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Modeling Police Patrol Routing and its Problem-Solving Technique Based on the Ant Colony Optimization Algorithm(case Study: Iran's Police)

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Abstract: In light of the fact that crime prevention is of high importance on the issue of ensuring security within a community which is mostly carried out by police patrols of the police stations, making decisions on taking the best approach to plan and schedule police patrol routing may result in the increase in efficiency of these patrols. In this study, in order to reach an optimal solution to the police patrol routing problem and enhance the efficiency of the patrols, a routing model was proposed based on the stochastic approach to vehicle routing problem and by utilizing the graph theory and mathematical modeling approach. Then, the meta-heuristic ant colony optimization algorithm was utilized to find an efficient way of solving the model. After determining the police patrol routes taking into account the obtained optimized values for the objective functions, the suitable number of patrollers and headways (i.e. time interval between two patrols) was also determined.

Key words: Stochastic Vehicle Routing Problem (SVRP), ant colony optimization algorithm, police patrol, headways, police station

INTRODUCTION

Human being has always been trying to prevent any illegal act that endangers public order by various means and to restore peace to the society and the area in which they live. Due to the importance of police patrols in crime prevention and limited resources available (e.g. manpower, vehicles, etc.), we are face with the issue of "planning patrol routing", so as to be able to maximize coverage of important locations (hot spots) at minimum cost (length of patrol route). In addition, as the police stations are in charge of providing the society with security and crime prevention, planning the police patrol routes can result in productivity and cost reduction of these patrols. Accordingly, in an attempt to make efficient use of the available resources and to decrease the costs, we are aimed at designing a specific patrolling route with maximize coverage of the hot spots at minimum cost for all the urban areas. In general, our goals in modeling the police patrol routing are as the following:

- Increasing community security and reducing crime
- Creating scientific and effective approach to police patrols and increasing the efficiency of police forces
- Efficient and effective use of police resources (manpower, vehicles, etc.)

Details of the aforementioned general objectives are to, for example, reduce the amount of time that an area remains without patrolling, win the confidence of people through constant presence of police in the city and in a timely manner, help to reduce air pollution and create a sense of goodwill in the society.

Theoretical framework

Police patrol: A "patrol" or "patrol officers" are a group of law enforcement officers that are assigned to monitor a specific geographic area. Their duties include responding to calls for service, making arrests, resolving disputes, taking crime reports and conducting traffic enforcement and other crime prevention measures. (Ebtehaj, 2009). The following items are among the most important duties of patrol officers:

- Fighting the evident crimes. (Golduzian 2008)
- Having presence in the hot spots of crime in the region
- Having immediate presence in the scenes of a serious offense
- Performing maneuver in hot spots of crime
- Controlling the main roads and district patrol control and detection of stolen vehicles using the offices and computers
- Supporting the implicit patrols
- Performing detection and prosecution operations
- Carrying out special missions on command

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Vehicle routing problem: The vehicle routing problem is a problem which asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver the goods to a given set of customers?" The vehicles repetitively go out of a warehouse and deliver the goods to the related customers and then return to the warehouse again. Each vehicle has a limited weight capacity and may also have a limited length of path to traverse. Only one vehicle is allowed to serve every customer. The problem is to find a set of routs to meet the restrictions and also minimize the distance traveled by vehicles. (Baker and Ayechew 2003). In practice this is equivalent to minimizing the total distance traveled or minimizing the number of vehicles used.

The variations and specializations of the vehicle routing problem: In reality, there are constraints within the vehicle routing problems, such as the following, that should be satisfied. However, the algorithmic approaches to the problem are adaptable to various constraints. The vehicle routing problems with different constraints takes different names which are as the followings:

- Capacitated Vehicle Routing Problem (CVRP): The vehicles have limited carrying capacity of the goods that must be delivered
- Vehicle Routing Problem with Time Windows (VRPTW): The delivery locations have time windows within which the deliveries (or visits) must be made
- Multiple Depots Vehicle Routing Problem (MDVRP). A company may have several depots from which it can serve its customers
- Vehicle Routing Problem with Pick-up and Delivering (VEPPD): there is the possibility that customers return some commodities to the depots
- Split Delivery Vehicle Routing Problem (SDVRP): It is allowed that the same customer can be served by different vehicles if it reduces overall costs
- Stochastic Vehicle Routing Problem (SVRP): Some of the items may be randomly selected, such as the number of customers, demands, time constraints or travel or service time
- Periodic Vehicle Routing Problem (PVRP): The deliveries may be done only on selected days

