

Inspecting the Collective Behaviours: A Survey for Football Teams

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Abstract: The collective behaviour on football teams has been widely studied in the last few years, after decades of a deeper study about the technical and physical demands of the football. Now a days a new approach for the match analysis has been possible using the new technological advances from the multiplayer tracking systems. Therefore, the aims of this study is to provide a summarized analysis of the tactical metrics proposed and used in the last few years, trying understand how can these metrics help the match analysis. Thus, the main goal is to provide a practical perspective for the collective behaviour analysis.

Key words: Football, collective behaviour, tactics, match analysis, metrics

INTRODUCTION

Sport performance consists of a complex series of interrelationships between wide varieties of performance variables (Borrie *et al.*, 2002). Therefore, structures and configurations of play should be considered as a whole rather than analyzed in an individual manner (Clemente *et al.*, 2012). Systems with many dynamically interacting elements can produce rich and varied patterns of behaviors that are clearly different from the behaviour of individual players (Duarte *et al.*, 2012). In this line of thought, McGarry *et al.* (2002) considered that the rich and varied patterns that arise in team sports are the result of the self-organization among many coupled oscillators (e.g., players). Some collective sport games use specific methods and metrics in order to analyze and evaluate the dynamical collective behaviour. However, despite the large quantity of research methods and techniques to model performance in sport, used singly or in combination, results have been somehow disappointing (Nevill *et al.*, 2008). Simple frequency data cannot necessarily capture the full complexity of the performance (Borrie *et al.*, 2002). Therefore, searching for identification and interpretation of substantial game behaviour, it is imperative to gather quantitative and qualitative information (Garganta, 2009). Consequently, it is necessary to develop novel concepts and tools from the systems approach and computer science, to handle such complexity (Balague and Torrents, 2005). Actually, it is fundamental improves the match analysis to provide relevant information for coaches and their staff.

Match analysis refers to the objective of recording and examining events occurring during competition (Carling *et al.*, 2005). The main goal of match analysis is to provide to the coach relevant information about the team and/or the individual performance of each player (Franks and McGarry, 1996), thus allowing to improve the quality of future planning (Hughes and Franks, 2004).

Thus, the information about the collective behaviour is one the most important goals of the match analysis. Thus is important try simplify the information, trying provide to coaches some practical applications about the tactical metrics. Therefore, this study aims develop a summarized survey about the recent tactical metrics and at the same time provide some practical applications about each metric. The study will be organized in a individual presentation of each metric, showing their concept and the practical application for the football analysis.

MATERIALS AND METHODS

Centroid: measuring the spatial centre

General concept: The centroid is the geometric centre calculation of the team. The usefulness of the team's centroid may be the potential to compute the in-phase relation among the two opposing teams in longitudinal and lateral directions (Bourbousson *et al.*, 2010b). Moreover, can be a useful metric to analyze the equilibrium point of the team, considering their distribution.

Prior approaches: One of the first applications of the centroid' method was performed by Frencken *et al.* (2011) on the Fifth World Congress on Science and Soccer. In the same year was proposed the geometric centroid with the same concept (Yue *et al.*, 2008). In both studies the centroid is based on the calculus of average position of all players of each team (excluding the goalkeeper):

$$\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix} = \frac{1}{\sum_{i=1}^N w_i} \begin{bmatrix} \sum_{i=1}^{11} w_i x_i \\ \sum_{i=1}^{11} w_i y_i \end{bmatrix} \quad (1)$$

