

Modelling the Asymmetric Volatility with Combine White Noise Across Australia and United Kingdom GDP Data Set

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Abstract: The objective of this investigation presents Combine White Noise (CWN) Model that outperform the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH). This study employed the GDP data set of two countries to compare the results of the new CWN Model with existing EGARCH Model. The empirical analysis for the two countries revealed that CWN proved to be more appropriate model. The inference of CWN yielded a reliable outcome of lower information criteria with higher log likelihood values in each country data evaluation while EGARCH revealed higher information criteria and lower log likelihood values when comparing the two models. CWN provided a better forecast output with lower forecast errors values in each country whereas EGARCH offered higher values of forecast errors. CWN estimation was efficient in both countries as the determinant of the residual of covariance matrix is approximately zero while AU has better estimation efficiency than UK. This will assist the policy makers to plan for reliable economy of a society.

Key words: Appropriate model, combine white noise, determinant residual covariance, forecast accuracy, log likelihood

INTRODUCTION

Asymmetric time series is the reaction to shocks and this reaction depends on two rules which can be either negative or positive shocks (Wecker, 1981). The examination of the degree to which the asymmetry is reflected in the variances series and its importance in the dynamics of the GDP data set (Albu *et al.*, 2015a). Albu *et al.* (2015b) employing proxy resolved by the differences in the variance estimated by two kinds of volatility models with normally distributed errors and skewness errors. The existence of the asymmetry in volatilities is captured by these differences.

Albu *et al.* (2015b) focus firstly on the asymmetric volatilities in European financial markets building on the outputs to examine the relationship between the normally distributed errors and skewness distributed errors. As examined in the analysis there is heavy tails in the distributions of the differences between the two cases. The outputs reveal that asymmetries in volatility are closer to co-movement in the European equity markets, based on their study. They integrate a series of GARCH initiates with the Markov switching approach for modeling.

Three GARCH in mean, namely, Glosten, Jannathan and Runkle GARCH, Exponential GARCH and Power GARCH are employed and perform very well in discovering asymmetric volatility in stock market. The findings reveal that; the volatility causes negative returns that sustain the volatility feedback effect and higher volatility is produced by negative return shocks than the positive shocks of equal magnitude which is in support of the leverage effect (Thakolsri *et al.*, 2015).

EGARCH and TGARCH identify the leverage effect in twenty nine countries with stock market price. Negative shocks returns produce more volatility than the positive shocks of an equal magnitude. Bad news has more effect on volatility of the value of data distribution than the good news (Elgammal and Najjar, 2015). The TGARCH and EGARCH obtain equal length on the conditional volatility of different lengths (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer and Hafner, 2014; Kamaruzzaman and Isa, 2015; Alhagyan *et al.*, 2015; Farnoosh and Ebrahimi, 2015; Mutunga *et al.*, 2015; Chang *et al.*, 2015). Leverage and asymmetry are of similar case (McAleer, 2014; McAleer and Hafner, 2014; Chang *et al.*, 2015). GARCH modelling the leverage effect is not possible because any restriction imposed will be

positivity restriction which has no leverage effect, since the negative correlation between returns shocks and subsequent shocks to volatility is the leverage effect (McAleer and Hafner, 2014; Kamaruzzaman and Isa, 2015; Chang *et al.*, 2015).

MATERIALS AND METHODS

The data sets of UK and AU were obtained from the Data Stream of Universiti Utara Malaysia library for Gross Domestic Product (GDP) quarterly data from 1960Q-2014Q4 and 1960Q3-2015Q2, respectively. Autoregression Model is described as:

$$y_t = \phi y_{t-1} + \varepsilon_t \tag{1}$$

allow ε_t to be a real value time of stochastic procedure with adequate information through time t to be I. Taking the GARCH Model to be:

$$\varepsilon_t | I_{t-1} \sim N(0, h_t) \tag{2}$$

$$h_t = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i} \tag{3}$$

$$= \omega + A(L)\varepsilon_t^2 + B(L)h_t$$

the EGARCH specification is:

