

Impact of Exercise on Diabetics Subjects

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Abstract: We revisit the generalized Mathematical model used for study of diabetes mellitus. We note the impact of different levels of physical activities on the stability of the disease. We observed that when we exercise the risk of becoming diabetics reduces.

Key words: Impact of exercise, diabetics , physical activities, mathematical model, stability of the disease

INTRODUCTION

Diabetes mellitus is composed of a heterogeneous group of disorders characterized by high blood levels. This disease is also characterized by polyuria, polydipsia and weight loss in spite of polyphagia, hyperglycemia and glucosuria. There are widespread biochemical abnormalities but the fundamental defects to which most of the abnormalities can be traced includes increase in liberation of glucose into the circulation from the liver and reduced entry of glucose into various peripheral tissue. (Ayeni and Adewale, 2003; Derrick and Grossman, 1976; Ganongy, 1999; Jennifer, 1998).

During exercise, the caloric needs of muscle are initially met by glucogenolysis in muscle and increased uptake of glucose. Plasma glucose initially rises with increase hepatic glucogenolysis but may fall with strenuous, prolonged exercise. After exercise liver glycogen is replenished by additional glyconeogenesis and a decrease in hepatic glucose output. Exercise has direct effects in carbohydrate metabolism the entry of glucose into skeletal muscle is increased during exercise in the absence of insulin. Its also increased insulin sensitivity of the muscle increase in sensitivity persists for several hours after exercise and regular exercise training can produce prolong increase in insulin sensitivity.

Exercise can precipitate hypoglycemia in diabetes not only because of the increase in muscle uptake of glucose but also because absorption of injected insulin is more rapid during exercise.

When the intensity of exercise is extremely high the plasma glucose level falls the first symptom are palpitations, sweating and nervousness due to autonomic discharge at lower plasma glucose levels. Later on, Neuroglycopenic symptoms begin to appear these include hunger as well as confusion and the other cognitive

abnormalities. At even lower plasma glucose level lethargy, coma, convulsions and eventually death occur. The onset of hypoglycemic symptoms calls for prompt treatment with glucose or glucose containing drinks such as orange juice.

Many scientists have studied the nature, characteristic and diagnosis of this disease; these studies are either experimental or theoretical. Davies in his study derived a mathematical model for the insulin dependent diabetes mellitus.

Harris (1988) investigated the nature of the disease, base on the study he was able to classify the disease into group. Ayeni *et al.* (2003) investigated the existence and uniqueness of Mbah generalized mathematical model of Davies. In this study we shall examine the impact of exercise on diabetic's subjects using the new generalized mathematical model.

MATHEMATICAL FORMULATION

According to Mbah (1998) the model is given as:

$$\frac{dx}{dt} = a_1 q e^{-k(t-t_0)} - a_2 xy - a_3 x \quad (1)$$

$$\frac{dy}{dt} = b_1 x - b_2 y + b_3 (\gamma t + \delta) \quad (2)$$

$$x(t_0) = x_0, y(t_0) = y_0$$

In his model, he used x and y to represent glucose and insulin, respectively, a_2, a_3, b_1 and b_2 are all constants associated with the mechanism which was determined experimentally.

In Eq. 1, a_2 measures the plasma membrane permeability of the glucose a_3 measures the body cell utilization of the plasma glucose.

Likewise in Eq. 2, b_1 measures the pancreatic response to glucose stimulation while b_2 measures the level of insulin degradation. The first expression in Eq. 1 denotes the quantity of glucose intake i.e. $a_1 q e^{-k(t-t_0)}$ and the last expression in Eq. 2 i.e., $b_3(\gamma t + \delta)$ was used to represent the quantity of insulin injected into the patients. Q is the quantity of initial glucose intake, k is the delay parameter associated with time taken to absorb this quantity of initial glucose intake in the blood we note that a_1 was not defined similarly b_3 , γ are not defined.

Following Mbah, we present a new generalized mathematical model for the study of diabetes mellitus as follows:

$$\frac{dc}{dt} = -a_1 c p - (a_2 + a_4)c + a_3 m w c e^{-q(t-t_0)} \quad (3)$$

$$\frac{dp}{dt} = b_1 c - b_2 p + b_3 m w p e^{-r(t-t_0)} \quad (4)$$

$c(t_0) = c_0, p(t_0) = p_0$

In this new mathematical model, c and p represent carbohydrate (glucose) and protein (insulin), respectively. In Eq. 3 a_1 measures the plasma membrane permeability of the carbohydrate into the cell in the presence of protein while a_2 measures the body cell utilization of the plasma carbohydrate in this model and a_3 is the concentration of glucose in carbohydrate foods intake. We denote the intensity of exercise with a_4 which measures the level of conversion of glycogen to glucose during exercise.

