Research Journal of Applied Sciences 2 (8): 892-899, 2007
ISSN: 1815-932X


# A Laboratory Study of the Effects of Porosity on the Deviation from Darcy's Law in Saturated Porous Media

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**Abstract:** Darcy's law for laminar flow of fluids in porous media is an essential assumption in determining permeability, magnitude and direction of seepage force and travel time of fluid in porous media. Sand samples of different porosities from riverbed were used as porous media, a modelled experiment was set up to determine the volume of water flowing across a unit cross sectional area per unit time in these saturated sand samples packed in a vertical transparent cylindrical tube of radius $1.85\times10^{-2}$m. Values of volume flux rate were determined for hydraulic gradient between 1.875 and 30.000 by using vertical flow form of Darcy's equation. Darcy's law is not perfectly obeyed by all samples used because both volume flux and seepage velocity increase as smaller rate than hydraulic gradient. The extent of deviation from Darcy's law increases with decreasing porosity. The velocity is zero in sand samples A-E for gradient ranges of $0.029< i < 0.758$. A plot of deviation from Darcy's law against porosity shows that deviation is related to porosity, with polynomial fitting of degree 2 with correlation coefficient of 0.99.

**Key words:** Volume flux rate, porous media, permeability, porosity, hydraulic gradient

## INTRODUCTION

A porous medium is any material with interconnected pores, which allows the passage of fluid such as water. The rate at which fluid can flow in a porous medium (or a material) depends on the material's porosity. Porosity can be defined as the ratio of the void space (or pores) in a material to the bulk volume of the material. Porosity is a fraction from 0 and 1, although it may also be expressed in percent by multiplying the fraction by 100. It depends on particle size, size distribution, packing configuration, shapes, continuity and turtosity of the pores (Brian, Robert, 2002).

A porous medium is said to be saturated if all the pores are completely filled with water under hydrostatic pressure. If the pores become filled with air instead of water, the saturation decreases. A porous medium is said to be homogenous if the permeability in a given direction is the same from point to point, while it is heterogeneous if the permeability varies from point to point. The term anisotropy is used to described material where permeability or conductivity at a point has a directional dependence but when permeability is the same in all directions, the material at that point is isotopic. Isotropy and homogeneity are often assumed in the analysis of groundwater problems.

The expression for flow in porous media is known as Darcy's law. It states that the velocity of flow is proportional to the hydraulic gradient. It holds when the water particles move in a smooth, orderly procession in the direction of flow, that is laminar flow (Barer *et al.*, 1972).

In this study, the fluid is assumed to be Newtonian and behaves as a continuum while the flow is laminar, steady, fully developed and incompressible

Stearns (1972) found that the presence of appreciable clay in a porous medium appears to be associated with deviation from Darcy's Law. He later made a generalized conclusion that percentage of a surface-active materials such as clay in porous media determines the extent of this deviation.

The purpose of this research is to consider a vital common property of soils which can be easily determined in the laboratory and from which this measure of deviation can be related with. Thus, porosity was chosen. This is necessary because other porous media apart from clay with smaller grain sizes can also be a surface-active material.

In addition, Swartzendruber (1962) predicted the existence of threshold gradients, at which seepage velocity is zero as well as origin. This deviation from Darcy's law indicates the possible existence of threshold gradients range.

Therefore, there is a need to look for this experimentally and if it is found to be so, it will be very useful in practical seepage control in man-made constructions.

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The determination of seepage velocity of fluid in the porous media, which is one of the major parameter in application of Darcy's law in solving environmental problems depends strongly on porosity media (List and Brooks, 2001; Barer *et al.*, 1972). Therefore there is a need to look at effects of porosity on the deviation from Darcy's law.

The objectives of this study are to:

- Verify the validity of Darcy's law.
- Investigate the effect of porosity on the deviation from Darcy's law in water-saturated porous media.
- Confirm experimentally the existence of threshold gradient which is the range of hydraulic gradient at which seepage velocity is zero.
- Determine how porosity affects the deviation from Darcy's law.
- These are necessary so that better understanding of the deviation from Darcy's law and their applications can be achieved especially for seepage control in man-made constructions.