Displaying vehicle routing problem as a graph: Vehicle routing problem is one of the most important issues in the operational research. The VRP branch is concerned with the graphs. Graph Theory, in turn, is one of the most extensive operational research topics. On the subject of vehicle routing problem, the road network is described using a graph on which there are arces and vertices representing road and junctions, respectively. The arces, or the roads, are connected to each other through the vertices, or the junctions. On the issues of graphs, the arces and vertices can reveal different realities. The graphs are identified by their arces and vertices; a graph is consisted of a set of vertices and a set of edges. A graph is displayed by a set of G = (V, E) or G (N, A) in which N or V represent the edges and E or A represent the vertices. Another point is the numbers written out on the vertices and edges. These numbers can have different meanings. With this brief introduction to the issue of the graphs, an explanation of the topics related to the vehicle routing problem can be presented. The vehicle routing problem can be expressed by a complete graph.

The proposed approaches to solve the vrp: Because of the nature of the vehicle routing problem, most approaches offered to solve this problem have used the approximate (meta-heuristic) solution techniques. Basically, meta-heuristic techniques to solve the vehicle routing problem are divided into three categories, based on the paper. (Kytojokia *et al*. 2007). hyper heuristic, meta-heuristic, heuristic. Ant colony optimization (ACO) is one of the meta-heuristic approaches to solve difficult problems combined that the successful application of it in VRP (Bullnheimer *et al*. 1999) has been witnessed.

Optimization: Optimization can be defined as allocation of the resources to the best possible way. In other word, optimization refers to the process of finding one or several feasible solution and in fact it is called to a branch of computational science that focuses on finding the best solution to the problem. (Konak and Coit and Smith 2006). Finding an optimal solution is called single objective optimization if an optimization problem involves only one objective function; and when it is concerned with mathematical optimization problems involving more than one objective function it is called multi-objective optimization.

The optimization algorithms: Optimization algorithms are search techniques that aim to find a solution for an optimization problem so that the desired quantity for optimization is optimized based on a defined set of constraints on the variables of the objective function.

The main components of optimization problems: Any optimization problem involves the following components:

- The objective function, which represents the amount that must be optimized
- The variables that affect the value produced by the objective function
- The constraints, which limit can the values that variables can adopt

Therefore, an optimization technique aims to assign valid values (of the allowable range for each variable) to the variables of the problem so that the objective function gets optimized and all the constraints of the problem get satisfied. To achieve this goal, the optimization algorithm seeks a solution from the nominated solutions (called S) in the search space.

Review literature: (Reis *et al*., 2006) in his article, take the first steps to determine the appropriate strategy for effective police patrols using GAPatrol, an instrument for the simulation of the evolutional multi-agent. One of the fundamental aspects that can be seen in their article is the features of GAPatrol that automatically discovers the hot spots of crime. The simulation finds the places like pharmacies, squares and lottery halls as the hot spots of crime. In addition, the proposed method using "education" prevents professional activities of criminals through greater allocation of police resources to better monitor the related areas.

(Ratcliffe *et al*., 2011) in a study examined the experiences of the police foot patrols in Philadelphia. They concluded in their paper that the police foot patrol promotes and increase the presence of police in the community and reduce the fear of crime, while, in general, it will not cause a reduction in the incidences of crime. After 12 weeks, the results indicated a significant decrease in the level of violent behavior in the hot spots of crime.

Moreover, in the field of police patrols optimization, the stochastic optimal patrolling routes were created by (Jiang *et al*., 2011) in which a composition of Getis-Ord Gi and Cross Entropy approach were employed. They also have used the main streets of the city as the hot spots of crime.

We can also refer to the paper published by (Ruan *et al*., 2005) in the field of police patrols in patrolling in a stochastic environment. They examined the hypothesis that patrols have preventive effect and also respond to calls of citizens while patrolling.

Kevin and Qiu (2010) proposed a new technique to determine optimal police patrol areas. This technique employs a traditional maximal covering formulation and an innovative backup covering formulation to provide alternative optimal solutions to police decision makers, the study investigates how solutions generated with maximal covering models can increase the level of police service by finding more spatially efficient allocations of law enforcement resources under varying scenarios. The models and technique proposed in this paper are tested by the Dallas police force and the results show a substantial improvement in the amount of police presence in the incidents of crime as well as total distance traveled.

Chawathe (2007) address the problem of planning police patrol routes to regularly cover street segments of high crime density, or hotspots of crime, with limited police forces. In this research, He developed a new police patrolling strategy based on Bayesian method and ant colony algorithm. In this strategy, virtual marker is laid to mark the visiting history of each crime hotspot and patrollers continuously decide which hotspot to patrol next based on pheromone level and other variables.

