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On the Response of Damped Rectangular Plates to Uniform Partially Distributed Moving Masses

E.O. Titiloye, M.S. Dada, J.A. Gbadeyan and A.S. Idowu
Department of Mathematics, University of Ilorin, Ilorin, Nigeria

Abstract: The response of damped rectangular plates to uniform partially distributed moving masses is investigated. Using a series solution for the dynamic reflection in term of the normal modes, the partial differential equation governing the behaviour of the model is reduced to an ordinary differential equation. This was solved using finite difference scheme. The results were presented in graphical and tabular forms.

Key words: Damped rectangular plates, uniform partially distributed, moving masses , simply supported

INTRODUCTION

Damping may be defined as the dissipation of energy in motion and the consequent reduction or decay of the motion. Hence damped vibration is any oscillation in which the amplitude of the oscillatory quantity decreases with time. Damping is usually due to friction. It is well known (Gbadeyon and Usman, 2003), that damping becomes important when the need to have a thorough understanding of the control and mechanical response of vibrating structures arises.

In this study, the problem of determining the dynamic response of a rectangular, damped, elastic plate carrying uniform partially moving load is investigated. The elastic plate is assumed to have uniform cross-sectional area. The effect of both rotatory inertia as well as shear deformation is assumed negligible. The moving partially distributed load is also assumed moving at uniform velocity. A constant damping coefficient is used throughout the analysis. There are different types of damping, however, viscous damping whose coefficient is assumed to be directly proportional to the mass distribution of the system is considered. The corresponding one-dimensional problem was earlier studied in Gbadeyon and Usman (2003). An asymptotic analysis of eigen frequencies of uniform beam with both structural and viscous damping coefficient has also been carried out in Hankum and Goong (1991) and Huang (1985). Also the analysis in this study is for simply supported plates. However, it also holds for any other boundary conditions that are of practical interest such as damped plates etc. Numerical examples are also presented.

THE MATHEMATICAL FORMULATION

The equation governing the response of damped, isotropic rectangular plates according to the classical theory of elastic plate is Timoshenkos and Woniowsky (1959)

$$D\nabla^4 W(x,y,t) + M_1 W_{tt},(x,y,t) + 2M_1 \gamma_2 W_t (x,y,t) = P(x,y,t) \quad (1)$$

Where,

$$D = \frac{Eh^3}{12(1-\nu)} \quad (2)$$

$$\nabla^4 = \nabla^2 \nabla^2 W = W,_{xxxx}(x,y,t) + 2W,_{xxyy}(x,y,t) + W,_{yyyy}(x,y,t) \quad (3)$$

W(x, y, t)	:	The deflection of the plate.
h	:	The thickness of the plate.
E	:	The Young's modulus.
υ	:	The poisson's ratio.
x.y	:	The rectangular Cartesian coordinates in the plane of the plate.
M_1	:	The mass density per unit area of the plate.
t	:	The time.

P(x, y,t) is the applied moving load on the plate, γ is the viscous damping coefficient, (,x) and (,t) denotes partial differentiation with respect to x and t, respectively.

It should be noted, however, that Eq. 1 was based on the assumption that:

- There is no deformation in the middle plane of the plates. This plane remains neutral during bending.

Corresponding Author: A.S. Idowu, Department of Mathematics, University of Ilorin, Ilorin, Nigeria

- Points of the plates lying initially on a normal-to-the middle plane of the plate remain on the normal to the middle surface of the plate after bending.
- The normal stresses in the direction transverse to the plate are negligible.