**Theory:** Hydraulic conductivity *K*, is the specific discharge per unit hydraulic gradient. It expresses the ease with which a fluid is transported through void space. It depends pn the solid matrix and fluid properties. The permeability *k*, of a porous medium is its fluid capacity for transmitting a fluid under the influence of a hydraulic gradient (Sherwani, 1978). It depends solely on the geometrical structure of the material i.e. porosity, grain size distribution, turtosity and connectivity.

Fluid flow through a porous material of permeability *k*, by Darcy is generally written as:

$$V_s = -\frac{k}{\mu}\nabla(P - \rho gz) \text{ (Frick and Taylor, 1978)} \quad (1)$$

which can be expressed as

$$V_s = -\frac{k}{\mu}\left(\frac{dp}{ds} - \rho g\frac{dz}{ds}\right) \quad (2)$$

where

| | | |
|---|---|---|
| s | = | Distance in the direction of flow, always positive; |
| Vs | = | Volume flux across a unit area of the porous medium in unit time; |
| z | = | Vertical coordinate, considered downward; |
| ρ | = | Density of the fluid; |
| g | = | Acceleration of gravity; |
| dp/ds | = | Pressure gradient along s at the point to point to what $V_s$ refers; |
| μ | = | Viscosity of the fluid; |
| k | = | Permeability of the medium; |
| dz/ds | = | Sin$\theta$, where $\theta$ is the angle between s and the horizontal $V_s$. |

Equation 2 can be expressed further as follows:

$$V_s = \frac{k}{\mu}\left(\rho g\frac{dp}{ds} - \frac{dz}{ds}\right) \quad (3)$$

$$\frac{V_s\mu}{k} = \rho g\frac{dz}{ds} - \frac{dp}{ds} \quad (4)$$

$$\frac{dz}{ds} = \sin\theta \quad (5)$$

then

$$\frac{V_s\mu}{k} = \rho g\sin\theta - \frac{dp}{ds} \quad (6)$$

$$\frac{V_s\mu}{k} - \rho g\sin\theta = -\frac{dp}{ds} \quad (7)$$

Integrating both sides

$$\int\left(\frac{V_s\mu}{k} - \rho g\sin\theta\right)ds = -\int_{p_1}^{p_2} dp \quad (8)$$

$$\left(\frac{V_s\mu}{k} - \rho g\sin\theta\right)s = P_1 - P_2 = P \quad (9)$$

$$\frac{V_s\mu}{k} = \frac{P}{S} + pg\sin\theta \quad (10)$$

If a porous medium is completely saturated with an incompressible fluid and is vertical then ( Fig. 1a and b). dz/ds = sin $90^0$ ($\theta = 90^0$ , sin $90^0 = 1$) (Fig. 1a)
Equation 10 reduce to

$$\frac{V_s\mu}{k} = \frac{P}{S} + pg \quad (11)$$

$$V_s = \frac{k}{\mu}\left(\frac{P}{s} + \rho g\right) \quad (12)$$

P = ρgh, s = L (Fig. 1b)