Also in Eq. 4, a_1 measures the pancreatic response to carbohydrate stimulation while b_2 measures the rate of protein degradation and b_3 is the concentration of insulin (Protein) intake.

We used $m w c e^{-q(t-t_0)}$ to represent the quantity of glucose intake. Where $m w c$ is the molecular weight of the initial carbohydrate intake; q is the delay parameters associated with time taken to absorb this quantity of carbohydrate in the blood and a_3 measures the concentration of the carbohydrate intake. Also, we used $m w p e^{-r(t-t_0)}$ to represent the quantity of protein intake. Where $m w p$ is the molecular weight of the initial protein injected. r is the delay parameter associated with time taken to absorb this quantity of initial protein intake.

We find that in excessive involvement in exercise the glucose in the blood gets easily depleted in this situation we find that body system must supply the required energy to be able to carry on with the body activities. To do this the stored glucose in the form of glycogen will be reconverted to glucose to restore the required blood glucose level. Therefore, in this model we denote the intensity of exercise with a_4 which measures the level of conversion of glycogen to glucose during exercise.

Considering the model Eq. 3 and 4

$$\frac{dc}{dt} = -(a_2 + a_4)c - a_1 c p + a_3 m w c e^{-q(t-t_0)} \quad (5)$$

$$\frac{dp}{dt} = b_1 c - b_2 p + b_3 m w p e^{-r(t-t_0)} \quad (6)$$

Let

$$x = \begin{bmatrix} c \\ p \end{bmatrix}$$

$$\frac{dx}{dt} = \begin{bmatrix} -(a_2 + a_4) - A_1 & \\ b_1 & -b_2 \end{bmatrix} \begin{bmatrix} c \\ p \end{bmatrix} + u(t) \quad (7)$$

Where

$$u(t) = \begin{bmatrix} a_3 m w c e^{-q(t-t_0)} \\ b_3 m w p e^{-r(t-t_0)} \end{bmatrix}$$

Again we have linearized $a_1 c = a_1 \bar{C}$ and replaced $a_2 \bar{C}$ by A_1 , solving (6) we have the characteristic equation

$$|A - \lambda I| = \begin{vmatrix} -(a_2 + a_4) - \lambda & -A_1 \\ b_1 & -b_2 - \lambda \end{vmatrix} = 0$$

Where

$$A = \begin{bmatrix} -(a_2 + a_4) - A_1 & \\ b_1 & -b_2 \end{bmatrix}$$

$$[-(a_2 + a_4) - \lambda][(-b_2 - \lambda)] - (b_1)(-A_1) = 0.]$$

$$= [b_2(a_2 + a_4) + b_2\lambda + [a_2 + a_4]\lambda + \lambda^2 + A_1 b_1 = 0.]$$

$$= b_2(a_2 + a_4) + b_2\lambda + [a_2 + a_4]\lambda + \lambda^2 + A_1 b_1 = 0.$$

$$= \lambda^2 + (a_2 + a_4 + b_2)\lambda + (b_2[a_2 + a_4] + A_1 b_1) = 0.$$

$$\lambda_1 = \frac{-(a_2 + a_4 + b_2) + [(a_2 + a_4 + b_2)^2 - 4[b_2(a_2 + a_4) + A_1 b_1]]^{1/2}}{2}$$

$$\lambda_1 = \frac{-(a_2 + a_4 + b_2) + [(a_2 + a_4 + b_2)^2 - 4[b_2(a_2 + a_4) + A_1 b_1]]^{1/2}}{2}$$

$$\lambda_2 = \frac{-(a_2 + a_4 + b_2) - [(a_2 + a_4 + b_2)^2 - 4[b_2(a_2 + a_4) + A_1 b_1]]^{1/2}}{2}$$

RESULTS

Case 1: When there is no exercise i.e., when the patients concerned does not exercise

$a_1 = 1.2, a_2 = 0.05, a_3 = 0.03, a_4 = 0$
 $b_1 = 0.05, b_2 = 2, b_3 = 0.01$
 Here $\lambda_1 = -0.03126985408$
 $\lambda_2 = -1.998730146$

Case 2: When the patients concerned involved in a light exercise. As above except $a_4 = 0.01$

Here $\lambda_1 = -0.0412763415$
 $\lambda_2 = -1.998723659$

Case 3: When the patients concerned involved in a medium exercise as above except $a_4 = 0.03$

Here $\lambda_1 = -0.061289517$
 $\lambda_2 = -1.998710483$

Case 4: When the patient concerned involved in a heavy exercise as above except $a_4 = 0.1$