In the case of distributed time load, Eq. 1 may be written as

$$D(W_{,xxxx}+2W_{,xxyy}+W_{,yyyy})+M_1W_{,tt}(x,y,t)+2M_1\gamma W_{,t}(x,y,t)$$

$$=\frac{1}{r}\left(M_2g-M^2\frac{d^2W}{d^2}\right)\left[H\left(x-vt+\tfrac{r}{2}\right)-H\left(x-vt-\tfrac{r}{2}\right)\right]\delta(y-y_L) \tag{4}$$

Where, r is the length of the load, H(x) is the Heaviside unit function, δ(x) is the Dirac-delta function, v is the velocity of the moving load, g is the acceleration due to gravity, M_2 is the mass of the load and also,

$$\frac{d^2W(x,y,t)}{dt^2}=W_{,tt}(x,y,t)+2vW_{,xt}(x,y,t)+v^2W_{,xx}(x,y,t) \tag{5}$$

The pertinent boundary conditions are:

$$\begin{aligned} W(0,y,t)&=0=W_{,xx}(0,y,t)\\ W(a,y,t)&=0=W_{,xx}(a,y,t)\\ W(x,0,t)&=0=W_{,yy}(x,0,t)\\ W(x,b,t)&=0=W_{,yy}(x,b,t)\end{aligned} \tag{6}$$

Finally, the initial conditions are

$$W(x,y,0)=W_{,t}(x,y,0)=0 \tag{7}$$

METHODS OF SOLUTION

Assuming a separation of variables solution in the from of a series, namely,

$$W(x,y,t)=\sum_{m=1}^{N}\sum_{m=1}^{N}Q_{mn}(t)W_n(x)W_m(y) \tag{8}$$

Where, $W_n(x)$ and $W_m(y)$ are the fundamental mode shapes of beams having the boundary conditions of the plate (Hankum and Goong, 1991). m and n are the number of contributed modes and $Q_{mn}(t)$ are the unknown functions of time.

By substituting Eq. 8 into 4 we have

$$D\{W_{,xxxx}+2W_{,xxyy}+W_{,yyyy}\}+M_1W_{,tt}(x,y,t)+2M_1\gamma W_{,t}(x,y,t)=\frac{1}{r}(M_2g-M_2(W_{,tt}(x,y,t)+2vW_{,xt}(x,y,t)+v^2W_{,xx}(x,y,t)))\left[H\left(x-vt+\tfrac{r}{2}\right)-\left(x-vt-\tfrac{r}{2}\right)\right]\delta(y-y_L) \tag{9}$$

Using Eq. 8 in 9 we obtain

$$\sum_{m=1}^{M}\sum_{n=1}^{N}[Q_{mn}(t)\{D(W_n^{iv}(x)W_m(y)+2W_n''(x)2W_m''(y)+W_m^{iv}(y)W_n(x))+M_1\ddot{Q}_{mn}(t)W_n(x)W_m(y)+2M_1\gamma\dot{Q}_{mn}(t)W_n(x)W_m(y)$$
$$=\frac{1}{r}\left\{M_2g-M_2\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\begin{array}{l}\ddot{Q}_{mn}(t)W_n(x)W_m(y)+2vW_n'(x)W_my\dot{Q}_{mn}(t)\\ +v^2W_n''(x)W_m(y)Q_{mm}(t)\end{array}\right)\right.\left[H\left(x-vt+\tfrac{r}{2}\right)-H\left(x-vt-\tfrac{r}{2}\right)\right]\delta(y-y_L) \tag{10}$$

For free vibration of elastic plate we have

$$D\{W_n^{iv}(x)W_m(y)+2W_n''(x)W_m''(y)+W_n(x)W_m^{iv}(y)\}=\lambda_{mn}W_n(x)W_m(y)M_1 \tag{11}$$

Where, $\lambda_{mn}=\omega^2_{mn}M_1$,n = 1, 2, 3… m = 1, 2, 3… are the eigenvalues for some specific boundary conditions.