Of the other papers on which this issue has been addressed is the study carried out by Chen and Yum (2010). In this paper, a definition of the security level has been proposed by defining the functions of security level. Accordingly, a balanced patrol districting solution has been proposed so that the multiple units assigned for patrolling patrol in the area in a coordinated manner.

In a study conducted by Dana A. (Steil *et al*., 2011) patrol routing algorithms were used to specify the route of the simulated mobile agents patrolling on a roadway system. Uncertainty in the patrol routing is very important and lack of adaptability and predictability can have negative effects on the service providers. The researchers have addressed the police patrol routing problem using different of patrol routing algorithms and they aimed at generating "optimal" patrol routes.

By looking at the papers which have utilized mathematical approaches to devise a routing model we can refer to the model presented by R. Wolfler Calvo (2000). In this paper, a single-objective mixed integer programming function was offered for police patrols at night. Because of the high complexity of the model, an innovative approach was used to solve the problem, in which the problem was converted to two sub-problems.

Also Lauri and Koukam (2008) have presented a paper in which they have used a mathematical approach and two-step innovative approaches to solve police patrol routing problem. In this paper, modeling police patrol operations have been investigated using graph theory and taking account of patrol cars.

The subject of police patrols has also been studied in Iran. Of the studies conducted in this regard is the paper entitled "The causes of the failure to implement GPS in police patrols in the holy city of Mashhad" (Rabie *et al*., 2009) by prof. Mohammad Akhbari *et al*. They studied the causes of failure in using GPS (Global Positioning System) in police patrols in the city of Mashhad. In this study, given the importance of the GPS system, the following four main hypotheses were investigated lack of equipment, poor training staff training programs lack of support from some employees and managers and 4. Inappropriate planning.

We can also refer to the research conducted by Javadian *et al*. (2010) entitled "factors affecting the success of police patrols in preventing of motor vehicle theft (Case Study: West Tehran)." They finally came to the conclusion that there is a meaningful relationship between the variable "disciplinary patrol" and the variables training, environmental features, amount of authority of the police agent officials, operating procedures, vehicle available to police and a timely notification.

Theoretical framework

Stochastic vehicle routing problem: On the issue of the police patrol routing, the routes must be scheduled in such a way that the patrols be able to cover maximum locations of the hot spots of crime at a minimum time. Therefore, the patrols should visit the routs with a strong possibility of incident of crime with a high frequency, so that the probability of the patrol being present in the proximity of the crime scene increases. While, the time parameter should also be taken into consideration. In fact, a complete patrolling circle should be designed in such a way that the routs with a high possibility of crime incident be visited at a minimum time. If we take the possibility of crime incident as customers, the police patrol routing problem is compatible with the stochastic vehicle routing problem on which the length of route (total travel time) must be minimized on a weighed graph.

Ant colony optimization algorithms: In this section, the ant colony optimization algorithms will be discussed. The ant algorithms are multi-functional systems that their behavior is inspired with the real ants and each function represents a superficial ant. These algorithms are successful sample of intelligent systems and are applied on the issues like the classical problem of retailer and relational distant networks.

An examination of ants shows that in the natural world, ants, at first, randomly choose the routes that lead to a specific food, upon returning the food to their colony lay down "pheromone" trails. If other ants find this route, they are likely not to keep travelling randomly but instead they follow the trail, return and reinforce it if they eventually find food. Over time, however, the pheromone trail starts to evaporate, thus reducing its attractive strength. The more time it takes for an ant to travel down the route and back again, the more time the pheromones have to evaporate. A short route, by comparison, gets marched over more frequently and thus the pheromone density becomes higher on shorter paths than longer ones. If there were no evaporation at all, the paths chosen by the first ants would tend to be excessively attractive to the following ones. In that case, the exploration of the solution space would be constrained. This simple idea is put to use to find solutions to difficult optimization problems.

MATERIALS AND METHODS

In this study, in order to solve the problem using graph theory and mathematical modeling approach, a model based on the approach of random vehicle Stochastic Vehicle Routing Problem (SVRP) is presented. Then, we present an effective technique for the solving problem using the meta-heuristic algorithms based on the ant colony system.

After determining the disciplinary (law enforcement) patrol routes according to the optimal values obtained for the objective functions, the optimal number of patrols and headway (interval between police patrols) are also determined to examine the problem of patrolling an area more effectively.

