Journal of Food Technology 5 (2): 139-142, 2007

Mathematical Modelling of the Garification of Fermented Cassava Mash

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Abstract: The drying kinetics of fermented cassava mash (gari), during the falling rate period, was studied in three dimensions, with respect to the progression of a drying front that has its geometry evolving axis-symmetrically in a direction towards the center of the particle and progresses at the same rate as the rate of removal of moisture from the particle. The model equation based on Fick's second law was solved numerically in spherical coordinates and the average moisture content for each incremental interval (in dimensionless terms) plotted against dimensionless time values. The moisture content versus time curve had a good fit ($R^2 = 0.9997$) with a model of the general form, MR = A exp ($-kt^n$). The model was validated by using it to fit kinetic data from the drying of cassava particulate in a fluidized bed at 50°C.

Key words: Gari, gelatinization, drying, fermented cassava mash, fick's equation

INTRODUCTION

Cassava is one of the major root tubers produced in the forest zones of Nigeria, with yields as high as 38 million metric tones of cassava per annum (Nwosu, 2006). In Nigeria, over 70% of the cassava yield is processed into gari a convenient, storage cupboard, staple food (Sanni and Olubamiwa, 2003).

The process involves roasting-alternate cooking and drying-of fermented cassava mash, causing the starch in it to gelatinize. During this process, the fermented cassava mash is dried, at a temperature above 70°C and from a moisture content of 45-53% to a moisture content of about 12% or less, for white gari and 10% or less, for yellow gari, to make it suitable for storage (Ajibola *et al.*, 1987; Sefa-Dedeh and Plange, 2001).

The drying of cassava is characterized by a short constant-rate drying regime, followed by a falling-rate regime. The later regime follows largely a diffusion mechanism of internal moisture movement (Akeredolu *et al.*, 2003). Literature is rife with drying kinetics data on cassava slabs and chips (Chirife, 1971; Igbeka, 1982; Akeredolu, 1987; Akeredolu *et al.*, 2003; Okpala *et al.*, 2003) in comparison to cassava particulate.

This research develops a theoretical model in three dimensions, which could be used to study the drying kinetics of fermented cassava mash during the falling rate period, based on gelatinization.

Theoretical base for the model: The basic kinetics in the drying of fermented cassava mash is that of thermal gelatinization in a limited water environment.

This idea explains the difference in the behaviour of gari in the presence of cold and hot water. Gari is insoluble in cold water but solubilizes in hot water to form a viscous paste that has adhesive properties.

Thus, at temperatures above the gelatinization temperature and in the presence of sufficient water, the gelatinization process that started during drying is completed.

Based on the above, some assumptions suitable for thermal gelatinization can be adapted for the kinetic study of the drying of fermented cassava mash.

Mathematical modelling: In order to successfully model the drying rate of fermented cassava mash (gari) during the falling rate period, a few assumptions have to be made. These assumptions, which are based mostly on literature review, will aid in the derivation of a mathematical model for the drying rate, which can be compared with existing models or matched with experimental data in an attempt to validate or otherwise reject the assumptions.

- The mechanism of the drying process is predominantly controlled by liquid diffusion due to concentration gradient, such that the moisture content alone models the drying rate.

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- The particles of the fermented cassava mash are assumed spherical in shape, after the process of sieving.
- The fermented cassava mash has uniform moisture content.
- The drying process moves on a uniform front towards the direction of higher moisture content-the direction of decreasing radius, that is, towards the center of the particle-with the geometry of the drying front evolving axis-symmetrically.

This idea is based on thermal gelatinization (Hayes *et al*., 2004).

- The resultant combined effect of particle agglomeration and shrinkage during drying is negligible (Hayes *et al*., 2004).