Substituting Eq. 11 into 10 we obtain

$$\sum_{m=1}^{M}\sum_{n=1}^{N}[Q_{mn}(t)\lambda_{mn}W_n(x)W_m(y)M_1+2M_1\gamma\dot{Q}_{mn}(t)W_n(x)W_m(y)+M_1\ddot{Q}_{mn}(t)W_n(x)W_m(y)]$$
$$M_1\ddot{Q}_{mn}(t)W_n(x)W_m(y)+2M_1\gamma\dot{Q}_{mn}(t)W_n(x)W_m(y)$$
$$=\frac{1}{r}\left\{M_2g-M_2\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\begin{array}{l}\ddot{Q}_{mn}(t)W_n(x)W_m(y)+2vW_n'(x)\\ W_m(y)\dot{Q}_{mn}(t)+v^2W_n''(x)W_m(y)Q_{mm}(t)\end{array}\right)\right.\left[H\left(x-vt+\tfrac{r}{2}\right)-H\left(x-vt-\tfrac{r}{2}\right)\right]\delta(y-y_L) \tag{12}$$

Multiplying both sides of Eq. 12 by $W_i(x)$ $W_j(y)$, taking the double integrals of both sides along the length and width of the plate and using the properties of orthogonal functions $W_n(x)$ and $W_m(y)$ we have

$$\lambda_{mn}\,\alpha M_1 Q_{mn}(t)+2\alpha M_1\gamma\dot{Q}_{mn}(t)$$
$$+\alpha M_1\ddot{Q}_{mn}(t)=\frac{1}{r}M_2 g\int_0^a W_i(x)$$
$$\left[H\left(x-vt+\frac{r}{2}\right)-H\left(x-vt-\frac{r}{2}\right)\right]dx\int_0^b aW_i(y)$$
$$\delta(y-y_L)+\sum_{n=1}\sum_{m=1}M_2\ddot{Q}_{mn}(t)$$
$$\int_0^a W_n(x)W_i(x)\left[\left(H\left(x-vt+\frac{r}{2}\right)-H\left(x-vt-\frac{r}{2}\right)\right]\right.$$
$$dx\int_0^b W_m(y)W_j(y)\delta(y-y_L)-$$
$$2\dot{Q}_{mn}v\int_0^a W_n'(x)W_i(x)\left[H\left(x-vt+\frac{r}{2}\right)-H\left(x-vt-\frac{r}{2}\right)\right]$$
$$dx\int_0^b W_m(y)W_j(y)\delta(y-y_L)$$
$$-v^2\dot{Q}_{mn}(t)\int_0^a W_n''(x)W_i(x)\left[H\left(x-vt+\frac{r}{2}\right)-H\left(x-vt-\frac{r}{2}\right)\right]dx$$
$$\int_0^b W_m(y)W_j(y)\delta(y-y_L) \qquad (13)$$

Equation 13 may be simplified as

$$\ddot{Q}_{mn}(t)+2\gamma\dot{Q}_{mn}(t)+\lambda_{nm}Q_{mn}(t)=\frac{1}{M_1 r}$$
$$\Bigg\{M_2 g\,W_j(y_L)\int_{vt-r/2}^{vt+r/2}W_i(x)dx$$
$$-\sum_{n=1}^{N}\sum_{m=1}^{M}\Bigg[\ddot{Q}_{nm}(t)W_m(y_L)W_j(y_L)\int_{vt-r/2}^{vt+r/2}W_n(x)W_i(x)dx+2v\dot{Q}_{nm}(t)W_m(y_L)W_j(y_L)$$
$$\int_{vt-r/2}^{vt+r/2}W_n^1(x)W_i(x)dx+v^2 Q_{nm}(t)W_m(y_L)W_j(y_L)$$
$$\int_{vt-r/2}^{vt+r/2}W_n''(x)W_j(x)dx\Bigg]\Bigg\} \qquad (14)$$

Equation 14 is the generalized ordinary coupled differential equation to be solved for some specific boundary conditions of the plate.