Here $\lambda_1 = -0.1313378559$
 $\lambda_2 = -1.996862145$

Cases	λ_1	λ_2	Remarks
Zero solution + no exercise	-0.03126985408	-1.998730146	Asymptotically stable
Zero solution + light exercise	-0.0412763415	-1.998723659	„
Zero solution + medium exercise	-0.061289517	-1.998710483	„
Zero solution + heavy exercise	-0.1313378559	-1.996862145	„
Zero solution but (body) cell at rest	0	-1.99875	unstable

Case 1: When there is no exercise i.e. when the patients concerned does not exercise

$a_1 = 1.2, a_2 = 0.05, a_3 = 0.03, a_4 = 0$
 $b_1 = 0.05, b_2 = 2, b_3 = 0.01$
 Here $\lambda_1 = -0.030456958$
 $\lambda_2 = -1.999543041$

Case 2: When the patients concerned involved in a light exercise. As above except $a_4=0.01$

Here $\lambda_1 = -0.040459291$
 $\lambda_2 = -1.999540709$

Case 3: When the patients concerned involved in a medium exercise as above except $a_4 = 0.03$

Here $\lambda_1 = -0.060464028$
 $\lambda_2 = -1.999535972$

Case 4: When the patient concerned involved in a heavy exercise as above except $a_4 = 0.1$

Here $\lambda_1 = -0.130481407$
 $\lambda_2 = -1.999518593$

Cases	λ_1	λ_2	Remarks
Zero solution + no exercise	-0.030456958	-1.999543041	Asymptotically stable
Zero solution + light exercise	-0.040459291	-1.999540709	„
Zero solution + medium exercise	-0.060464028	-1.999535972	„
Zero solution + heavy exercise	-0.130481407	-1.999518593	„
Zero solution but (body) cell at rest	0	-1.99955	unstable

DISCUSSION

Figure 1 and 2 shows the effects of different intensity of exercise on Diabetes patient. We observed that exercise

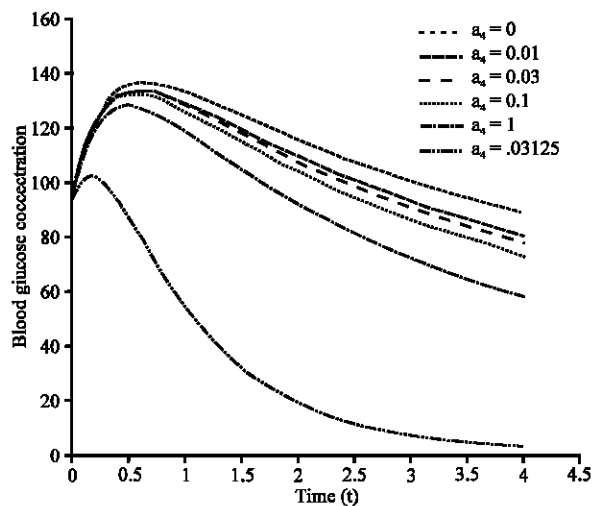


Fig. 1: Graph of glucose coccentration against time (t) for $a_1 = 1, 2, a_2 = 0.05, a_3 = 0.03, b_1 = 0.05, b_2 = 2, b_3 = 0.01, mwc = 180, mwp = 0$

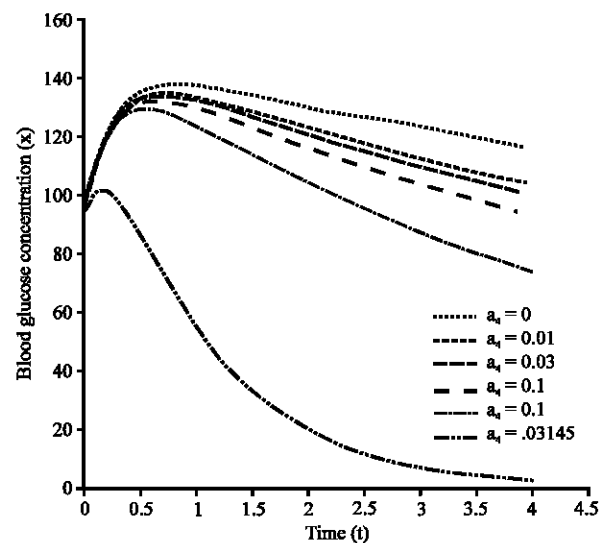


Fig. 2: Graph of glucose coccentration against time (t) for $a_1 = 1, 2, a_2 = 0.03, a_3 = 0.03, b_1 = 0.03, b_2 = 2, b_3 = 0.01, mwc = 180, mwp = 0$

has a significant effect on diabetes patients. From the cases considered it was seen clearly that the higher the intensity of exercise the less risk of becoming diabetics. In the last case considered we observed that when the body is at rest for too long such that the peripheral cell utilization of the glucose is negative, the characteristics equation becomes unstable which mean that the patients stand a high risk of becoming diabetes.

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