RESULTS

Modeling police patrol routing: There are networks of patrolling routes for law enforcement units. These networks consist of a number of arces representing routes and vertices representing junctions. Each arc has its distinctive features. The features of each acre are:

- The travel time $T^{t\,(i,j)}$: is the sum of the travel times of the arcs on the shortest path from i to j on the original road graph. This value is held constant for each arc
- The possibility of crime incident in each arc per unit of time$P_{oc\,(i,j)}$: the possibility crime incident in each arc follows a specific function that is assumed to be constant in this study
- The average delay caused by the queue: SD_{ij} on every route there is the possibility that one or two route be obstructed due to the reasons like queuing, that can occur at red lights, stop signs, bottlenecks, or any design-based or traffic-based flow constriction. Due to the existence of these bottlenecks vehicles that move upstream will face road capacity reduction. In such a situation, these vehicles will try to pass through the remaining lines until the time that the bottleneck gets unblocked, so, these cars will arrive at the queue and will face an additional delay. This delay depends on a variety of parameters including the distance between car in the queue and bottleneck, percentage of decrease in the volume of traffic and so on

In solving this problem, all the parameters for each arc are assumed equal in both directions. Also, all the routes are assumed two-way street i.e. in each arc (i, j) every vehicle is allowed to move from i to j and vice versa.

The purpose of this scheme is planning a patrolling route for police stations. This plan has three major parts that can severely affect the performance of the patrols. These include:

- Determining the optimal route for patrols
- Determining the appropriate number of fleet
- Setting the time interval

In the following, each of these three parameters will be introduced and examined.
The introduction of the parameters used:
i, j: the vertices of the start and end point of the arcs of the network.
n: the number of vertices on the network.
$T_{t(i,j)}$: The net travelling time of an arc (without taking the delays into consideration).
$T_{t(i,j)}$ Total traveling time of an arc (travel time along a route after a vehicle has left the starting arc, which includes the delay time plus the net traveling time)
D: The length of the considered work shift.
T_p: Total travelling time of the constructed circle.
Tp^{best}: The travelling time of the best (shortest) circle.
h: The interval of the patrols.
DE_q: The delay due to queuing.
SD_{ij}: The average time of delay due to queuing for the cars travelling the arcs.
$P_{oc(i,j)}$: The possibility of crime incident in arc (i,j) per time unit.

Identifying the optimal patrolling routes: The routes should be planned in such a way that patrols can visit the highest number of arcs with a high possibility of crime incident at a minimum time. To this end, the arces with a higher possibility of crime incident should be visited more than the other areas.

But the time parameter should also be taken into consideration. In fact, a full circle should be designed in such a way that, arcs which are hot spots of crime be most visited locations in a shorter time period. This cycle must have the following characteristics: he designed cycle should include all arcs which means that no arc should remain without a visit because there is a possibility of crime incident in each arc.

Total scheduled to return to patrol should not exceed the considered work shift, there is a time limit for planning. But what should be noted is the fact that the travelling time of each arc will change in case of incident of crime, in other word, the net travelling time of an arc($T_{t(i,j)}$) is also a function of The possibility of crime incident in arc ($T_{oct(i,j)}$). Therefore, during planning the patrolling routes, total traveling time of an arc, represented by the parameter $T_{p(i,j)}$ should be taken into consideration (i.e. travel time along a route after a vehicle has left the starting arc, which includes the delay time plus the net traveling time)

On the routing problem, there are a set of routes that are defined by each patrol. The goal is to reach to an optimized patrolling circle so that the following functions get optimized:

Minimizing travelling time of circle: travelling is the sum of all the arcs in the designed cycle Maximizing the number of visits to the possible offenses in the designed cycle: this value is obtained from the total number of sites visited in arcs of the cycle. In this case, if we obtain the possibility of crime incident in every arc per time unit by $P_{oc(i,j)}$ and the travelling time of each arc by $T_{p(i,j)}$ the number of visits to the places with a higher possibility of crime incident during travelling each arc is obtained by the following equation:

$$l_{i,j} = 0.5 \times P_{oc(i,j)} \times T_{p(i,j)} \quad (1)$$

The coefficient 0.5 in the equation (1) represents the fact that there is 50% possibility a crime may occur in front of the patrol. On the other hand, the main limitation in solving the problem is the total travelling time which should not exceed the length of a work shift.