Based on the assumption that the moisture content alone models the drying rate, Fick's second law in 3 dimensions can be written as:

$$\frac{\partial \omega}{\partial t} = D\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2}\right) \tag{1}$$

The assumption that the fermented cassava mash particles are spherical in shape after sieving, allows us to apply the transformation for spherical coordinates;

$$x = r\sin(\phi)\cos(\theta) \tag{2}$$

$$y = r\sin(\phi)\sin(\theta) \tag{3}$$

$$z = r\cos(\phi) \tag{4}$$

The Fick's equation in three dimensions, in spherical coordinates, reduces to:

$$\omega_t = \frac{D}{r^2}\left(\begin{array}{l} 2r\omega_r + r^2\omega_{rr} + \dfrac{\sin(\phi)}{\cos(\phi)}\omega_\phi + \\ \omega_{\phi\phi} + \dfrac{\omega_{\theta\theta}}{\sin^2(\phi)} \end{array}\right) \tag{5}$$

Applying the assumption that the drying front evolves axis-symmetrically, that is, independent of ϕ and θ, gives;

$$\frac{\partial \omega}{\partial \phi} = \frac{\partial \omega}{\partial \theta} = 0 \tag{6}$$

Equation 5 becomes;

$$\frac{\partial \omega}{\partial t} = D\left(\frac{2}{r}\frac{\partial \omega}{\partial r} + \frac{\partial^2 \omega}{\partial r^2}\right) \tag{7}$$

Dimensionless model: If we let

$$t' = (tD/R) \tag{8}$$

$$r' = (r/R) \tag{9}$$

$$\omega' = \left(\frac{\omega - \omega_o}{\omega_f - \omega_o}\right) \tag{10}$$

Applying our dimensionless variables to Eq. 7, we have the Fick's equation in dimensionless form;

$$\frac{\partial \omega'}{\partial t'} = \frac{2}{r'}\frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r'^2} \tag{11}$$

Numerical solution:

$$\frac{\partial \omega'}{\partial t'} \cong \frac{\omega'_{m+1} - \omega'_m}{\Delta t'} \tag{12}$$

$$\frac{\partial \omega'}{\partial r'} \cong \frac{\omega'_{n+1} - \omega'_n}{\Delta r'} \tag{13}$$

$$\frac{\partial^2 \omega'}{\partial r'^2} \cong \frac{\omega'_{n+1} - 2\omega'_n + \omega'_{n-1}}{(\Delta r'^2)} \tag{14}$$

$$r' = k(\Delta r') \tag{15}$$

Substituting Eq. 12-15 into 11 and rearranging gives;

$$\omega'_{m+1,n} = \frac{(1+2/k)\,\omega'_{m,n+1} + (M - 2/k - 2)\,\omega'_{m,n} + \omega'_{m,n-1}}{M} \tag{16}$$

Where

$$M = (\Delta r')^2 / \Delta t' \tag{17}$$

Conditions for convergence and stability that are binding for this case can be given as;

$$1 + 2/k \geq 0 \text{ and } M - 2/k - 2 \geq 0$$

Stability and convergence conditions can be met for all positive real values of k but not for k = 0. At k = 0, Eq. 16 becomes indeterminate.

However, at r = 0, $\partial\omega'/\partial r' = 0$ because of symmetry and

$$\lim_{r'\to 0}\left(2/r\right)\left(\partial\omega'/\partial r'\right) = \left(2\partial^2\omega'/\partial r'^2\right)_{r'=0}$$

(Mickley *et al*., 1957).

Then at r′ = 0

$$\left(\frac{2}{r'}\frac{\partial\omega'}{\partial r'} + \frac{\partial^2\omega'}{\partial r'^2}\right)_{r'=0} = \frac{3\partial^2\omega'}{\partial r'^2} = \frac{6\omega'_{m,1} - \omega'_{m,0}}{\left(\Delta r'\right)^2} \qquad (18)$$

Substituting Eq. 12 and 18 into 11 gives;

$$\omega'_{m+1,0} = \frac{\omega'_{m,1} + \left(M-6\right)\omega'_{m,0}}{M} \qquad (19)$$

Equation 19 gives the numerical solution to the fick's Equation for k = 0, that is, at r′ = 0, while, Eq. 16 gives the solution of the fick's equation for $k \geq 0$.

Boundary conditions: Since we are considering the falling rate period, the surface of the particle is already dry, therefore;

At r′ = 1, ω′ = 1 for all time $t' \geq 0$

and at t′ = 0, ω′ = 0 for all r′< 1

RESULTS AND DISCUSSION

The numerical solutions obtained were computed for K = 10 and M = 10 and average values for each incremental time interval obtained. The dimensionless moisture content or Moisture Ratio (MR = 1-ω′) was plotted against dimensionless time (t′) and the values fitted with an exponential function. The plot and model obtained are shown (Fig. 1).