$$V_s = \frac{k}{\mu}\left(\frac{\rho gh}{s} + \rho g\right) \quad (13)$$

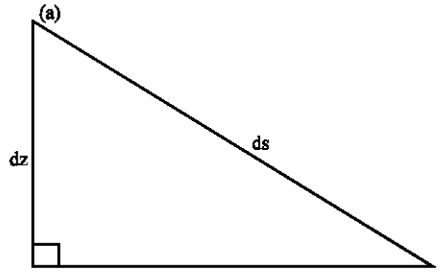


Fig. 1a: Illustration of dz/ds

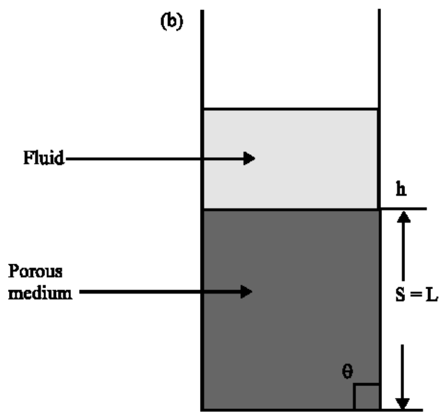


Fig. 1b: Illustration of θ = $90^0$ with horizental for vertical flow

$$V_s = \frac{\rho g}{\mu} k \left( \frac{h}{L} + 1 \right) \tag{14}$$

$$K = \frac{\rho g}{\mu} k \text{ (Domenico } et\ al.\text{, 2000)} \tag{15}$$

Where,

K = Hydraulic conductivity ($ms^{-1}$);
k = Permeability ($m^2$);
μ = Viscosity of fluid ($Nsm^{-2}$);
ρ = Density of fluid ($kgm^{-3}$) and
μ/ρ = Kinematic viscosity (for water) = $1\times 10^{-6}$ $m^2s^{-1}$

The hydraulic conductivity contain properties of both medium and fluid with units $ms^{-1}$ and characterizes the capacity of a medium to transmit water, whereas the permeability with unit $m^2$ characterizes the capacity of the medium to transmit any fluid. The two properties are related by Eq. (15).

By using Eq. 15 in 14, Eq. 14 becomes

$$V_s = K \left( \frac{h}{L} + 1 \right) \tag{16}$$

$$V_s = q = \frac{Q}{A} = K \left( \frac{h}{L} + 1 \right) \text{(Jacob and Arnold, 1990)} \tag{17}$$

Where,

Q = Volumetric flow rate ($m^3s^{-1}$)
A = Average cross-sectional area perpendicular to the line of flow ($m^2$)
q = Volume flux ($ms^{-1}$)
h = Head constant (or hydraulic head) (m)
L = Flow path length of the sample (m)

Thus, Eq. 17 can be written in simple form as

$$q = K \left( \frac{h}{L} + 1 \right) \text{(Ghildyal and Tripathi, 1987)} \tag{18}$$

or

$$q = Ki \tag{19}$$

Where

$$i = \left( \frac{h}{L} + 1 \right) = \text{Hydraulic gradient}$$

While the volume flux *q*, has the units of the velocity, it is not the velocity of the water in the pores. The matrix takes up some of the flow area. The average pore water velocity is termed the seepage velocity, *v* and is given as:

$$V \frac{Q}{A\phi} = \frac{q}{\phi} \text{ (Jacob and Arnold, 1990)}$$

Where,

v = Seepage velocity ($ms^{-1}$)
q = Volume flux ($ms^{-1}$) and
ϕ = Porosity of the media

The maximum pore velocity ia a function of the pore geometry and cannot be easily predicted except for simple shapes. However, it can be determined in the laboratory by volumetric method.

## MATERIALS AND METHODS

Sand samples were collected from the riverbed of two different rivers within the University of Ibadan. Sizeable quantities of these samples were washed and rinsed in order to remove organic particles and unwanted grains and brought to the laboratory. Thereafter, the sand samples sun dried and later placed in an oven. After, the samples were allowed to cool down, the stony particles and pepples were removed. Five different sieves were used to sieve the available sand samples in order to obtain samples of different grain sizes. The porosity $\phi$ of each sample was determine by volumetric approach in the laboratory.

The porosity of each samples was determined by volumetric approach. In the laboratory measurement of porosity, it is necessary to determine only two of the basic parameters (bulk volume, pore volume and grain volume):

$$\text{Bulk volume} = \text{grain volume} + \text{pore volume} \tag{21}$$

$$\text{Total porosity}, \phi = \frac{\text{Bulk volume - Grain volume}}{\text{Bulk volume}} \tag{22}$$

$$\phi = \frac{\text{Pore volume}}{\text{Bulk volume}} \tag{23}$$

In this research, bulk and grain or matrix volumes were determined volumetrically by measured 3 mL of dried sand sample using a 10 mL measuring cylinders. It was ensured that the measuring cylinder was tapped with a solid object and the sand inside get re-arranged and compacted before the value of the volume was recorded. This is necessary in order to maintain steady volume. A similar measuring cylinder was half-filled with water and the volume was noted. The sands was then pored into the water and the final volume of the components of the cylinder (water and sand) is recorded.

