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Designing a Variable Step Size For the Successful Implementation of $P(EC)^m$ and $P(EC)^mE$ Mode

Jimevwo Godwin Oghonyon, Oluwole Odetunmibi and Abiodun Opanuga
Department of Mathematics, Covenant University, Ogun State, Ota, Nigeria

Abstract: The successful implementation of $P(EC)^m$ and $P(EC)^mE$ mode otherwise known as block predictor-corrector methodsis solely dependent on the principal local truncation error of both predictor and corrector methods of the same order. The development of this platform is built on formulating a class of $P(EC)^m$ and $P(EC)^mE$ mode that possesses the same order but different k-step. Nevertheless, designing of a variable step size on $P(EC)^m$ and $P(EC)^mE$ mode attracts a lot of computational benefits which guarantees convergence, step size control, tolerance level and error minimization.

Key words: $P(EC)^m$ and $P(EC)^mE$ mode, convergence, tolerance level, step size control, principal local truncation error

INTRODUCTION

Imagine that there is a need to evaluate the standard initial value problem by an implicit multistep method. So then at each step of evaluation for y_{n+k} the implicit system is stated as:

$$y_{n+k}+\sum_{j=0}^{k-1}\alpha_j y_{n+j}=h\beta_k f\left(x_{n+k}+y_{n+k}\right)+h\sum_{j=0}^{k-1}B_j f_{n+j} \quad (1)$$

Commonly, this is carried out by the fixed point iteration:

$$y_{n+k}^{[v+1]}+\sum_{j=0}^{k-1}\alpha_j y_{n+j}=h\beta_k f\left(x_{n+k},y_{n+k}^{[v]}\right)+h\sum_{j=0}^{k-1}B_j f_{n+j}, \quad (2)$$
$$y_{n+k}^{[0]}\text{arbitrary}, v=0,1,...$$

which by $h<1/(|\beta_k|L|)$ will converge to the unique solution of Eq.1 with the understanding that:

$$h<\frac{1}{\left(|\beta_k|L\right)}$$

Where, L is the Lipschitz constant of f with regards to y. For non-stiff problems, this limitation on h is not very important; in practical application, circumstances of accuracy position a majorly more protective restraint on h. Though Eq. 2 will converge for discretional $y^{[0]}_{n+k}$, a piece iteration requests for single appraisal of the function f and computing can evidently be preserve if there is estimable hypothesis as potential for $y^{[0]}_{n+k}$. This is done in a convenient manner by utilizing a distinguish explicit multistep method to supply the initial hypothesis $y^{[0]}_{n+k}$.

Thusly, the explicit method, the predictor and the implicit method the corrector, the two unitedly constitute a predictor-corrector pair. This turns out to be vantage in possessing the predictor and corrector of the same order which ordinarily means that the stepnumber of the predictor has to be peachier than that of the corrector. Preferably than address the ramification of possessing two distinct stepnumbers as sited in (Lambert, 1973, 1991).

According to Dormand (1996), it is very important that a numeric process shall obtain dependable solutions to differential Equations. The practical application of step size control established on local error estimates has departed a long way towards achieving this target. Numerous trial problems do give global errors relative to defined local error tolerance and thus, in virtually pragmatic examples, there may be sure-footed of the elongation of this property. Not with standing in some positions, it is favourable to have a straight estimate of the global error of the numeric solution. Virtually numeric analysts consider this as cumbersome and/or a computational costly process.

More methods have been formulated to offer global error estimation. A distinctive procedure, frequently adopted when local error control is apply is named tolerance reduction. This relies on the presumption of tolerance proportionality being rectify. Resolving the differential equations above the sort after interval, a novel solution is found employing a reduced or increased

Corresponding Author: Jimevwo Godwin Oghonyon, Department of Mathematics, Covenant University, Ogun State, Ota, Nigeria

tolerance. The variance in the solution, acquired at like points can be utilized to estimate global error. Subsequently changing step-sizes do not eventually give corresponding result points if not dense result is used the technic is sometimes practiced rather inexpertly. All the same the technic is rather effectual for non-stiff systems. A more taxonomic approach is the classical Richardson extrapolation scheme which demands a second solution with halved or with double step-sizes. This method is reasonably dependable but pricy.

Peradventure the most dependable procedure for global error computing is established on a parallel solution of an associated system of differential equations. These are built to possess a solution met by the real global error of the principal system of equations (Dormand, 1996). Moreover, for sensible convergence of computing methods, the differential equation problem:

$$y'(x) = f(x, y),\ y(a) = \alpha, x \in [a,\ b] \quad \text{and } f: R \times R^m \to R^m \tag{3}$$