$$\log h_t = \alpha + \beta |z_{t-1}| + \delta z_{t-1} + \gamma \log h_{t-1}, |\gamma| < 1 \tag{4}$$

where, $z_t = \varepsilon_t / \sqrt{h_t}$ is the standardized shocks $z_t \sim iid(0, \alpha)$ with $|\gamma| < 1$ as the stability test. The impact is asymmetric if $\delta \neq 0$. But if leverage is present then $\delta < 0$ and $\delta < \beta < -\delta$. While both β and δ must be positive in which the variances of two stochastic processes are. Then, modeling leverage effect is not attainable (McAleer, 2014; McAleer and Hafner, 2014). The Combine White Noise model is deduced from the unequal variances (heteroscedastic errors) behaviors in the process of estimation being exhibited by EGARCH Model. To overcome with the heteroscedasticity challenges, the standardized residuals of EGARCH errors are transformed into equal variances (white noise). Every equal variances series are converted to models by regression models.

Moving average process is employed for the estimation of these white noise series which is called combine white noise:

$$Y_1 = \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + \dots + \theta_{1q}\varepsilon_{1,t-q}$$

$$Y_2 = \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + \dots + \Phi_{2q}\varepsilon_{2,t-q}$$

...

$$Y_j = \varepsilon_{jt} + \phi_{j1}\varepsilon_{j,t-1} + \phi_{j2}\varepsilon_{j,t-2} + \dots + \phi_{jq}\varepsilon_{j,t-q}$$

$$Y_{jt} = \sum_{j=1}^q \theta_j \varepsilon_{j,t-q} + \sum_{j=1}^q \Phi_j \varepsilon_{j,t-q} + \dots + \sum_{j=1}^q \phi_j \varepsilon_{j,t-q}$$

$$= A(L)\varepsilon_t + B(L)\varepsilon_t + \dots + \varepsilon_t [A(L) + B(L) + \dots] \tag{5}$$

$$Q_{\varepsilon t} = U_t \tag{6}$$

The can be written as:

$$Y_t = U_t U_t \sim N(0, \sigma_c^2) \tag{7}$$

where, $A(L)+B(L)+\dots = Q$ which are the matrix polynomial. U_t is the error term of combine white noise model and the combination of equal variances is σ_c^2 . The combine variances of the combine white noise is:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \dots \tag{8}$$

Bayesian Model averaging output revealed the best two variances in the best two models to be employed. It follows as:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{9}$$

write σ_c^2 as the combine white noise:

$$\sigma_c^2 = w^2 \sigma_1^2 + (1-w)^2 \sigma_2^2 + 2\rho w \sigma_1(1-w)\sigma_2 \tag{10}$$

where, W represents the balanced weight for the model and minimum σ_c^2 emerges when the equation is differentiated with respect to W and equate to zero in order to get:

$$w = \frac{\sigma_c^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2} \tag{11}$$

where the correlation ρ is the intra-class correlation coefficient used for a reliable measurement.

RESULTS AND DISCUSSION

The UK GDP and AU GDP data graph indicated upward trend which were characteristics of non-stationary

Table 1: Histogram-normality and ARCH tests for UK and AU data

| Tests | Coefficient/value UK data | Probability | Coefficient/value AU data | Probability |
|---------------------|---------------------------|-------------|---------------------------|-------------|
| Normal test | | | | |
| Standard deviation | 0.966867 | | 1.055567 | |
| Skewness | 0.375454 | | 0.364743 | |
| Kurtosis | 7.014953 | | 3.949680 | |
| Jarque-Bera | 152.2389 | 0.000000 | 13.08565 | 0.001440 |
| ARCH tests | | | | |
| F-statistic | 0.060187 | 0.006400 | 4.908379 | 0.000300 |
| Obs. R ² | 10.060730 | 0.005300 | 22.57658 | 0.000400 |

