Research Journal of Applied Sciences 11 (11): 1432-1435, 2016
ISSN: 1815-932X

Group Chain Sampling Plans Based on Truncated Life Test for Inverse Rayleigh Distribution

Mohd Azri Pawan Teh, Nazrina Aziz and Zakiyah Zain
Department of Mathematics and Statistics, School of Quantitative Sciences,
UUM College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia

Abstract: This study introduces group chain sampling plans for inverse Rayleigh distribution when the life test is truncated at a pre-assumed time. The design parameters such as the number of optimal groups and operating characteristic values are calculated by satisfying the consumer's and producer's risks at a certain specified quality level. Quality level is defined in terms of mean with assumption that the test termination time and acceptance number are pre-fixed. An example is provided for illustrative example.

Key words: Group chain sampling plan, consumer's risk, producer's risk, inverse Rayleigh distribution, operating characteristic values

INTRODUCTION

Over the decades, there was so much development in the area of acceptance sampling. One basic development for the attribute sampling plan is the introduction of single sampling plan, followed by chain sampling plan, group sampling plan and group chain sampling plan.

The single sampling plan was introduced by Epstein (1954) for exponential distribution. For the single sampling plan, it succeeded to discriminate between a good lot and a bad lot. However, the plan itself has two drawbacks. The first drawback is the probability of lot acceptance start to drop very quickly when acceptance number is zero or when acceptance number is one. The second drawback is it does not provide a platform for multiple inspections.

The chain sampling plan was proposed by Dodge (1955) to solve the drawbacks in the single sampling plan. For the chain sampling plan, it is looking at the cumulative of the defective products from the preceding lots as the operating procedure. The chain sampling plan solved the drawback of the single sampling plan. That is, the probability of lot acceptance is more accurate while maintaining the minimum sample size. However, the plan has one drawback. That is, it does not provide a platform for multiple inspections.

The group sampling plan was further developed by Aslam and Jun (2009) for inverse Rayleigh distribution. The operating procedure for the group sampling plan is several products will be put into a group and inspection will be conducted simultaneously. Therefore, this operating procedure helps to reduce the inspection time and cost. Plus, the group sampling plan does provide a platform for multiple inspections. However, the plan has one disadvantage. That is, the probability of lot acceptance starts to drop quickly when acceptance number is zero or when acceptance number is one.

The group chain sampling plan was initiated by Mughal *et al.* (2015) for Pareto distribution of the 2nd kind. The group chain sampling plan was a combination of the two established plans; group sampling plan and chain sampling plan. The operating procedure for the group chain sampling plan is several products will be placed into a group and it is looking at the cumulative of defective products from the preceding lots. The group chain sampling plan succeeded in reducing the inspection time and cost. At the same time, the probability of lot acceptance is more accurate while maintaining the minimum sample size. Besides that, it does provide a platform for multiple inspections.

Therefore, in this study, we develop a group chain sampling plan for sentencing lots when the lifetime of a product follows inverse Rayleigh distribution. Tables are constructed for the selection of optimal group parameters.

MATERIALS AND METHODS

Inverse Rayleigh distribution: Inverse Rayleigh distribution has been used widely especially in the area of Bayesian estimator. The distribution has been used for Bayesian and non-Bayesian estimator (Soliman *et al.*, 2010), Bayes estimator for singly and doubly type 2

Corresponding Author: Nazrina Aziz, Department of Mathematics and Statistics, School of Quantitative Sciences, UUM College of Arts and Sciences, Universiti Utara Malaysia, 06010 Sintok, Kedah, Malaysia

censored data (Feroze and Aslam, 2012) and Bayesian estimator when the data are left censored (Sindhu *et al.*, 2013). In the acceptance sampling plan, Rosaiah and Kantam (2005) and Rao (2012) developed the single sampling plan using the Inverse Rayleigh distribution. For inverse Rayleigh distribution, the Cumulative Distribution Function (CDF) is given by:

$$F(t;\sigma) = \exp(-\frac{\sigma^2}{t^2}), t > 0 \quad (1)$$

where, σ is the scale parameter. The mean of the distribution is written as:

$$\mu = \sqrt{\pi\sigma} \quad (2)$$

Probability of failure for a product, P during the test termination time, t_0 is given by Eq. 4 where t_0 is a multiple of the specified mean life, μ_0 and specified constant, a, written as:

$$t_0 = a\mu_0 \quad (3)$$

therefore, can be estimated by:

$$p = \exp\left[-\frac{1}{\pi a^2}\left(\frac{\mu}{\mu_0}\right)^2\right] \quad (4)$$