Wherein the position of the *i*th player is defined as (*x_i*, *y_i*). Using the centroid calculus, three measures were derived: the *x* distance (m), representing the longitudinal displacement *y* distance (m), representing the lateral displacement and radial distance (m), comprising both longitudinal and lateral displacements. After, in 2010 using the same basis of a geometric spatial centre was proposed a new calculus approach (Lames *et al.*, 2010):

$$C = (x_{max} - y_{max}) - (x_{min} - y_{min}) \quad (2)$$

Using this approach the goalkeeper still remains excluded from the calculation and in some way just considers two players to analyze the equilibrium point. The third prior concept was developed based on the weighted calculation (Clemente *et al.*, 2013). This metric follows the main principles of the centroid proposed by Frencken *et al.* (2011). Nevertheless, using this weighted concept was possible consider the goalkeeper due to its preponderance in the defensive phase. Moreover, considering the ball location was allowed providing weights for each player's influence in which the higher weight is assigned to the player closer to the ball and the lower weight was assigned to the player farther from it. In other words, the relevance of each player to the team's centroid, i.e., weight was based on the Euclidean distance from each player to the ball as:

$$W_i = 1 - \frac{\sqrt{(x_i - x_b)^2 + (y_i - y_b)^2}}{d_{max}} \quad (3)$$

Where:

(*x_b*, *y_b*) = Corresponds to the position of the ball
d_{max} = The Euclidean distance of the farthest player to the ball at each iteration (Clemente *et al.*, 2013)

Practical considerations: Let us show an example of the centroid applied on the 7-a-side football match (Fig. 1). The teams' centroids are indicated in the middle of the circumferences. Team A players are represented

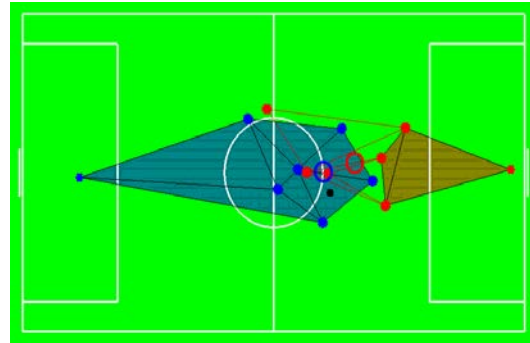


Fig. 1: Example of the centroid calculation applied on 7-a-side football match

by blue filled circles and Team B players by red filled circles. The ball is represented by a black filled circle.

The team's centroid allows understanding that the equilibrium point of the team's A is closer to own goal, thus maintaining the 'regular' defensive state, i.e., an equilibrium relationship between both teams. Whenever that the team's offensive centroid overtake the defensive team's centroid can be considerate a dangerous situations, i.e., a non-equilibrium relationship.

Previous studies showed that the goals suffered on open-play are correlated with this proximity between team's centroid and with this non-equilibrium relationship (Frencken *et al.*, 2011). For a football application this kind of graphical representation allows a systematic analyze about the equilibrium point of the team. This can be essential to understand the defensive adjustment to the opponent team.

Moreover, based on this non-equilibrium relationship, can be develop a 'warning system' providing online information about the frequency that the team is exposed to the dangerous situations. Using this information and their regularity over the match, coaches may perform some changes on the team's structure to decrease the exposure to the dangerous.

RESULTS AND DISCUSSION

Stretch index: measuring the dispersion

General concept: The stretch index measures the space expansion or contraction of the team on the longitudinal and lateral directions (Bourbousson *et al.*, 2010). The stretch index is measured based on the centroid position, thus is sum of each player's dispersion in both axis. This metric can be used to analyze the team's dispersion in defensive and offensive status.

Prior approaches: The pioneering study of the stretch index metric (Bourbousson *et al.*, 2010b) was developed

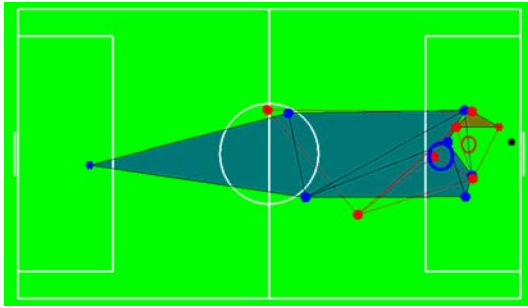


Fig. 2: Example of the stretch index calculation applied on 7-a-side football match

from a systemic analysis of two professional basketball teams competing in the French championship in a 5-a-side game. The concept was:

$$SI = \frac{1}{2} \times \sum_{i=1}^n |x_i - x_c| \tag{4}$$

where (x_c) summarizes the distances of all players from the team centroid and (x_i) the team's centroid based on the player's positions (Bartlett *et al.*, 2012).