Porosity can be determined as follows:

| | | |
|---|---|---|
| Volume of sand (bulk volume) | = | A (mL) |
| Volume of water | = | B (mL) |
| Volume of mixture of water and sand | = | C (mL) |

$$\text{Therefore total porosity}, \phi = \frac{\text{Pore volume}}{\text{Bulk volume}} \tag{24}$$

In an experiment set up in the laboratory, volume flux rate q, for each sample at different hydraulic gradients, a saturated sand sample was transferred to the transparent cylindrical tube of cross-sectional area $2.69 \times 10^{-4} m^2$ (Fig. 2). To ascertain uniform compaction throughout the sample, the screened end was blocked so as to prevent the water passing through when the sample was being transferred. A continuous steady supply of water was fed through the sand samples of length L and at height h a hole was drilled, this enabled the height to be maintained, as excess water got drained through an overflow arrangement. The volume of water discharged, Q through the sample for a period of 60 sec after steady state has been attained at constant head was measured by measuring cylinder. It must be noted that the length $L$ is varied in order to obtain different hydraulic gradients, i. Measurements were made at hydraulic gradients of i = 1.875, 3.750, 7.500, 15.000 and 30.000; for each sample.

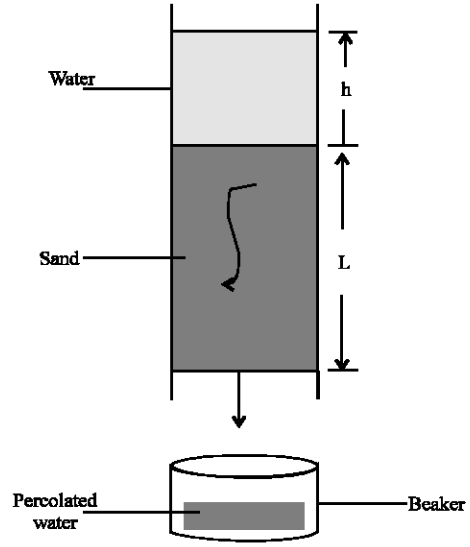


Fig. 2: Sand model for vertical flow under head h. (Jacob, 2001)

## RESULTS AND DISCUSSION

Table 1 shows the values of discharge volume, Q at each hydraulic gradient for each sample A-E. Table 2 shows the values of volume flux rate, q at each hydraulic gradient computed from the values of discharge volume, Q by using Eq. 17, the volume flux rate, q was determined knowing that q = Q/A, where $A = \pi (d/2)^2$ and d is the diameter of the cylindrical tube used given as $1.85 \times 10^{-2}$m.

Using Eq. 18, the slope of the graph of volume flux rate, q against hydraulic gradient i equals hydraulic conductivity, K. Thus, from the equation of the volume flux rate-gradient curve for each sample (Fig. 3-7), the value of hydraulic conductivity are determined and presented in Table 3. Permeability was determined from the respective values of hydraulic conductivity by using Eq. (15). Table 4 shows the seepage velocity, V value for each gradient by using Eq. (20), by dividing each value of volume flux rate, q by the porosity of the medium. Table 2 and 4 clearly show that both the

Table 1: Experimental determined values of volume of discharge Q for samples at various hydraulic gradient (h +L/L) for 60 sec