Design of the group chain sampling plan: The group chain sampling plan is applied with the following steps:

- For each lot, find the optimal number of g groups and allocate r items to each group such that the sample size is given by
- Accept the lot when and reject the lot if
- Accept the lot if and continue the inspection if no defectives are found in the preceding i lots

The group chain sampling plan is characterized by two design parameters g and i. The probability of rejecting a good lot is known as producer's risk while the probability of accepting a bad lot is known as consumer's risk. When deciding the parameters of the proposed sampling plan we make use of the consumer's risk. Normally, the consumer's risk is expressed by the consumer's confidence level. Suppose the confidence level is p* then the consumer's risk becomes $\beta = 1\text{-}p^*$. The number of optimal groups in the group chain sampling plan is determined by using:

Table 1: The number of optimal groups for the group chain sampling plan for inverse Rayleigh distribution

			α					
β	r	i	0.7	0.8	1.0	1.2	1.5	2.0
25	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	1	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	3	2	2	2	1	1
	3	2	2	2	1	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	4	3	2	2	2	1
	3	2	3	2	2	1	1	1
	4	3	2	2	1	1	1	1
	5	4	2	1	1	1	1	1

$$P_a(P) \le \beta \quad (5)$$

The probability of lot acceptance for the group chain sampling plan is given by:

$$p_a(p) = (1-p)^{gr} + grp(1-p)^{gr-1}(1-p)^{gri} \quad (6)$$

In Table 1, the number of optimal groups is presented satisfying Eq. 5 when β = 0.25, 0.10, 0.05, 0.01; α = 0.7, 0.8, 1.0, 1.2, 1.5, 2.0; r = 2(1)5 and i = 1(1)4 . These design parameters are consistent with the previous literatures (Mughal *et al.*, 2015; Teh *et al.*, 2016a, b) for developing the group chain sampling plan using inverse Rayleigh distribution. Once the optimal number of groups is calculated, the probability of lot acceptance can be calculated by using different values of quality level. For a fixed and the operating characteristic values as a function of the mean ratio, μ/μ_0.

RESULTS AND DISCUSSION

Operating Characteristic (OC) function: The probability of lot acceptance can be defined as a function of the deviation of specified mean life from the true mean life. This function is called Operating Characteristic (OC) function of the sampling plan. Once the optimal number of groups is obtained, the probability of lot acceptance can be calculated at different values of quality level.

Illustrative example: Table 1 shows the number of optimal groups for the group chain sampling plan for inverse Rayleigh distribution at different quality levels. For

Table 2: The operating characteristic values having, for the group chain sampling plan for inverse Rayleigh distribution

β	g	α	2	4	6	8	10	12
0.25	1	0.7	0.0464	0.7992	0.9993	1.0000	1.0000	1.0000
	1	0.8	0.0966	0.9402	1.0000	1.0000	1.0000	1.0000
	1	1.0	0.2767	0.9981	1.0000	1.0000	1.0000	1.0000
	1	1.2	0.5448	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.5	0.8851	1.0000	1.0000	1.0000	1.0000	1.0000
	1	2.0	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000
0.10	1	0.7	0.0464	0.7992	0.9993	1.0000	1.0000	1.0000
	1	0.8	0.0966	0.9402	1.0000	1.0000	1.0000	1.0000
	1	1.0	0.2767	0.9981	1.0000	1.0000	1.0000	1.0000
	1	1.2	0.5448	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.5	0.8851	1.0000	1.0000	1.0000	1.0000	1.0000
	1	2.0	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	2	0.7	0.0021	0.5477	0.9972	1.0000	1.0000	1.0000
	1	0.8	0.0966	0.9402	1.0000	1.0000	1.0000	1.0000
	1	1.0	0.2767	0.9981	1.0000	1.0000	1.0000	1.0000
	1	1.2	0.5448	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.5	0.8851	1.0000	1.0000	1.0000	1.0000	1.0000
	1	2.0	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000
0.01	2	0.7	0.0021	0.5477	0.9972	1.0000	1.0000	1.0000
	2	0.8	0.0093	0.8219	0.9999	1.0000	1.0000	1.0000
	1	1.0	0.2767	0.9981	1.0000	1.0000	1.0000	1.0000
	1	1.2	0.5448	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.5	0.8851	1.0000	1.0000	1.0000	1.0000	1.0000
	1	2.0	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000

instance, if the consumer's risk is 0.05, the number of testers is 4, the allowable acceptance number is 3 and the specified constant is 0.7, then the number of optimal groups is 2.

