this research, a new idea come out with the generating half wing of butterfly and distinct Hamiltonian circuit via. starter sets method under exchanged strategy. This new method did not use wing strategy to develop half wing of butterfly and Hamiltonian circuit. We provide an example for case n = 4 and 5 to exemplify the algorithms. Furthermore, the calculation for order of complexity is also presented for generating distinct half wing and hamiltonian circuit using starter set method.

Key words: Half Butterfly method, complete graph, starter sets method, distinct circuit, butterfly, formulate, movement

INTRODUCTION

In this study, we shall follow the standard ideas and definitions of a graph. We shall define a distinct circuit is a closed loop on a graph where every node (vertex) is visited exactly once. This kind of circuit is called as Hamiltonian Circuit (HC). We denote the complete graph as K_n.

The research on decomposing a graph in HC is a thought-provoking process due to the fact that the number of HC for any graph with n vertices is n (Riaz and Khiyal, 2006). Numerous study focused on developing fast algorithms for finding HC (Riaz and Khiyal, 2006; Babar *et al*., 2006; Chalaturnyk, 2008). In 2017, a research done by Darus *et al.* (2017)'s proposed the idea of theoretical of decomposing K_n into distinct HC. The method that is been used is called Half Butterfly Method (HBM). This HBM was inspired from Gopal *et al*. (2007) in solving bipartite graph where they applied shift and rotate strategy and the butterfly strategy. The advantage of HBM is visualisation of half wing of butterfly. Wing strategy is a main step in HBM. Other advantage is from visualization of distinct circuit, we easily list all n! permutation by applying concept of mirror image. In the reseacrh by Darus *et al.* (2017), they proposed the following.

Theorem 1: Let, G be a complete graph. Then G is decomposable into (n-1)!/2 distinct Hamiltonian circuits with different paths for all n≥3. This theorem applied concept of arrangement distinct element and map the element into circuit to obtain HC. With same process, circuit can be represented as (n-1)/2 distinct half wings.

Thus in this study, we provide a new algorithm using starter sets method under exchange restriction to visualize the graphic representation of distinct HC and half wing. We will use the, example, for case n = 5 to illustrate the algorithms. In starter sets algorithm, we attempt to fix first element using starter sets method to generate distinct HC in complete graph and half wings for any n≥4. The starter sets method was introduced by Ibrahim *et al.* (2010) and later was improved by Karim *et al.* (2011). The improvement and application of starter sets method can

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be found by Karim *et al.* (2011). All implementation was for generating permutation. We start with some basic terminologist and definition where needed along this study.

MATERIALS AND METHODS

Preliminary definitions: The following definitions will be used throughout this study.

Definition 1: Let K_n = (V, E) A function g = $\begin{pmatrix} x_1 & x_2 & x_3 & ,... & x_n \\ x_j & x_{j+1} & x_{j+2} & ,... & x_{j+n} \end{pmatrix}$ maps the vertices $x_i \rightarrow x_j$ where x_i, x_j ϵV and $1 \leq$ I, $j \leq n$.

Example 1: Let, g = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, Then 1→2, 2→3, 3→4 and 4→1. Thus, we write the mapping for g as (1, 2) (2, 3) (3, 4) (4, 1).

Definition 2: Let, A be a circuit with direction (1, 2, 3, ..., n-1, n, 1). Then, a circuit B is the mirror image of A, if the direction of B is (n, n-1, ..., 3, 2, 1, n).

Definition 3: Let, A and B be two circuits with n vertices. If B is the mirror image of A then B is isomorphic to A.

Example 2:

Definiton 4: A starter set is a set that is used as a basis to enumerate other permutation.

Definition 5: The i-exchange is a restriction of element in i th position, $2 \leq i \leq n-2$ exchange to the right starting with element in n-2 position until 2 th position recursively, for permutation.

Example 1: Let, a permutation (1 2 3 4 5), thus, $2 \leq i \leq 3$ which i = 2 and 3 only. We start with select element in (n-3)th position, i.e., element '3' in 3rd. Thus we exchange '3' to the right until its located at (n-1) th position:

Starter set 1: (1 2 3 4 5)
Starter set 2: (1 2 4 3 5)
Starter set 3: (1 2 4 5 3)

Next we select element in 2nd position, i.e., element '2'.

Starter set 1	Starter set 2	Starter set 3
(1 2 3 4 5)	(1 2 4 3 5)	(1 2 4 5 3)
(1 3 2 4 5)	(1 4 2 3 5)	(1 4 2 5 3)
(1 3 4 2 5)	(1 4 3 2 5)	(1 4 5 2 3)
(1 3 4 5 2)	(1 4 3 5 2)	(1 4 5 3 2)

The process is highly dependence to previous circuit. In other word, it is done under recursive way. In next study, we demonstrate procedure to generate three and 12 distinct half wing for case n = 4 and 5, respectively, using starter sets method under exchange restriction.