Table 2: UK data ARCH, GARCH and CWN coefficients, information criteria and log likelihood

| Variables | α | β | δ | γ | AIC | BIC | HQ | LL |
|-----------|----------------------|----------------------|---------------------|---------------------|---------|---------|---------|---------|
| ARCH | 0.334938 (0.0003) | 0.424743 (0.0000) | | | 2.68436 | 2.74646 | 2.70944 | -288.60 |
| EGARCH | 0.291288 (0.0000) | 0.218189 (0.0106) | 0.09329 (0.1228) | 0.98997 (0.0000) | 2.35147 | 2.37644 | 2.46014 | -249.31 |
| CWN | | | | | -0.4444 | -3.3515 | | 383.158 |

The coefficient of the mean equation is α . β and δ are the coefficients of the variance equations, γ is the coefficient of the log of variance equation. Probability Value (PV) in parentheses

Table 3: AU data ARCH, GARCH and CWN coefficients, information criteria and log likelihood

| Variables | α | β | δ | γ | AIC | BIC | HQ | LL |
|-----------|--------------------|--------------------|--------------------|-------------------|---------|---------|---------|----------|
| ARCH | 0.13645 (0.000) | 0.13623 (0.006) | | | 2.90733 | 2.96942 | 2.93241 | -312.89 |
| EGARCH | -0.0462 (0.448) | -0.0157 (0.811) | 0.02031 (0.422) | 1.0106 (0.000) | 2.65324 | 2.76191 | 2.69713 | -282.20 |
| CWN | | | | | 11.1777 | 11.3635 | | -1211.97 |

series. The inspection of the volatility clustering, long tail skewness and excess kurtosis were the description of heteroscedasticity. The data set were transformed in returns series for both countries data set. As reported in Table 1, UK data disclosed right tail skewness, excess kurtosis and Jarque-Bera test was highly significant which indicated non-normality with standard deviation less than one.

The AU data also disclosed right tail skewness, excess kurtosis and Jarque-Bera test was significant with non-normality and standard deviation greater than one. The AU data distribution was wide spread compare to UK data distribution. Skewness were both positive in the two countries. The excess kurtosis in AU was not as high as in UK. The Jarque-Bera in UK was significantly a little higher than AU Both of them were non normal.

Table 1 disclosed ARCH presence in both countries data distribution. In normal tests, AU data distribution indicated non normality which was the characteristics of heteroscedasticity whereas, ARCH tests indicated ARCH effect in the data distribution.

Table 2 and 3 disclosed the least values of information criteria and log-likelihood high values which offered the right model among ARCH and other GARCH family models. Since EGARCH Model had least values of AIC, BIC and HQ with high log-likelihood values. UK data have a better estimation with least values of information criteria and high log likelihood values for ARCH and EGARCH estimation than the AU data

estimation. In Table 2 and 3 the CWN disclosed a better estimation UK data CWN in Table 2 have a better estimation with least values of information criteria and high log likelihood values, than the AU data estimation in Table 2.

Leverage is not viable using GARCH family model; because, the statistical properties for modeling leverage effect is not available (McAleer, 2014; McAleer and Hafner, 2014). Heteroscedasticity has not been exclusively erased in the existing models (White, 1980; Antoine and Lavergne, 2014; Uchoa *et al.*, 2014). The weaknesses of EGARCH were strengthened by disintegrating EGARCH Model (GARCH errors) into white noise series. White noise models were modeled by regression models. Bayesian model averaging output revealed the first best models of which the best two models were chosen (Asatryan and Feld, 2015). Linear regression with autoregressive errors were fitted including zero mean and variance one (Higgins and Bera, 1992). The BMA chosen models agreed with the fitted linear regression. Table 4 for UK data disclosed that the two models had unequal variances because its value was $p < 0.05$. Table 5 for AU data disclosed that the two models had equal variances because its value was $p > 0.05$ (Lim and Loh, 1996; Boos and Brownie, 2004; Bast *et al.*, 2015).