Table 2 illustrates the operating characteristic values when the number of testers is 4 and the allowable acceptance number is 3. For example, the probability of lot acceptance is 0.5477 when the consumer's risk is 0.05, the number of optimal groups is 2, the specified constant is 0.7 and the mean ratio is 4.

Suppose that an experimenter is interested that the true unknown mean life is at least 1000h. The experiment is designed such that it will be terminated at 700h with the consumer's risk of 0.05. Based on the consumer's risk values and time termination ratios, the number of optimal groups is determined by using the group chain sampling plan based on truncated life test. Assume that the lifetime of a product follows inverse Rayleigh distribution. If the experimenter designs the experiment based on the number of testers, r = 4 and the preceding samples, i = 3, then the number of optimal groups is 2. Therefore, the design parameters for the group chain sampling plan are (a, r, i, g) = (0.7, 4, 3, 2). This mean the experimenter needs to select a random sample of size 8 from the submitted lot and place 4 products to each of the 2 groups. The lot will be accepted if not more than one defective is recorded within 700 hours and no defectives products are found in the next 3 subsequent samples. If the experiment has the same design parameters, the probability of lot acceptance increases from 0.5477-0.9972 when the mean ratio increases from 4-6 as shown in Table 2.

CONCLUSION

In this study, we developed a group chain sampling plan based on truncated life test when the lifetime of a product follows Inverse Rayleigh distribution. The group chain sampling plan is applied to the distribution with the purpose of reducing the cost and inspection time. For the proposed plan, the sample size for the plan is $n = g \times r$. The results have shown that the number of optimal groups decreases as the specified constant increases. The operating characteristic value increases when the quality level increases and reaches the maximum value of 1 μ/μ_0 when is >2. Our proposed plan can be further applied to other lifetime distributions as well as other quality and reliability characteristics.

GLOSSARY OF SYMBOLS

g: Number of optimal groups
r: Number of testers
n: Sample size
d: Number of defective products
i: Allowable acceptance number
α: Producer's risk
β: Consumer's risk
$P_a(P)$: Probability of lot acceptance
μ/μ_0 :Mean ratio
t_0: Test termination time
p: Quality level

REFERENCES

Aslam, M. and C.H. Jun, 2009. A group acceptance sampling plan for truncated life test having weibull distribution. J. Appl. Stat., 36: 1021-1027.

Dodge, H.F., 1955. Chain sampling inspection plans. Ind. Qual. Control, 11: 10-13.

Epstein, B., 1954. Truncated life tests in the exponential case. Ann. Mathe. Stat., 25: 555-564.

Feroze, N. and M. Aslam, 2012. On posterior analysis of inverse Rayleigh distribution under singly and doubly type II censored data. Intl. J. Probab. Stat., 1: 145-152.

Mughal, A.R., Z. Zain and N. Aziz, 2015. Time truncated group chain sampling strategy for pareto distribution of the 2^(nd) kind. Res. J. Appl. Sci. Eng. Technol., 10: 471-474.

Rao, K., 2012. Acceptance sampling plans for percentiles based on the inverse rayleigh distribution. Electron. J. Appl. Stat. Anal., 5: 164-177.

Rosaiah, K. and R.R.L. Kantam, 2005. Acceptance sampling based on the inverse Rayleigh distribution. Econ. Quality Control, 20: 277-286.

Sindhu, T.N., M. Aslam and N. Feroze, 2013. Bayes estimation of the parameters of the inverse rayleigh distribution for left censored data. ProbStat Forum, 6: 42-59.

Soliman, A., E.A. Amin and A.A. Abd-ElAziz, 2010. Estimation and prediction from inverse Rayleigh distribution based on lower record values. Applied Math. Sci., 4: 3057-3066.

Teh, M.A.P., N. Aziz and Z. Zain, 2016a. Group chain sampling plans based on truncated life tests for log-logistic distribution. Intl. J. Appl. Eng. Res., 11: 8971-8974.

Teh, M.A.P., N. Aziz and Z. Zain, 2016b. Time truncated group chain sampling plans for rayleigh distribution. Global J. Pure Appl. Math., 12: 3693-3699.