| h + L/L(Hydraulic gradient) | Discharge volume $Q * 10^{-6}$ ($m^3$) A | Discharge volume $Q * 10^{-6}$ ($m^3$) B | Discharge volume $Q * 10^{-6}$ ($m^3$) C | Discharge volume $Q * 10^{-6}$ ($m^3$) D | Discharge volume $Q * 10^{-6}$ ($m^3$) E |
|---|---|---|---|---|---|
| 1.875 | 1.79 ±0.02 | 3.16 ±0.02 | 3.60±0.04 | 4.74±0.06 | 9.42±0.07 |
| 3.750 | 3.19±0.03 | 6.19 ±0.04 | 7.10±0.03 | 9.71±0.05 | 18.99±0.09 |
| 7.500 | 4.19±0.07 | 12.00 ±0.03 | 13.20±0.05 | 19.68±0.12 | 38.13±0.06 |
| 15.000 | 8.20±0.11 | 23.40 ±0.10 | 25.60±0.04 | 39.47±0.04 | 76.39±0.10 |
| 30.000 | 22.80±0.14 | 50.41 ±0.12 | 55.60±0.14 | 79.15±0.13 | 152.93±0.15 |

Table 2: Experimental determined values of volume flux rate q for samples at various hydraulic gradient (h +L/L)

| h +L/L (Hydraulic gradient) | Volume flux rate $q * 10^{-4}$ ($m\ s^{-1}$) A | Volume flux rate $q * 10^{-4}$ ( $s^{-1}$) B | Volume flux rate $q * 10^{-4}$ ( $s^{-1}$) C | Volume flux rate $q * 10^{-4}$ ( $s^{-1}$) D | Volume flux rate $q * 10^{-4}$ ( $s^{-1}$) E |
|---|---|---|---|---|---|
| 1.875 | 1.11± 0.02 | 1.98 ±0.02 | 2.23±0.04 | 2.94±0.06 | 5.84±0.07 |
| 3.750 | 1.98± 0.03 | 3.84 ±0.04 | 4.40±0.03 | 6.02±0.05 | 11.77±0.09 |
| 7.500 | 2.60± 0.07 | 7.44 ±0.03 | 8.18±0.05 | 12.20±0.12 | 23.64±0.06 |
| 15.000 | 5.08 ± 0.11 | 14.51 ± 0.10 | 15.87±0.04 | 24.47±0.04 | 47.36±0.10 |
| 30.000 | 14.14± 0.14 | 31.25 ± 0.12 | 34.47±0.14 | 49.07±0.13 | 94.81±0.15 |

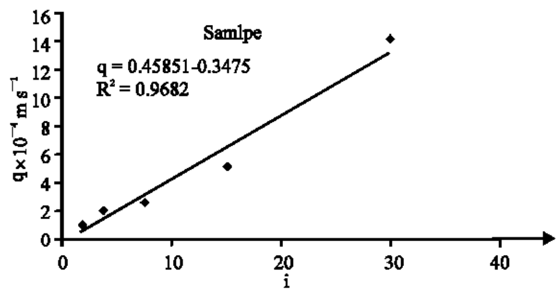


Fig. 3: Graph of q against i (sample A)

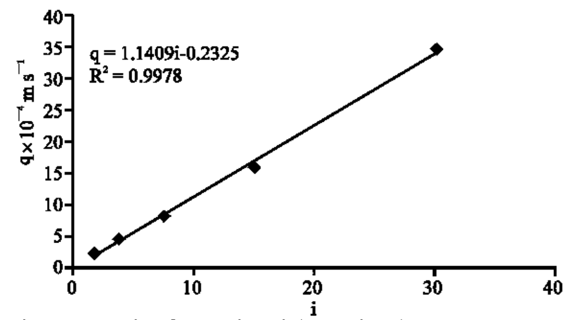


Fig. 5: Graph of q against i (sample C)

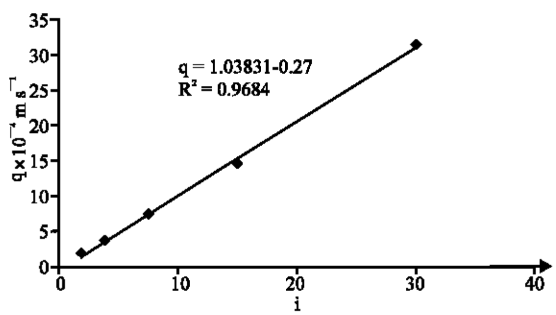


Fig. 4: Graph of q against i (sample B)