The model from the numerical solution obtained assumes the general form:

$$MR = A \exp(-kt^n) \qquad (20)$$

A practical application: Ogunleye (2003) studied on the garification of fermented and dewatered cassava particulate at 50°C in a fluidized bed. The data below was extracted from his research work (Table 1).

The first 30 min fall into a constant drying rate period and fit a linear model with $R^2 = 0.9978$. The falling rate period, which commences after the thirtieth minute, was modelled using the general form ; MR = A exp (-kt^n) to give the following values.

A = 1.9388, k = 0.0538 and n = 0.92, with $R^2 = 0.9927$

Table 1: Cassava particulate drying kinetics data

Time (mins)	0	10	20	30	40	50	60
MR	1	.832	.680	.544	.388	.288	.20
Time (mins)	70	80	90	100	110	120	
MR	.14	.096	.06	.04	.032	.028	

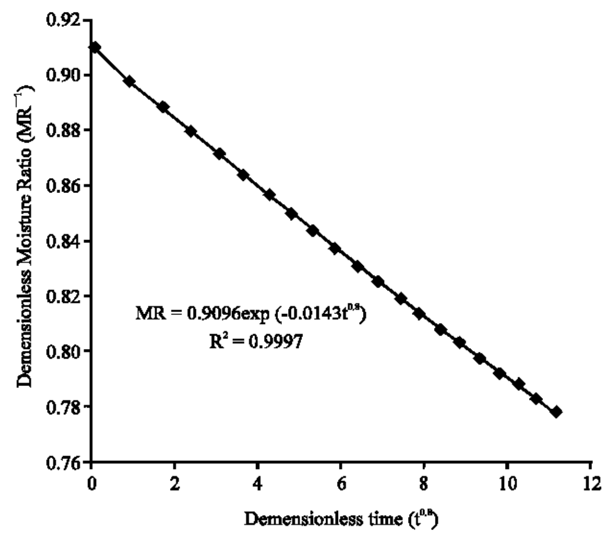

Fig. 1: Model of Moisture Content as a function of time

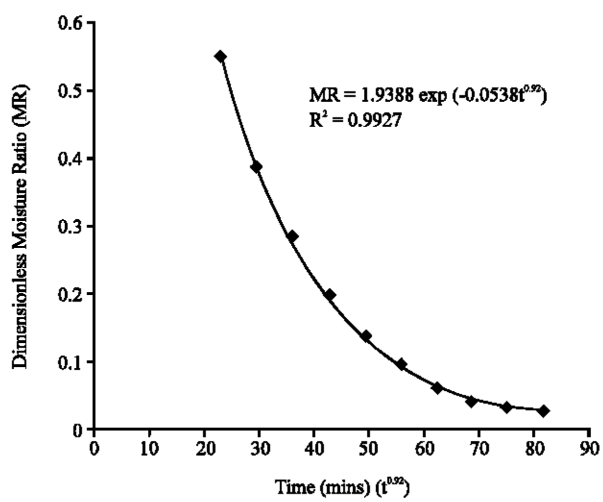

Fig. 2: Model of drying of cassava particulate at 50°C

CONCLUSION

Analysis of the experimental data above revealedv (Fig. 2) a good fit ($R^2 = 0.9924$) for n = 0.8, which is the value of n obtained from the numerical solution of Fick's second law. Thus, the value of n is not likely to vary considerably, such that an average value could be used instead.

Satimehin (2003) worked on a similar model for thin-layer drying of gelatinized white yam. The model had the value of A fixed as unity and values of n obtained ranged from 0.7721 to 0.9588. He studied the effect of process variables on the value of k using an average value of 0.92 for n.

This model can be used to study the effect of variables like humidity and temperature on the drying kinetics of fermented cassava mash, by studying how these variables affect the parameters in the model (A, k), with the value of n kept constant at an average value.

NOMENCLATURE

t	=	Time	s
D	=	Moisture diffusivity	$m^2\ s^{-1}$
r	=	Radius of spherical region	m
x	=	Length co-ordinate	m
y	=	Length co-ordinate	m
z	=	Length co-ordinate	m
R	=	Particle radius	m

Greek letters

ω	=	Moisture content	$g\ H_2 0\ g^{-1}$ dry matter
ω'	=	A measure of dryness	

Subscripts

O	=	Initial
f	=	Final

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