THE METHOD OF SOLUTION FOR SPECIFIED BOUNDARY CONDITIONS

For simply supported rectangular pates, the edges conditions can be expressed as

$$W(0,y,t)=W(a,b,t)=\frac{\partial^2 W(0,y,t)}{\partial x^2}=\frac{\partial^2 W(a,b,t)}{\partial x^2}=0 \qquad (15)$$

$$W(x,0,t)=W(x,b,t)=\frac{\partial^2 W(x,0,t)}{\partial y^2}=\frac{\partial^2 W(x,b,t)}{\partial y^2}=0 \qquad (16)$$

Again, the initial conditions are

$$W(x,y,0)=\frac{\partial W(x,y,0)}{\partial t}=0 \qquad (17)$$

The normalized deflection curves for simply supported boundary conditions for a rectangular plate of dimensions a by b Gbadeyan and Dada (2001) is:

$$\left(\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\right)\left(\sqrt{\frac{2}{a}}\sin\frac{m\pi y}{b}\right)$$
$$W_n(x)W_m(y)=\frac{2}{\sqrt{ab}}\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b} \qquad (18)$$

n = 1, 2, 3, ..., m = 1, 2, 3, ... where unity is the normalized constant. The eigen values are obtained by substituting Eq. 18 into 11 which yields

$$\lambda mn = D\pi^4\left[\frac{n^2}{a^2}+\frac{m^2}{b^2}\right] \qquad (19)$$

The exact governing equations for simply supported rectangular plate is obtained when Eq. 18 is substituted into the generalized governing Eq. 13 as

$$\ddot{Q}_{nm}(t)+2\gamma\dot{Q}_{nm}(t)+\lambda_{nm}Q_{nm}(t)=$$
$$\frac{1}{M_1 r}\Bigg[\frac{2M_2 g}{\sqrt{ab}}\sin\frac{i\pi y_L}{b}\int_{e_1}^{e_2}\sin\frac{i\pi x}{a}dx$$
$$-\sum_{n=1}^{N}\sum_{m=1}^{M}\Bigg[\left(\frac{2}{\sqrt{ab}}\right)^2\sin\frac{m\pi y_L}{b}\sin\frac{i\pi y_L}{b}\int_{e_1}^{e_2}\sin\frac{n\pi x}{a}\sin\frac{i\pi x}{a}dx$$
$$+\frac{8n\pi}{a^2 b}v\dot{Q}_{mm}(t)\sin\frac{m\pi t_L}{b}\sin\frac{j\pi y_L}{b}\int_{e_1}^{e_2}\cos\frac{n\pi x}{a}\sin\frac{i\pi x}{a}dx$$
$$-\frac{4\pi^2 n^2}{a^3 b}v^2 Q_{mm}(t)\sin\frac{m\pi y_L}{b}\sin\frac{j\pi y_L}{b}\int_{e_1}^{e_2}\sin\frac{n\pi x}{a}\sin\frac{i\pi x}{a}dx\Bigg]\Bigg] \qquad (20)$$