RESULTS AND DISCUSSION

Starter sets method: Basically, here are 3 main steps such as follows:

- Step 1: Fixing the 1st element under permutation
- Step 2: Generating the (n-1)!/2 distinct circuits using i the exchanged restriction
- Step 3: Finding the mapping
- Step 4: Drawing the circuit as Hamiltonian circuit/half wing of butterfly

Let, start with n = 4

Step 1: Fixing the 1st element under permutation (1 2 3 4)

Step 2: Generating 3 distinct starter sets $2 \leq i \leq 2$. Thus, i = 2 only. We exchange element in 2nd place, '2'. Then we get 3 distinct starter sets as follows:

(1 2 3 4)
(1 3 2 4)
(1 3 4 2)

Step 3 and 4: Finding the mapping and drawing the HC and half wing for n = 5:

Step 1: Fixing the 1st element under permutation:

(1 2 3 4 5)

Step 2: Generating 12 distinct starter sets, $2 \leq i \leq 3$. start with i = 3, three distinct starter sets were obtained by selecting element in 3rd position to (n-1) th position:

1st starter set, S_1 : (1 2 3 4 5)
2nd starter set S_2: (1 2 4 3 5)
3rd starter set, S_3 : (1 2 4 5 3)

Table 1: HC and its wing for K4

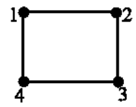
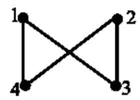
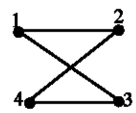
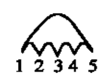
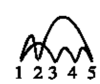
Starter set	Mapping	HC	Half wing
(1 2 3 4)	1→2→3→4→1 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$		
(1 3 2 4)	1→3→2→4→1 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$		
(1 3 4 2)	1→3→4→2→1 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$		

Table 2: The 12 distinct states sets

S_1	S_2	S_3
(1 2 3 4 5)	(1 2 4 3 5)	(1 2 4 5 3)
(1 3 2 4 5)	(1 4 2 3 5)	(1 4 2 5 3)
(1 3 4 2 5)	(1 4 3 2 5)	(1 4 5 2 3)
(1 3 4 5 2)	(1 4 3 5 2)	(1 4 5 3 2)

Table 3: Half wing from S_1

S_1	Mapping	Half wings
(1 2 3 4 5)	1→2→3→4→5→ $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$	
(1 3 2 4 5)	1→3→2→4→5→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$	
(1 3 4 2 5)	1→3→4→2→5→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$	
(1 3 4 5 2)	1→3→4→5→2→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}$	

Next for i = 2, we choose element in 2nd position to move the right to (n-1) position recursively. After we obtained 12 distinct starter sets, the process is stop.

Step 3 and 4: Finding the mapping and drawing distinct half wing (we give example for S_1 in Table 1 and 2 only). From Table 3, we also can produce distinct Hamiltonian circuit.

Table 3 and 4 show the similar process of mapping but different result in term of visualizations. For half wing representation, the elements is arrange as a linear order and then do the connection based on mapping direction. Meanwhile for HC, we referred to complete graph representation and draw the edges among the vertex.

The crucial part in this algorithm is to generate all distinct starter set where the total is (n-1)!/2. The exchange restriction is chosen due to its minimal operation compare to cycle operation. For example.

Under exchange operation, only two elements are alternated for any n. For cycle operation (n-1) elements

Table 4: Hamiltonain circuit from S_1

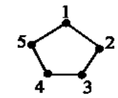
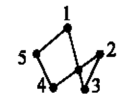
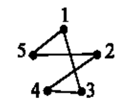
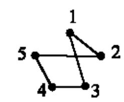
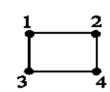
S_1	Mapping	Hamiltonian circuit
(1 2 3 4 5)	1→2→3→4→5→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$	
(1 3 2 4 5)	1→3→2→4→5→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 2 & 5 & 1 \end{pmatrix}$	
(1 3 4 2 5)	1→3→4→2→5→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$	
(1 3 4 5 2)	1→3→4→5→2→1 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 1 \end{pmatrix}$	

Table 5: Difference between exchanged restriction and cycle operation

Exchanged restriction	Cycle operation
(1 2 3 4 1)	(1 2 3 4 1)
(1 3 2 4 1)	(1 3 4 2 1)
(2 and 3 exchange)	(3 4 2-cycled)
(1 3 4 2 1)	(1 4 2 3 1)
(2 and 4 exchange)	(4 2 3-cycled)