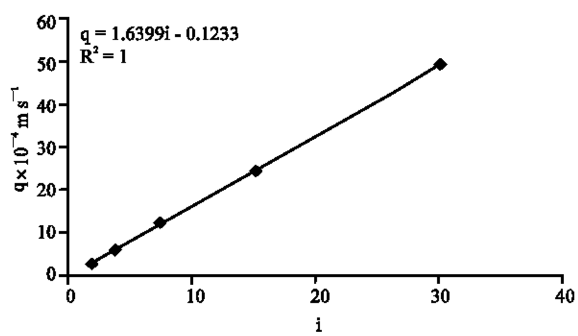


Fig. 6: Graph of q against i (sample D)

volume flux rate and seepage velocity increase as the hydraulic gradient increases.

Volume flux increases at smaller rate than hydraulic gradient, especially at the gradient of 1.875-7.5, but at gradients above 7.5, volume flux increases at greater rate than gradients for samples a, b and c. however, volume flux increases in nearly direct proportional rate with gradient for samples d and e for all gradients.

Seepage velocity increases at smaller rate than hydraulic gradient at gradient of 1.875-7.5 for samples A, B and C, but increases at higher rate than gradient at gradient of 15.0 and 30.0. Seepage velocity increases in nearly direct proportional rate with gradient at gradient 7.5 and above.

This shows that apart from porosity, hydraulic gradient influences the deviation from Darcy's law. This could be investigated in the further work.

The results from the experiment show that generally volume flux, is related with hydraulic gradient, *i* for all sample as:

$$q = C_1 i + C_2 \qquad (25)$$

Where

q = Volume flux
I = Hydraulic gradient and
$C_i$ and $C_2$ = Are constant

instead of "original' Darcy's equation

$$q = C_i I \qquad (26)$$

Table 3: Value of porosity, hydraulic conductivity and permeability for various samples

| Sample | Porosity | Hydraulic conductivity ( $s^{-1}$) | Permeability ($m^2$) | Measure of deviation | Thresholdgradient range |
|---|---|---|---|---|---|
| A | 0250 ± 0.010 | $0.46\times10^{-4}$ | $0.47 \times 10^{-11}$ | 0.76 | 0≤i<0.758 |
| B | 0.333 ± 0.002 | $1.04\times10^{-4}$ | $1.06\times10^{-11}$ | 0.26 | 0≤i<0.263 |
| C | 0.364 ± 0.001 | $1.14\times10^{-4}$ | $1.16 \times 10^{-11}$ | 0.20 | 0≤i<0.203 |
| D | 0.400 ± 0.001 | $1.63\times10^{-4}$ | $1.67\times10^{-11}$ | 0.08 | 0≤i<0.075 |
| E | 0.420 ± 0.010 | $3.16\times10^{-4}$ | $3.23 \times 10^{-11}$ | 0.03 | 0≤i<0.029 |

Table 4: Experimental determined values of seepage velocity V for samples at various hydraulic gradient (h +L/L)

| h +L/L (Hydraulic gradient) | Seepage velocity $V*10^{-4}$ ( $s^{-1}$) A | Seepage velocity $V*10^{-4}$ ( $s^{-1}$) B | Seepage velocity $V*10^{-4}$ ( $s^{-1}$) C | Seepage velocity $V*10^{-4}$ ( $s^{-1}$) D | Seepage velocity $V*10^{-4}$ ( $s^{-1}$) E |
|---|---|---|---|---|---|
| 1.875 | 4.44±0.02 | 5.88±0.02 | 6.13±0.04 | 7.35±0.06 | 13.91±0.07 |
| 3.750 | 7.91±0.03 | 11.52±0.04 | 12.09±0.03 | 15.05±0.05 | 22.03±0.09 |
| 7.500 | 10.39±0.07 | 22.34±0.03 | 22.48±0.05 | 30.50±0.12 | 56.28±0.06 |
| 15.000 | 20.34±0.11 | 43.56±0.10 | 43.60±0.04 | 61.17±0.04 | 112.76±0.10 |
| 30.000 | 56.54±0.14 | 93.85±0.12 | 94.70±0.14 | 122.67±0.13 | 225.74±0.15 |