Nevertheless, this concept is based on non-weighted centroid (Frencken *et al.*, 2011). Therefore, based on the weighted centroid, Clemente *et al.* (2012a, b) developed the weighted stretch index. Similarly to the team's centroid, a weighted team's stretch index metric may then be calculated as:

$$Sind = \frac{\sum_{i=1}^N w_i d_i}{\sum_{i=1}^N W_i} \tag{5}$$

where, d_i is the Euclidean distance between player i and the team's centroid, i.e.:

$$d_i = \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2} \tag{6}$$

Within this context, the stretch index can be obtained by computing the mean of the distances between each player and the centroid of the team. Thus, this metric represents the mean deviation of each player on a team from its centroid.

Practical considerations: Let us provide an example for the stretch index on 7-a-side football match (Fig. 2). The size of the team's centroid (i.e., circumference) proportionally varies based on team's stretch

indexes. Team A players are represented by blue filled circles and Team B players by red filled circles. The ball is represented by a black filled circle.

The stretch index metric allows quantify the player's dispersion to the centroid point. Therefore, can be a useful indicator to analyze the offensive length and width principle and the defensive concentration principle (Costa *et al.*, 2010). Both principles represent the general principles and collective behaviours based on the ball possession status. Usually, teammates should decrease the dispersion during the defensive state and increase in the offensive moments. This is a common principle to all teams. Thus, an expectable result from the stretch index is an inverse correlation between both teams (Bartlett *et al.*, 2012). Also, is expected a significant stretch index value during the defensive moment, comparing with the offensive moments (Clemente *et al.*, 2013).

For the football application, the stretch index allows an easy observation about the team's dispersion. A high dispersion during defensive moments can allow opportunities to the opponent team penetrate. Moreover, a regular high value during the defensive phase can identify a non-positive pattern. Therefore, further developments can adopt a 'warning system' to inform the coaches about the tendency to high dispersion values during the defensive moments. Using this information could be possible to the coaches adjust the team's behaviour, if necessary.

Surface area: measuring the coverage space

General concept: The team's surface area represents the overall team position (Frencken *et al.*, 2011). The surface area can be defined as the total space covered by a team, referred to as the area within the convex hull (Frencken *et al.*, 2011).

Prior approaches: The first similar concept was proposed in 2004 with the name coverage area (Okihara *et al.*, 2004). The same concept was suggested by Moura *et al.* (2012), using the convex hull area to analyze the team covers. The convex hull of a set of points (i.e., each player's position on the same team in each instant) on a plane is the smallest convex set containing; if is finite, the convex hull is always a polygon whose vertices are a subset of (Preparata and Shamos, 1985). The convex hull was computed by Moura *et al.* (2012) using the Quickhull technique (Barber *et al.*, 1996). Thus, at each instant the convex hull of the team was divided in triangles to aid the calculation of the convex hull area (i.e., summing the areas of all triangles within the convex hull).