CWN emerged as appropriate model for estimation and forecasting in both countries according to Table 6 in comparison with EGARCH Models. In AU the CWN and EGARCH had minimum forecast errors values when

Table 4: Levene's test for equal variances for UK data (independent samples test)

| Variables | Levene's test for equality of variances | | t-test for equality of means | | Mean difference | SE difference | 95% Confidence interval of the difference | | |
|-----------------------------|---|-------|------------------------------|---------|-----------------|---------------|---|----------|---------|
| | F-value | Sig. | t-test | df | | | Sig. (2 tailed) | Lower | Upper |
| B equal variances assumed | 5.504 | 0.019 | 1.133 | 438.000 | 0.258 | 0.01545 | 0.01364 | -0.01135 | 0.04226 |
| Equal variances not assumed | | | 1.133 | 255.502 | 0.258 | 0.01545 | 0.01364 | -0.01140 | 0.04231 |

Table 5: Levene's test for equal variances for UK data independent samples test

| Variables | Levene's test for equality of variances | | t-test for equality of means | | Mean difference | SE difference | 95% Confidence interval of the difference | | |
|-----------------------------|---|-------|------------------------------|---------|-----------------|---------------|---|--------|--------|
| | F-value | Sig. | t-test | df | | | Sig. (2 tailed) | Lower | Upper |
| B equal variances assumed | 0.045 | 0.833 | -2.993 | 438.000 | 0.0003 | -01409 | 0.00471 | 02334 | 0.0048 |
| Equal variances not assumed | | | -2.993 | 424.759 | 0.003 | -01409 | 0.00471 | -02335 | -00480 |

Table 6: The summary of GARCH and CWN models estimation and forecasting evaluation for UK and AU data set

| Variables | CWN UK data | GARCH | CWN AU data | GARCH |
|---------------------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Estimation residual diagnostic | | | | |
| Stability test (Lag structure) | Stable | Stable | Stable | Stable |
| Correlogram (square) residual | covariance stationary | Stationary | Covariance stationary | Stationary |
| Portmanteau tests | No autocorrelation | No autocorrelation | No autocorrelation | No autocorrelation |
| Histogram-normality tests | Not normal | Not Normal | Not normal | Appear normal |
| ARCH test | No ARCH effect | No ARCH effect | No ARCH effect | No ARCH effect |
| Dynamic forecast evaluation | | | | |
| RMSE | 0.167297 | 0.653369 | 0.0333325 | 0.489917 |
| MAE | 0.040005 | 0.408789 | 0.007404 | 0.366493 |
| MAPE | 1.427953 | 169.7009 | 1.233974 | 107.6098 |
| Residual diagnostic | | | | |
| Correlogram (square) residual | Stationary | Stationary | Stationary | Stationary |
| Histogram-normality tests | Not normal | Not normal | Not normal | Appear normal |
| Serial correlation LM tests | No serial correlation | No serial correlation | No serial correlation | No serial correlation |
| Heteroscedasticity test | No ARCH effect | No ARCH effect | No ARCH effect | No ARCH effect |
| Stability diagnostic | | | | |
| Ramsey reset tests | Stable | Stable | Stable | Stable |
| Determinant residual covariance | 0.000104 | 5.75E-06 | | |

comparing with UK forecast errors. Forecasting was better using AU GDP data than using UK GDP data as revealed by the results of the empirical analysis. The determinant of the residual of covariance matrix values revealed that CWN was efficient in the two countries but AU was more efficient with the value closer to zero.

CONCLUSION

CWN offered a better and efficient estimation than the EGARCH estimation. CWN met the conditions except Levene's test of equal variances using UK data. While CWN met all the necessary conditions using AU data. Despite the fact, that the CWN for UK failed the Levene's test of equal variances, the results of CWN estimation outperformed the existing models estimation.

The CWN estimation produced the best outputs as CWN has the least values of forecast errors which were better outputs when weigh against the GARCH model dynamic evaluation forecast errors in both UK and AU (Ismail and Tuan Muda, 2006; Fildes *et al.*, 2011; Lazim, 2013). CWN estimation was efficient in both countries as the determinant of the residual of covariance matrix were

approximately zero while AU has better estimation efficiency than UK. The empirical analysis for the two countries gave assurance that CWN proved to be more appropriate model. Every data that has conditional heteroscedasticity can be modeled by CWN. The estimated CWN have better estimation than the existing models and equally improved the forecast precision. This is a better assurance for policy makers to have reliable economic planning.

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