If we take the possibility of crime incident in each arc($P_{oc(i,j)}$) as customers' demand police patrol routing problem will be compatible with the stochastic vehicle routing problem, therefore, the total travelling time on the route should be minimized in the weighted graph. After considering the different approaches to solve the stochastic vehicle routing problem, it was tried to change the stochastic variables of the problem to constant values and then solve the problem. Also in this research, the values of the travelling time which is the function of possibility of crime incident in each arc have been calculated by the same approach. The objective functions of the problem can be defined as the following:

$$\text{Min} = \sum_{i,j \in R_i} Tp_{(i,j)xi,j} \quad (2)$$

$$\text{Min} = \sum_{i,j \in R_i} Tp_{(i,j)} X_{i,j} = \sum_{i,j \in R_i} \left(0.5 \times P_{oc(i,j)} T_{p(i,j)}\right) X_{i,j} \quad (3)$$

If represent the work shift with the parameter D, the limitation that the patrolling time should not exceed the length of the considered work shift is calculated as follows:

$$\sum_{i,j \in R_i} Tp_{(i,j)} X_{i,j} \le D \quad (4)$$

In addition, each of the routes on the network should be visited at least for one time. The following limitation ensures that each route has been visited by the police patrol at least for one time:

$$\sum_{i,j \in R_i} Tp_{(i,j)} \ge 1 \quad (5)$$

$$X_{(i,j)} \in \{0,1\} \forall i, j^{\in} R_i \quad (6)$$

(R_iIncludes all the arces of the designed circle)

As can be seen, both the objective functions and the first limitation depend on the stochastic variable of arc's travelling time ($T_{p(i,j)}$), therefore, in order to solve the problem the travelling time of each arc should be calculated, which is a function of possibility of a crime incident in each arc.

Extracting an appropriate equation to determine the travelling time: Delay due to queuing ((DE↓q): in addition to being a function of the possibility of a crime incident in the arc, depend on the location of the crime in the arc. The travelling time each arc consists of the total travelling time and the delay due to queuing:

$$T_{p(i,j)} = T_{t(i,j)} + DE_{q(i,j)} \quad (7)$$

Using the analytical relations

- Calculating the DE_q:

If $P_{oc(i,j)}$ represent the possibility of crime incident in the arc (i, j) per time unit which cause obstruction on the route and $T_{t(i,j)}$ represent the the travelling time of arc, the possibility of obstruction and reduction in the speed of police patrols in the arc (i, j) cab calculated by the following equation:

$$P_{p(i,j)} = P_{a(i,j)} \times T_{t(i,j)} \quad (8)$$

It should be mentioned that, in the above equation instead of travelling time, the arc travelling time should be used. However, in order to prevent more complications in the next equations and due to the minor differences, the travelling time has been used instead of the arc travelling time. Therefore, the average delay due to queuing for each arc is determined and is shown by the parameter of SD_{ij}. So, the delay due to queuing is obtained by the following equation:

$$DE_{q(i,j)} = 0.5 \times P_{a(i,j)} \times T_{t(i,j)} \times SD_{ij} \quad (9)$$

Therefore, according to the equation (7), the total travelling time of each arc is obtained by the following equation:

$$T_{p(i,j)} = T_{t(i,j)} + 0.5 \times P_{a(i,j)} \times T_{t(i,j)} \times SD_{ij} \quad (10)$$

- Using the uncertainty model

In solving the stochastic vehicle routing problem to estimate the true value of the parameters depending on the stochastic demands, the analysis of the the uncertainty model is used. There are two models in this regard:

- Demand Base Risk Modeling
- Capacity Base Risk Modeling

As there is no capacity limitation in this problem, the demand base risk modeling is used. Moreover, as stated earlier, to this problem, the possibility of a crime incident in each arc is assumed as the demand in the stochastic vehicle routing problem. In the demand base risk modeling, the person's demand is obtained through the following equation:

$$\bar{d}_i = \lambda d_i^{min} + (1-\lambda) d_i^{max} \quad (11)$$

The y = 1 represents the risk free condition under which the defeat situation never happens, on the other hand, the y=0 creates a suitable condition for solving the problem. If we assume the possibility of crime incident in each arc as customers' demand and as the minimum and maximum of this possibility can be 0 and 1 respectively, so:

$$\begin{aligned} d_i^{min} &= P_{oc}^{min} = 0 \\ d_i^{max} &= P_{oc}^{max} = 1 \\ \lambda &= 0.5 \\ \bar{d}_i &= 0.5 \times 0 + (1-0.5) \times 1 = 0.5 \end{aligned} \quad (12)$$

As can be seen, this equation leads us to the same .05 coefficient. In order to calculate the travelling time, which depends on the crime incident on a route, the .05 coefficient can be used to remove the uncertainty condition of the problem and this leads to the same Eq: (10):

Determining the suitable number of fleet: The number of the fleet of police patrol is a decision variable that must be determined by police commanders. Perhaps, the total distance traveled by the fleet is the best index that can represent law enforcement costs, because the greater the distance traveled, the greater would be the car wear, the number of patrol cars and the cost of fuel and increased costs related to the use of law enforcement. Therefore, this problem can be stated as follows:

$$\frac{\frac{\text{thenumberofpossiblecrimeincidends}}{\text{thenumberofpossiblecrime}}\ \text{duringaworkshiftonthewholenetwork}}{\text{incidentsonthebestcircleselected}} \quad (13)$$