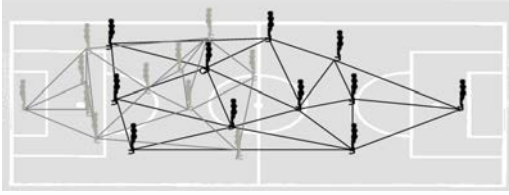


Fig. 3: Example of the surface area calculation applied on 11-a-side football match

Similar concept using the term ‘surface area’ was used by Frencken *et al.* (2011) and Duarte *et al.* (2012a). The surface area of each team was calculated as the area of a triangle, considering the Cartesian coordinates (Duarte *et al.*, 2012b):

$$A = xB \times yA - xA \times yB \tag{7}$$

$$B = xC \times yB - xB \times yC \tag{8}$$

$$C = xA \times yC - xC \times yA \tag{9}$$

$$\text{Area}(A,B,C) = \frac{|A + B + C|}{2} \tag{10}$$

The convex hull is calculated determining firstly a pivot point, in this case, the lowest-value player. If there were multiple, then the player with the highest -value was the pivot point. Then, the angle from the pivot to each player was calculated. Players were sorted by angle and removed if not part of the convex hull (Frencken *et al.*, 2011). An arbitrary point within the convex hull, here the centroid, was taken to create a triangle with the player that was designated as pivot and one of the remaining players. Therefore, the area was calculated by adding the triangles of consecutive points of the convex hull and the centroid (Frencken *et al.*, 2011). Nevertheless, the ‘surface area’ approach calculus does not consider the ball possession or their location, i.e., only considering the player’s location.

Practical considerations: Let us shows an example for the surface area of one team (Fig. 3). The surface area allows understand the general area covered by one team. This can be useful to identify the opportunities to play provided by the player’s distribution. Also, this metrics allow understand the principles that quantify by the teammate’s distribution (e.g., stretch index and team’s spread). Thus, the large correlation values found by Clemente *et al.* (2013) between stretch index and surface area are normal.

Effective area of play: measuring the effective triangulations

General concept: The effective area of play is based on the surface area concept. Nevertheless, this metrics allows measure the effective triangulations based on the principles of offensive and defensive coverage. Using this approach is possible analyze the triangulations with and without overlap between both teams (Clemente *et al.*, 2013). Thus, only the effective defensive triangulations will be considered in the cases of overlap. Therefore, will not be considered the non-effective triangulations for the total area covered.

Prior approaches: The first step on effective area is to calculate all the non-overlapping triangles formed by the players of the same team (Clemente *et al.*, 2013). The main condition to this is to generate, at first, the triangles with smaller perimeters. After generating all triangles of each team, the next step is to consider all triangles of each team without interception.

For the overlap triangulations was defined a simple principle. If a defensive triangle has a perimeter superior to 36 m; it will be nullified by the offensive triangles since there are no guarantees that the defensive players will be able to intercept the ball. After considering the triangles without interception, it is necessary to consider all triangles of the team that does not have the ball possession (i.e., defensive team) with perimeters inferior to 36 m (Clemente *et al.*, 2013). Therefore, the algorithm considers all the defensive triangles that have this condition, overlapping the interceptive offensive triangles. At last, all offensive triangles that are not intercepted by the defensive triangles with perimeter inferior to 36 m are considered. Consequently, the algorithm calculates all triangles, thus calculating the respective effective areas of both teams at every instant.

Practical considerations: Let us provide an example about the effective area of play on 7-a-side match (Fig. 4). Team A players are represented by blue filled circles and Team B players by red filled circles. The ball is represented by a black filled circle. The effective areas of Team A and B are represented by the blue and red regions, respectively.

The effective area of play can be an important concept to determine the offensive and defensive coverage. These principles of play are truly important to support the teammates. Only an efficient relationship between player’s distribution can optimize the tactical behaviour. Thus, for this example was adopted the value of 36 m to determine the maximum perimeter for an effective triangulation. Nevertheless, this value can be

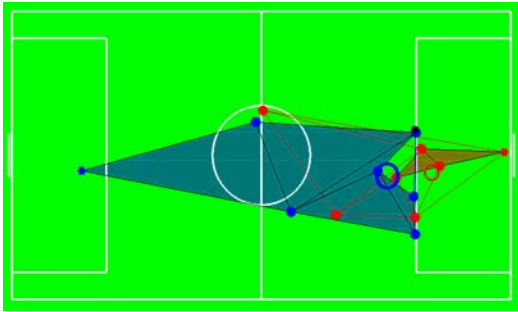


Fig. 4: Example of the effective area of play calculation applied on 7-a-side football match

changed considering the team own properties. In the example provided by Fig. 4 is possible analyze that the red team (in defensive phase) just ensures two effective triangulations. The large space between teammates allows an opportunity to the offensive team explore. Thus, the information about the effective area of play, mainly in the defensive phase can be essential to understand the tactical relationship between teammates.