Table 6: The equivalence starter set and their visualization

Starter set	Equivalence starter sets	Half wing	HC
(1 2 3 4)	(1 4 3 2)		

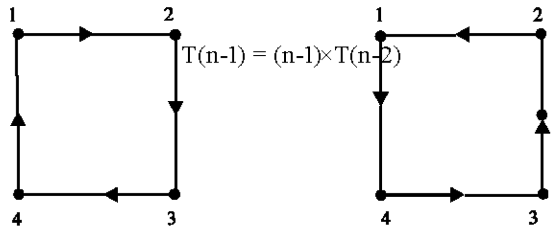
are to be cycled. However, both restrictions are dependent on the previous results. Furthermore, cycle operation can be used for starter sets method. The advantage of our starter sets method is it did not generating the equivalence starter sets. It's already proven on our previous studies. Let demonstrate the equivalence starter set and their visualization. In next study, we present the complexity of the algorithm for determining the distinct half wing and Hamiltonian circuit using starter sets method under exchange strategy.

Complexity of algorithm: Combinatorial algorithm related to permutation generation always fall under non polynomial times. In other word, n is bigger, the time computation become longer. Even we proposed new algorithm, we have to show the complexity of producing the starter sets. Given the implementation of starter sets procedure which contains one recursive call as follows Generate initial permutation as random (Table 5 and 6):

Starter (K)

```
1.  BEGIN
2.  IF (K=2)
3.  FOR (i = 1 to n)
4.  DO mapping and Print the half wing/Hamiltonian circuit
5.  ENDFOR
6.  RETURN
7.  ENDIF
8.  TEMP = K-1
9.  FOR (i = temp  to n) DO
10. OLD ←NUM[i]
11. NUM[i]←NUM[i+1]
12. NUM[i+1]←OLD
13. STARTER (K)
14. ENDFOR
15. END PROCEDURE
```

The critical study is lines 9-14. There is one loop and in that loop, there is recursive call on less (temp gets smaller). The time process of exchanging two elements in line 10-12 is constant. So, the time complexity is defined by a recurrence equation is (Fig. 1):



$T(n-1) = (n-1)\times T(n-2)$

Fig. 1: Circuit B ia a mirror image of cicuit A

This recurrence when expanded gives:

$$T(n) = (n-1)\times(n-2)\times,\ldots\times3\times2 = (n-1)!$$

Thus order of complexity for our program is estimates as O((n-1)!) where its non polynomial time.

CONCLUSION

Therefore, it is proven that the distinct half wing and HC with different path can be produced using distinct starter sets under exchange restrictions. We can't concluded this is only a method but its put an alternative method to visualize the HC and half wing. The difference of our method is we did not produce a similar half wing and HC. For our future work, we will explore in term of factorial number system to represent the half wing and HC.

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REFERENCES

Babar, G.M., M.S.H. Khiyal and A. Saeed, 2006. Finding hamilton circuit in a graph. Proceedings of the 2006 International Conference on Scientific Computing (CSC 2006), June 26-29, 2006, Las Vegas, Nevada, pp: 238-244.

Chalaturnyk, A., 2008. A fast algorithm for finding Hamilton cycles. MSc Thesis, University of Manitoba, Winnipeg, Canada.

Darus, M.M., H. Ibrahim and S. Karim, 2017. Some new theoretical work on half butterfly method for hamiltonian circuit in complete graph. Mat., 33: 113-118.

Gopal, A.P., K. Kothapalli, V.C. Venkaiah and C.R. Subramaniam, 2007. Various one-factorizations of complete graphs. Theor. Algorithmic Res., 1: 1-12.

Ibrahim, H., Z. Omar and A.M. Rohni, 2010. New algorithm for listing all permutations. Mod. Appl. Sci., 4: 89-94.

Karim, S., H. Ibrahim and H. Sapiri, 2015. A new method for starter sets generation by fixing any element. Proceedings of the 2015 International Conference on Research and Education in Mathematics (ICREM7), August 25-27, 2015, IEEE, Kuala Lumpur, Malaysia, ISBN:978-1-4673-7506-1, pp: 120-123.

Karim, S., H. Ibrahim, Z. Omar and H. Sapiri, 2016. A new starter sets generation method by fixing any element under exchange restriction. Intl. J. appl. Math. Stat., 54: 132-141.

Karim, S., Z. Omar and H. Ibrahim, 2011. Integrated strategy for generating permutation. Intl. J. Contemp. Math. Sci., 6: 1167-1174.

Riaz, K. and M.S.H. Khiyal, 2006. Finding hamiltonian cycle in polynomial time. Inform. Technol. J., 5: 851-859.