Where,

$$e_1 = vt-\frac{\partial}{2}\text{ and } e_2 = vt\frac{\partial}{2}$$

Equation 20 is further simplified by evaluating the integrals as

$$\ddot{Q}_{nm}(t)+2\gamma\dot{Q}_{nm}(t)+\lambda_{nm}Q_{nm}(t)= \frac{1}{M_1 r}\Bigg[\frac{4M_2 ga}{\sqrt{ab}\,n\pi}\sin\frac{i\pi y_L}{b}\sin\frac{i\pi vt}{a}\sin\frac{i\pi r}{2a} - M_2\sum_{n=1}^{N}\sum_{m=1}^{M}\Bigg[\left(\frac{2}{\sqrt{ab}}\right)^2\sin\frac{m\pi y_L}{b}\sin\frac{i\pi y_L}{b}\,\ddot{Q}_{nm}(t)\left(\frac{a}{\pi d_1}\cos\left(\frac{d_1\pi vt}{a}\right)\sin\left(\frac{d_1\pi r}{2a}\right)\right) - \frac{a}{\pi p_1}\cos\left(\frac{p_1\pi vt}{a}\right)\sin\left(\frac{p_1\pi r}{2a}\right)\Bigg) \frac{8n\pi}{a^2 b} v\dot{Q}_{mm}(t)\sin\frac{m\pi t_L}{b}\sin\frac{j\pi y_L}{b}\left(\frac{a}{p_1\pi}\sin\left(\frac{p_1\pi vt}{a}\right)\sin\left(\frac{p_1\pi r}{2a}\right)-\frac{a}{d_1\pi}\sin\left(\frac{d_1\pi vt}{a}\right)\sin\left(\frac{d_1\pi r}{2a}\right)\right) - \frac{4\pi^2n^2}{a^3b}v^2Q_{mm}(t)\sin\frac{m\pi y_L}{b}\sin\frac{j\pi y_L}{b}\left(\frac{a}{\pi d_1}\cos\left(\frac{d_1\pi vt}{a}\right)\sin\frac{d_1\pi r}{2a} - \frac{a}{\pi p_1}\cos\left(\frac{p_1\pi vt}{a}\right)\sin\left(\frac{p_1\pi r_1}{2a}\right)\right)\Bigg]\Bigg] \quad (21)$$

Where, $d_1 = n - i$, $p_1 = n + i$ and $n \neq i$ and

$$\ddot{Q}_{nm}(t)+2\gamma\dot{Q}_{nm}(t)+\lambda_{nm}Q_{nm}(t)= \frac{1}{M_1 r}\Bigg[\frac{4M_2 ga}{\sqrt{ab}\,n\pi}\sin\frac{i\pi y_L}{b}\sin\frac{i\pi vt}{a}\sin\frac{i\pi r}{2a} - \sum_{n=1}^{N}\sum_{m=1}^{M}\Bigg[\frac{4}{ab}\sin\frac{m\pi y_L}{b}\sin\frac{i\pi y_L}{b}\ddot{Q}_{nm}(t)\left(\frac{r}{2}-\frac{a}{2\pi n}\cos\left(\frac{2n\pi vt}{a}\right)\sin\left(\frac{n\pi r}{a}\right)\right) + \frac{8n\pi}{a^2b}v\dot{Q}_{mm}(t)\sin\frac{m\pi t_L}{b}\sin\frac{j\pi y_L}{b}\left(\frac{a}{2n\pi}\sin\left(\frac{2n\pi vt}{a}\right)\sin\left(\frac{n\pi r}{2a}\right)\right) - \frac{4\pi^2n^2}{a^3b}v^2Q_{mm}(t)\sin\frac{m\pi y_L}{b}\sin\frac{j\pi y_L}{b}\left(\frac{r}{2}-\frac{a}{2n\pi}\cos\left(\frac{2n\pi vt}{a}\right)\sin\left(\frac{n\pi r}{a}\right)\right)\Bigg]\Bigg],\ n=i \quad (22)$$

Clearly, closed form solutions to Eq. 21 and 22 are not possible. However, one can seek numerical solutions. The numerical method considered is a central difference method.

RESULTS AND DISCUSSION

In this study, numerical results are presented in graphical and tabular forms. The effects of viscous damping and other parameters such as velocity of the moving load were discussed.