This metric allows for an understanding of the proximity of players and if they can collectively perform an effective opposition that would create a defensive unit with a tactical principle which is essential for the opposition quality. Territorial Domain: measuring the difference of the number of players.

General concept: An instant visualization of team's advantage or disadvantage considering the number of players in each area could be an important information to understand the most prominent sectors and how team's behave in order to achieve their advantages in specifics situations. Therefore, the territorial domains aims provide information about this relationship between both teams.

Prior approaches: A study on elite teams of 11-a-side football game was recently presented by Vilar *et al.* (2012), wherein the numerical advantage and disadvantage on specific football spaces was analyzed, thus showing a team's pattern to focus on defensive stability, i.e., teams allocates more players than their opponents in sub-areas closer to their own goal to ensure a higher security. The field was segmented into 7 channels. For each field was analyzed at each instant the numerical vantage or disadvantage, thus identifying the tendency to be more or less offensive or defensive.

Practical considerations: The territorial domain can be a useful tool to understand the regions with numerical advantage or disadvantage. Let us provide one example (Fig. 5).

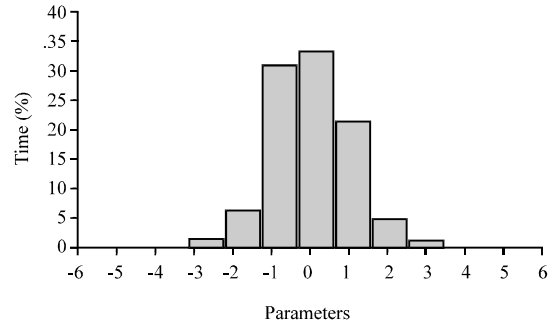


Fig. 5: Example of the territorial domain in the sector 1 (offensive midfield) applied on 7-a-side football match

In the sector 1 (Fig. 5), almost on 35% of time the teams have the same number of players. Nevertheless, there is a more time on numerical disadvantage. The territorial domain can help coaches to analyze the critical areas and support some strategy adjustments to optimize the team's process. Sometimes, the midfield 'fighting' is one of the most important processes to recover the ball and begin the offensive process. Therefore, using the territorial domain can be possible analyze if the teams are in numerical equilibrium or there are some non-equilibrium, thus resulting in non-efficient processes.

A high percentage of numerical disadvantage can be a 'warning information' for the coach and staff. Using this information can be possible analyze if this situation are in line with the team's strategy or if is a non-functional process, increasing the danger to suffer the goal.

Network: measuring the player's connectivity

General concept: Using the graph theory approach some works has been suggested, despite of their few generalization (Bourbousson *et al.*, 2010a; Passos *et al.*, 2011; Duarte *et al.*, 2012). Actually, the network as a single analysis cannot provide a powerful quantitative analysis. Nevertheless, some metrics can be applied on the network analysis, providing better information. This information can be based on the macro level of connectivity between all teammates, identifying macro team's structure.

Possible approaches: Few metrics have been proposed to evaluate the team's network (Lusher *et al.*, 2010). Thus, following will be suggested three 'macro' analysis metrics to analyze the offensive structure of football teams.