The number of the fleet of police patrol =

Taking into account the best cycle, the number of possible crimes visited during a work shift is obtained after determining the best cycle made and also, after solving the model as the optimal patrolling route. Therefore, using the concepts and parameters presented the following relationship is obtained:

$$V = \frac{\left(\sum_{i,j} P_{oc(i,j)}\right) \times D}{S^{best}} \quad (14)$$

Determining the headway: In addition to the route and the number of suitable fleet, the patrol police headway (interval between police patrols) is also highly effective on their performance. In cases of low, high or irregular these intervals, the network is not properly covered and the queuing phenomenon will occur in patrol vehicles, at any case reducing their productivity. The best network coverage will occur only when the system is programmed so as to patrols can cross an arc in a balanced and relatively equal interval. This concept can be represented as follows:

$$\text{The suitable intervals between police patrols} = \frac{\text{the travelling time of the best arc}}{\text{the determined number of the fleet}} \quad (15)$$

Using the presented concepts and parameters the following equation is obtained to calculate the headways of the patrols:

$$h = \frac{T_p^{best}}{V} \quad (16)$$

Selecting the problem solving algorithm: After determining the appropriate model for routing problem, a suitable way of solving this model should be adopted. As model created for police patrols are full of complexity, the appropriate way to solve this model is by using meta-heuristic methods.

As previously mentioned, the ant group optimization (ACO) due to the constructive nature of its solutions and the movement of ants on a tree-like structure has been proved to be useful in solving vehicle routing problem. On the other hand, applying the local search methods to improve the obtained solutions is also effective. Therefore, the algorithm designed to solve the problem of ACO algorithms have been used.

Also, given the fact that an ant colony is unable to optimize multiple objectives and patrol routing problem has two objectives, which must be optimized simultaneously, in order to solve this problem with two objective functions the idea of Multi- Cast Ant Colony System (MCACS) is used.

In this way, two groups of ants are trying to improve their own objectives in their own way, but the ants of each colony laying down pheromone trails on their own way, while the the ants of the two colony can see each other's pheromone trails. Therefore, it is expected that the process of the division of labor increase the efficiency of the algorithm. (Eshragh, 2005).

Implementation of the algorithm: Based on the aforementioned explanation and what has been described in the explanation of ACS, in the first place, multiple ant algorithms is implemented for the defined problem:

Transition rule: The possible relationship for the selection of the vertice j is the same for both ant groups. If the ant cross the vertice i, the vertice j is selected from the N_i, with the probability of P_{ij}.

$$p_{ij} = \frac{\left[\tau_{ij}\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}}{\sum_{j=1}^{m}\left\{\left[\tau_{ij}\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}\right\}} \quad (17)$$

The major difference of the written algorithm to the stage with the other existing algorithms is that it is possible to define vertice. In typical ACO algorithms for VRP, ant in the vertice i, choose the vertice j from the vertices that has not previously visited. While in this case, the destination vertice is selected according to the arces that have not been yet visited. According to the mentioned definition for a cycle, the aim is to visit all the arces of each arc may be visited several times. The aim is to guide the ants to visit more of curves with the possibility of further crime. Also, the aim is to navigate the ants to visit the arcs with a higher possibility of crime incident more frequently. Also, it is noteworthy that a condition is applied to the problem that in the first round, an arc that has been visited by ants once, won't be visited unless there is no other option.

As noted earlier, in this algorithm any of the ants individually try to improve the function of their own, so each group has its own meta-heuristic function to select the next arcs. The objective function for the first group ants is to minimize travel time; therefore, the charm function of the first group of ants is obtained from the following equation:

$$\eta_{ij}^{1} = \frac{1}{T_{P_{ij}}} \tag{18}$$

For the second group of ants, the objective function is to maximize the probability of visiting the crime scene and the locations with a high risk of crime. For this reason, the number of crimes visited when crossing an arc is taken as the charm function; so, the charm function for the second group of the ants is obtained through the following equation:

$$\eta_{ij}^{2} = P_{OC_{ij}} \times T_{P_{ij}} \tag{19}$$

The above equation represents the number of possible crimes visited by police patrol while crossing an arc (i, j). At this point, the parameters are obtained by extracting the next option. As mentioned earlier, ACS also uses a pseudo-random rule, known as probe. To this end, the rotating wheel method is used in this algorithm and one of the obtained probabilities for the possible arces is selected. The advantage of using this technique instead of a random independent parameter in ACS is the process of exploring the next option also depends on its quality and the arc with better quality, will have a greater chance of being selected.