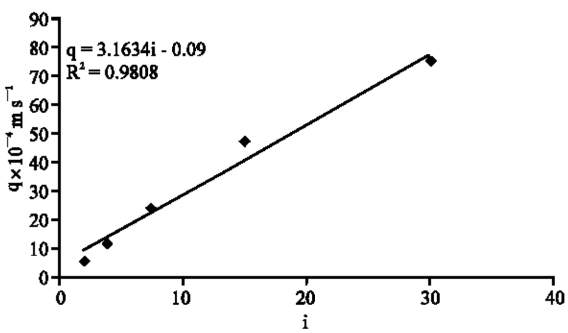


Fig. 7: Graph of q against i (sample E)

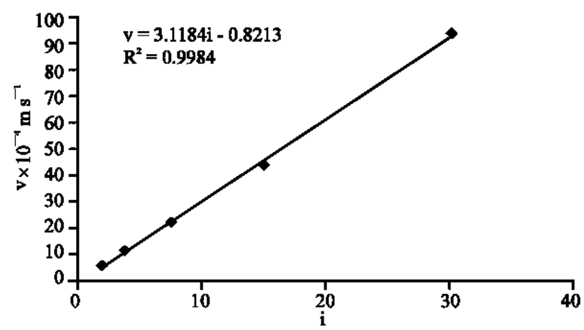


Fig. 9: Graph of v against i (sample B)

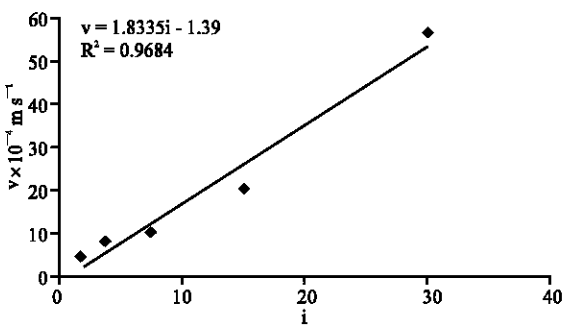


Fig. 8: Graph of v against i (sample A)

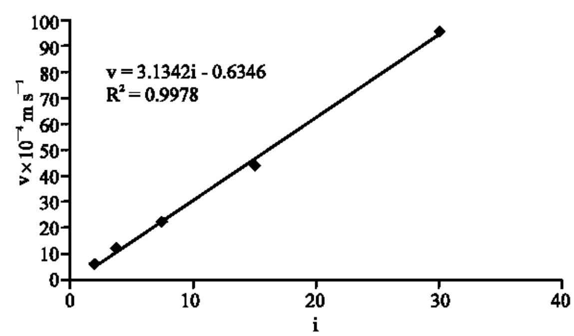


Fig. 10: Graph of v against i (sample C)

this shows that Dracy's law is not perfectly obey by all the samples.

In this experiment, the presence of constant, $C_2$ could not be taken as experimental error or random error because it followed a definite pattern for all the samples. It was found to be negative for all the samples, (Fig 3-7) in which samples A has the highest value and samples E with the least value. The measure of this deviation is the gradient intercept, $i'$.

Settingq = 0 from Eq. 25, yields

$$I = i' = C_2/C_1 \quad (27)$$

It was found that $i'$ increases with decrease in porosity (Table 4). This is in support of Sterns's results, because it was found that the increasing in percentage of clay in porous media decreases its porosity and this in turn increases the extent of the deviation.

Also, since it has been found that the deviation from Dary's law is not due to experimental error, this implies that there is existence of other higher gradients apart fro zero gradient at which seepage velocity is zero (Fig. 8-12). However, this threshold gradient range depends on the porosity of the media. The range increases with decreasing porosity (Table 4).