Network density: More players connected means higher density. To simplify the notation, let us use the function vectorize Matrix (Horvath, 2011) which turns an $n \times n$

dimensional symmetric matrix A into a vector whose components correspond to the upper diagonal entries of A (Horvath, 2011):

$$\text{Vectorize Matrix (A)} = (A_1, A_2, \dots, A_{n-1, n}) \quad (11)$$

Thus, the network density is defined as the mean off-diagonal adjacency and is closely related to the mean connectivity as observed in the following Eq. 12 (Horvath, 2011):

$$\begin{aligned} \text{Density} &= \text{mean}(\text{vectorize} \\ \text{Matrix (A)}) &= \frac{\sum_i \sum_{j>i} A_{ij}}{n(n-1/2)} \quad (12) \\ &= \frac{\text{mean}(k)}{n-1} \approx \frac{\text{mean}(k)}{n} \end{aligned}$$

where, k denotes the vector of node connectivities.

Network heterogeneity: The network heterogeneity is closely related to the variation of connectivity across players (Albert *et al.*, 2000; Watts, 2002). As Horvath's work (Horvath, 2011), it is herein defined as the coefficient of variation of the connectivity distribution:

$$H = \sqrt{\frac{n \sum k_i^2 - (\sum k_i)^2}{(\sum k_i)^2}} \quad (13)$$

A high heterogeneity of the player's network means that the teams exhibits a high level of sub communities and there is collectively, a low level of cooperation between players.

Network centralization: The network centralization can be give by the following Eq. 14 (Horvath, 2011):

$$\begin{aligned} \text{Centralization} &= \frac{n}{n-2} \left(\frac{\max(k)}{n-1} - \frac{\text{mean}(k)}{n-1} \right) \quad (14) \\ &= \frac{n}{n-2} \left(\frac{\max(k)}{n-1} - \text{Density} \right) \approx \frac{\max(k)}{n} - \text{Density} \end{aligned}$$

The centralization will be 0 for a network where each node has the same connectivity and closer to 1 with high heterogeneity. Should be highlighted that a regular grid network where has centralization 0.

Practical considerations: Following will be shown a network example (Fig. 6). The size of the nodes (i.e., players) represents the frequency that each player participated in offensive plays. The size of the arrows represents the level of connectivity between nodes.

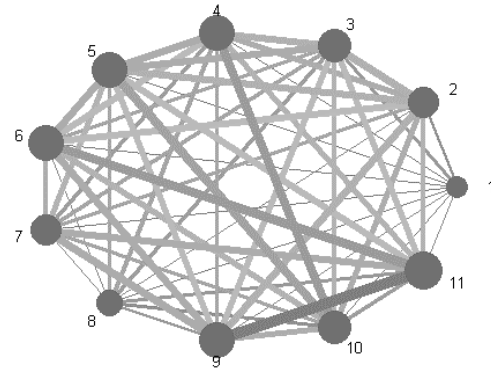


Fig. 6: Example of the network (1st match) applied on the 11-a-side match

Figure 6 shows the network of the 1st experimental match. The density value was 0.3970. The heterogeneity value was 0.3429. The centralization value was 0.1599. To compare the values, let us, show the Fig. 6 with a 2nd experimental match. It is possible analyse that the density in first match was lower, thus, more ambiguous. This can be also analyzed looking for the arrow's size. The network heterogeneity shows a high value, thus suggesting most tendencies to emerge clusters inside the team.

For all three metrics the values are just 'warning information' that allow understand how teammates interacts each other. Using this information can be possible observe the evolution over time and how team's change their behaviour. Moreover, can be possible test some players and their contribution to the team's properties.

CONCLUSION

The aim of this study was to provide some collective metrics that can be applied to the football match analysis. All of these metrics can be performed in an online fashion using automatic tracking systems. On the other hand, using the manual tracking systems can be performed the analysis on the post-match fashion. Anyway, the collective metrics analyzed can be useful information that can complement the football training programmes as well as support the strategies adopted for the matches. In sum, complementing the notational and kinematic analysis with collective metrics will be possible a higher understanding about the football process.

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