Table 1: Deflection W(a/2, b/2) in (mm) for various r

Time (ts)	r = 0.05	r = 0.5	r = 0.1
0	0	0	0
0.0667	0.1338	0.1337	0.1333
0.1333	0.2055	0.2053	0.2047
0.2	0.2908	0.2905	0.2896
0.2667	0.3688	0.3684	0.3673
0.3334	0.4438	0.4434	0.4420
0.4	0.5137	0.5132	0.5116
0.4667	0.5780	0.5774	0.5756
0.5334	0.6360	0.6353	0.6334
0.6	0.6870	0.6863	0.6841
0.6667	0.7304	0.7297	0.7274
0.7334	0.7659	0.7651	0.7628
0.8	0.7930	0.7922	0.7897
0.8667	0.8114	0.8105	0.8080
0.9334	0.8209	0.8200	0.8175
1.0001	0.8214	0.8205	0.8180
1.0667	0.8129	0.8120	0.8095
1.1334	0.7955	0.7947	0.7922
1.2001	0.7693	0.7686	0.7662
1.2667	0.7348	0.7340	0.7318
1.3334	0.6922	0.6915	0.6893
1.4001	0.6420	0.6413	0.6394
1.4667	0.5848	0.5842	0.5824
1.5334	0.5211	0.5206	0.5190
1.6001	0.4517	0.4512	0.4499
1.6668	0.3770	0.3766	0.3754
1.7334	0.2961	0.2958	0.2949
1.8001	0.2015	0.2013	0.2007
1.8668	0.0497	0.0496	0.0494
1.9334	0	0.0783	0.07810
2	0	0	0

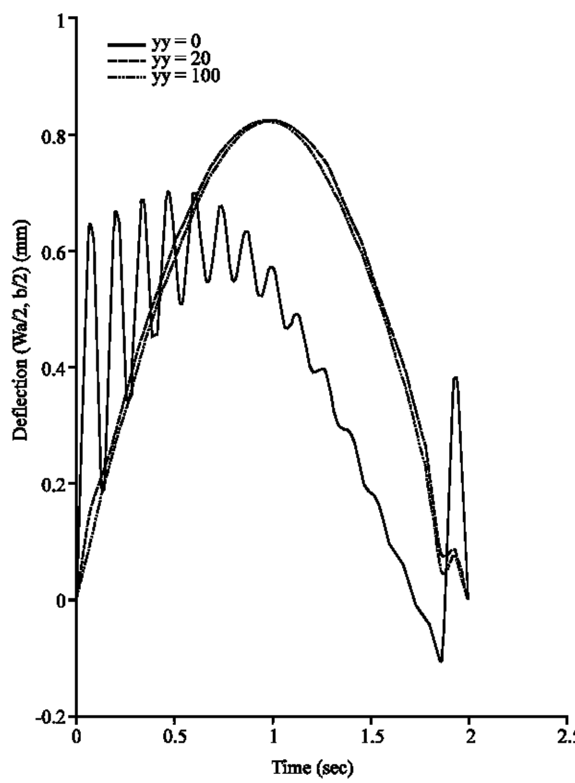

Fig. 1: The deflection in mm at different time for various values of yy

The mathematical model discussed here in is a rectangular plate of dimension a = 10 m by b = 5 m. The other parameters have been defined as follows: Poisson ratio $\upsilon = 0.2$, $E = 2.109\times10^7 \text{N m}^{-2}$, $v = 1.5 \text{ m s}^{-1}$ and $\gamma = 0.0$, 20 and 100. The various lengths of the load are r = 0.05 0.5 and 1.0.

Figure 1 shows the deflection curve at the middle of the plate for various value of yy, where yy = γ. It is observed that for fixed value r = 0.5, the amplitude of the maximum deflection of the mid-span of the plate increases with increase in yy. The absence of damping coefficient shows higher frequency of vibration as depicted in Fig. 1 for yy = 0.

Table 1 shows the deflection of the simply supported plate for various values of r for a fixed length yy. It is observed that for a fixed value of yy, the maximum amplitude of the deflection of the plate decreases with increase in r.

CONCLUSION

The analysis of damped rectangular plates to uniform partially distributed moving masses has been carryout in this paper. The governing partial differential equation for the dynamic of damped rectangular plates was simplified and reduced to a set of coupled ordinary differential equations. The resultant equations were solved using finite difference method. Discussion for simply supported plates was presented. The result shows that increase in damping coefficient (γ) yields an increase in the amplitude of the middle deflection of the plate. The effect of the distribution of the line load on the plate was discussed.

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