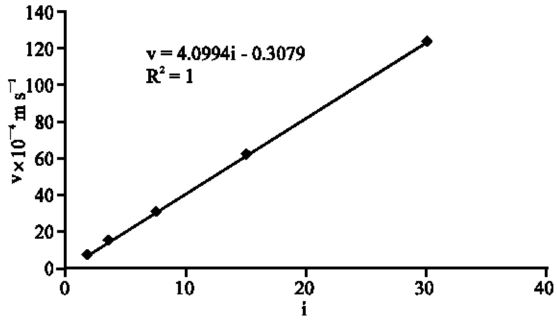


Fig. 11: Graph of v against i (sample D)

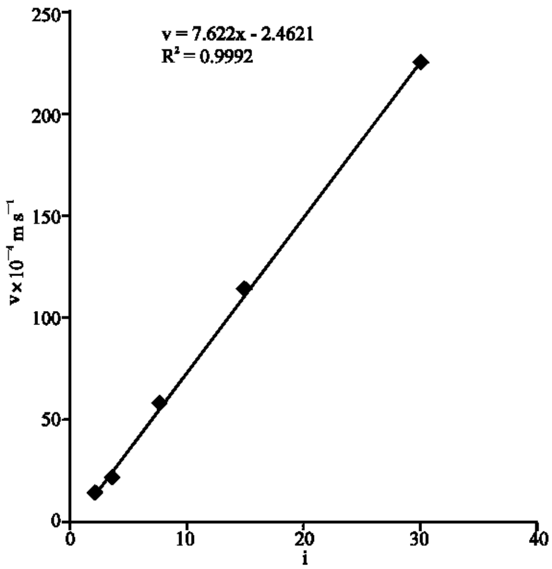


Fig. 12: Graph of v against i (sample E)

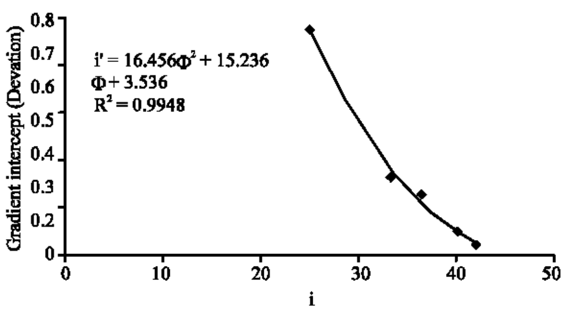


Fig. 13: Graph of gradient intercept against porosity

The plot was prepared between measure of deviation and porosity and found to be related in polynomial of degree two (Fig. 13) with relation $i' = 16.456\Phi^2 + 15.236\Phi + 3.536$ with coefficient of 0.9948. This shows that they are not linearly related. Also, this relation will be of help in predicting the extent of the deviation expected of a medium of a known porosity. In addition, it will be of help in determine or prediction of range of threshold gradient for a medium of a known porosity.

## CONCLUSION

At the end of the study, it was found that

- Darcy's law is not perfectly obeyed by all samples used.
- Deviation from this law was observed because apart from zero gradient, volume flux and seepage velocity were also found to be zero at higher gradient. This could not be attributed to random error because it followed a definite pattern for all samples.
- This deviation provides the existence of minimum threshold gradient before flow takes place in a medium and
- The deviation from Darcy's law increases with decreasing porosity with polynomial fitting of degree two with relation

$$i' = 16.456\,\phi^2 + 15.235\,\phi + 3.536$$

where

$I'$ = Gradient intercept, which is the measure of deviation and $\phi$ = porosity

It can be seen from the result that permeability of a medium increases with increasing porosity and the deviation from Darcy's law is related to porosity, with polynomial fitting of degree two with correlation coefficient of 0.99 indicating that porosity is not linearly related to deviation. Also, it has been found that apart from zero-gradient a practical situation at which velocity, $v$ equals zero can be achieved at higher gradients by using a material of lower porosity. These gradients called threshold gradient can be determined from velocity-gradient curve.

It was found in the course of the experiment that hydraulic gradients influences the deviation from Darcy's law, therefore this could be investigated in the further work. It may also be necessary for future work to incorporate the deviation into the Darcy's laws as a way of modification; so that a general equation that will take care of this deviation can be formulated.

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