Pheromone update: As mentioned before, both groups of ants are gradually choosing the optimal route to complete a full cycle through the transition rule (equation 6-5) and pseudo-random rule. At this point, the ants optimize the route depending on the quality of the response they receive. But the answer that each ant receives consists of two parts; each separately affects the efficiency of the route:

The total travelling time on the circle ( ):

represents all the arcs of the circle

$$T_P = \sum_{i,j \in R_i} T_{P_{ij}} \tag{20}$$

The number of possible crime incidents visited along the circle (I): The sum of the survey made arcs of the cycle and the following relationship is obtained. This number is equal to the sum of the number of possible crime incidents visited while crossing each arc along the circle which is obtained by the equation (1) as follows:

$$I = \sum_{i,j \in R_i} I_{i,j} = \sum_{i,j \in R_i} (0.5 \times P_{OC_{i,j}} \times T_{P_{i,j}}) \tag{21}$$

R_i Represents all the arcs of the circle

As stated earlier, the optimality of the achieved result depends on both of the factors. Therefore, a new function should be defined that is dependent on two above factors so as to be able to compare the obtained results with this new function. The defined function to solve the problem represents the number of possible crime incidents visited in a work shift and is obtained by the following equation:

$$S = \frac{D}{T_p} \times I \tag{22}$$

It should be noted that a work shift (D) should be longer than (T_p). All ants of both colonies build their own cycles each get to a specific T_p and I. Then, according to the specified work shift for return, they calculate the parameter S. this parameter can be a measure to evaluate the obtained results by each ant. At this point, the optimality of the arces should be increased by increase in the amount of pheromone on the routes of the network.

At the beginning of the algorithm a fixed amount of initial pheromone) are laid on all the arcs, which mean that all the routes have the equal chance to be chosen at the first try and the choices are absolutely random. Then, after all the ants have made their cycles and obtained their respective answers, the pheromone on the arcs change in subsequent iterations so that the ants make a better choice by observing the ant pheromone on the routs. In ACS, this process, which is called pheromone update, is done in two ways:

Local pheromone update: To avoid accumulation of pheromones in the arcs and thereby strengthening exploring technique in new areas and to prevent stagnation, ants of each colony update the pheromones available on the arc while passing it. At this stage, the available on each arc evaporate with a coefficient and then a coefficient of the first pheromone on the arcs is added to the pheromone available on the arcs.

This minimizes the diversity of the pheromone on the different arcs and makes the probability of each arc of the network stronger. The local pheromone update is carried out by the following equation:

$$(1-\rho)\tau_{ij}(t) + \rho\tau_0 \rightarrow \tau_{ij}(t) \tag{23}$$

Table 1: The advised parameters for the different ACO algorithms (Dorigo and Stutzle 2003)

ACO algorithm	α	β	p	m	τ_0
AS	1	2 to 5	0.5	N	$\frac{m}{C^{nn}}$
E A S	1	2 to 5	0.5	N	$\frac{e+m}{\rho C^{nn}}$
AS rank	1	2 to 5	0.1	N	$\frac{0.5\times r(r-1)}{\rho C^{nn}}$
M M A S	1	2 to 5	0.02	N	$\frac{1}{\rho C^{nn}}$
ACS	--	2 to 5	0.1	N	$\frac{1}{nC^{nn}}$

The value of w$\tau_0$is obtained by the following equation:

$$\tau_0 = \frac{1}{nS^0} \tag{24}$$

n represents the number of the vertices and s^0 is a constant value for the answer. Usually, of the answers obtained by algorithm the nearest neighborhood is used which is S^{nn} (Gambardella *et al*., 1999).

This choice is because of its easy implementation. Thus, considering the definition of circle in this study, the implementation of the nearest neighborhood leads to an endless circle. Because, by selecting each arc, there are various arces with an equal distance that can also be selected. Therefore, the problem is solved by applying a model of local search of the nearest neighborhood which leads the choice to an end of the cycle. Of course, the value of τ_0 is also changeable in the written algorithm.

After the ants of each colony made their own cycles the parameter S is calculated for all the ants in the colony and through the following equation the pheromone is updated on the traveled edges by the ants that have obtained the best answer:

$$\tau_{ij}(t+1) = (1-\varphi)\tau_{ij}(t) + \varphi \times S^c_{opt}(t) \tag{25}$$

{The arces (i, j) are the best arc made by the colony c in t iteration} In order to avoid accumulation of pheromone and the stagnation phenomena, first the pheromone available on the arces of the best circle evaporates and then, pheromone is laid down on the same arc corresponding to the best result obtained. But what must be considered is the quality of the solution generated. In the sense that, if an ant is located in a repetitive cycle and visit an arc with a high optimality, the value of S will increase regularly and the ant will be in better position while, the solution obtained won't be of a good quality or indeed worthless executively.

When solving examples of VRP with a complete graph, there is no limit to the route choice and each ant can choose a route between two arces. Also, for problems with a fixed graph the ant returns to the warehouse when it has no other vertice to choose from. But as noted, the definition of patrol routing problem requires that ant have no limit in revisiting an arc. Even the problem tends to navigate the ant to re-visit the arcs with a higher optimality. However, this should not cause the ant to travel in a repetitive sub-optimal cycle. For this reason, the following conditions are used in which an ant doesn't visit an arc that has visited before, unless:

- The ant does not have other alternative except the repetitive arces and the traverse is incomplete
- The ant has travelled all the arces but has not reached the starting point yet

In these conditions, the ant continues choosing the repetitive arces based on the transition rule. However, by utilizing the penalty methods, the algorithm is designed in such a way that makes the trace of the ant fade in proportional to the number of arcs visited more than once with the coefficient of penalty ratio. So, the formula (25) is improved as the following:

$$\tau_{ij}(t+1) = (1-\varphi)\tau_{ij}(t) + \varphi \times S^c_{opt}(t)\left[1 - \text{P.R.} \times \frac{B^c(t)}{\text{E.C.}}\right] \tag{26}$$

After the pheromone on the whole network is updated by the above equation, the network is ready for the next iteration.

As can be seen, the routes with a higher density of pheromone generate better objective function i.e. the patrollers who pay a visit to a more probable crimes on the travelled arc. Table 1

It can be seen that, a lot of parameters play a rule on the obtained results from the above procedure including.

τ_0, p, φ, β, αAlthough specific values are not advised for these parameters in the different ACO algorithms, it is emphasized on the same reference that the sensitivity of the implemented meta-heuristic algorithms to these parameters can be variable and should be examined. Table 1 After reaching to the last condition the best route is specified. This optimal route is related to the ant that has visited more probable crimes along a single work shift.

CONCLUSION

In this study, the police patrol modeling was carried out by an ingenious method inspired by the Stochastic Vehicle Routing Problem (SVRP) and Multi-Cast Ant Colony System (MCACS). This method has been able to produce encouraging results and to develop an effective approach to the police patrol routing problem. The values obtained by various methods are shown in the following Table 2

The above table includes the results produced by multi-cast ant colony system in the first column, the results produced by ant colony system with time windows in the second column, the results produced by ant colony system with the probability incident occurrence in the third column and the results produced by GRASP in fourth column. As illustrated by the table above, the algorithm designed based on multi-cast ant colony system yields better results (the number of visits to probable crime incidents and the number of fleet of patrollers) in comparison to the other algorithms.

The following are some of the features of this project:

- Appling the vehicle routing problem approach to police patrol routing modeling with a more emphasis on hot spots of crime (areas of high crime density)
- Due to the probabilistic nature of the problem in terms of the time and location of crime occurrence along an arc, an uncertainty demand model was used to reach a solution and the function of probable crime occurrence distribution along an arc was considered as a specific probable value
- Determining the optimal number of fleet of patrollers in each city area and the ability to modify this plan according to the parametric changes in the model
- Using meta-heuristic algorithms to solve mathematic model of the patrol routs

Table 2: A comparison of the results of different methods

The applied algorithem the result	MCACS	ACS -TIME	ACS-INC	GRASP
The number of probable incidents visited on a single work shift	2.99	2.65	2.80	2.56
The number of required fleet of patrollers	25	28	26	29

SUGGESTIONS

The suggestions for further research: As the existing literature on the subject of police patrol routing and scheduling is sparse, further studies can be carried out in a various fields. The following suggestions for further research, amongst the rest, can yield invaluable results.

- Appling mixture of different approaches to the problem: for example, a mixture of ant colony optimization and the other meta-heuristic algorithms such as genetic algorithm or the other local search techniques
- Using the function of probable crime occurrence distribution along an arc instead of considering a specific probable value along an arc and use of the uncertainty demand model
- Dynamic planning for the patrols in order to be able to change the direction of the patrollers to crime scene area
- Planning for a linear route rather than a full circle, or dividing a large network of routes into several smaller areas to make it possible for the system to plan police patrols in a single work shift

Seemingly, planning and modeling police patrols in a network of routes can promote level of security in the network. Therefore, the probability of crime occurrence reduces in the arces. To re-plan the process, the existing one can be iterated. On the whole, it seems that a new method is required to update the information and new patrolling routes in a specific time period.

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