Must possess a unparalleled solution. Therefore, the assumptions is adopted as stated below. The computational solution to is mostly written as:

$$\sum_{i=1}^{j} \alpha_i y_{n+i} = h \sum_{i=1}^{j} \beta_i f_{n+i} \tag{4}$$

where, the step size is h, $\alpha_j = 1$, α_i, i = 1, ... j, β_j are unknown constants which are uniquely fixed such that the formula is of order j as discussed in (Akinfenwa *et al*., 2013; Oghonyon *et al*., 2015 a, b). According to Lambert (1973, 1991) a solution to is sought after and assume that $f \in R$ is sufficiently differentiable on $x \in [a, b]$and satisfies a global Lipchitz condition, i.e., there is a constant $L \geq 0$ such that:

$$\left|f(x, y) - f(x, \bar{y})\right| \leq L\left|y - \bar{y}\right|,\ \forall y,\ \overline{y \in R}$$

Under this presumptuousness, Eq. 3 assured the existence and uniqueness defined on $x \in [a, b]$ as well as regarded to fulfill the Weierstrass theorem, see for example (Gear, 1971; Lambert, 1973; Xie and Tian, 2014) for details. Where a and b are finite and $y^{(1)}$ $[y^{(i)}_1, y^{(i)}_2, ..., y^{(i)}_n]^T$ for i = 0 (1)3 and $f = [f_1, f_1, ..., f_n]^T$, originate in real life applications for problems in science and engineering such as fluid dynamics and motion of rocket as presented by (Mehrkanoon *et al*., 2010).

MATERIALS AND METHODS

Definition (global error): The global error of the numeric solution is specified as:

$$\varepsilon_n = y_n - y(x_n),\ n = 0, 1, ..., N \tag{5}$$

and a numeric procedure for resolving is said to be convergent if:

$$\lim_{h \to 0}\left(\max_{0 \leq n \leq N} \|\varepsilon_n\|\right) = 0$$

Thusly, in a convergent procedure, the global error will incline to zero together with the step size. From a pragmatic viewpoint, this connotes that the global error will decreased as the step size is decreased (Dormand, 1996).

Definition (local truncation error): Assuming the numeric method (one-step method) has an increase equation:

$$y_{n+1} = y_n + h\phi(x_n,\ y_n,\ h)$$

which may be put into new order as:

$$0 = y_n h\phi(x_n,\ y_n,\ h) - y_{n+1}$$

This equation is not fulfilled by replacing the differential equation's analytic solution value y (x_n) for y_n and the variance is specified to become the local truncation error:

$$t_{n+1} = y(x_n) + h\phi(x_n,\ y(x_n), h) - y(x_{n+1})$$

So, therefore the local truncation error is the measure by which the analytic solution runs out to fulfil the numeric formula. The local truncation error, or numerical error of a numeric method is used in the analysis of the global error of the procedure. Nevertheless, a more pragmatic measure from a computing viewpoint is the error per step or local error (Dormand, 1996).

Agreeing with (Uri and Linda, 1998) like the Runge-Kutta methods errors arrived at each step are often not difficult to approximate than the global error. Hence, even if the global error is more significant, the local truncation error is the one that general-purpose multistep ciphers normally approximate in order to control the step size and to determine the order of the method to be used. The local truncation error is associated to the local error by:

$$h_n\left(|d_n| + O(h^{p+1})\right) = |I_n|\left(1 + O(h_n)\right)$$

Hence, to control the local error, multistep ciphers attempt to estimate and control h_n d_n. In the practical application of the multistep method, selecting a suitable value for the steplength is the most cumbersome problem. A limit for the global truncation error does not, in general, supply enough basis for selecting h. Rather, the idea of discovering an interval for h which ascertains that the global truncation error does not increase in a definite sense. All the same it is very important to select h such that the local truncation error is satisfactorily small. The practical application of a limit for the local truncation error is confounded by the pragmatic difficultness of determining a limit for $|y^{(p+1)}(x)|$. Moreover, the usage of predictor-corrector method in a suitable mode can avert estimating higher derivatives by applying Milne's device to approximate the principal local truncation error. Employing the step-control policy: the step length h will be selected such that:

- The principal local truncation error as approximated by Milne's device stays at each step less than a pre-assigned tolerance (stopping criteria)
- H rests within an interval of absolute or relative stability and
- The condition $h < 1/L|\beta_k|$ is met

The step-control policy might anticipate for a reduction or allow an increment in h as the computing continues (Lambert, 1973, 1991). Hence, from the above definitions and demonstration, designing a variable step size for the successful implementation of $P(EC)^m$ and $P(EC)^mE$ mode necessitates that some touchstones must be gratified to assure the execution of this method as sited in (Dormand, 1996; Lambert, 1973, 1991). Therefore, the principal objective of this study will be to design a variable step size for the successful implementation of $P(EC)^m$ and $P(EC)^mE$ mode for solving ODEs.

The residuum of this study is hatch out as follows: in $p(EC)^m$ and $p(EC)^mE$ mode implementations. The local truncation error of predictor-corrector mode. Estimation of $P(EC)^m$ and $P(EC)^mE$ mode. Lastly, displays concluding notes as interpreted in (Akinfenwa *et al*., 2013).

$P(EC)^m$ and $P(EC)^mE$ mode implementations: The predictor-corrector pair is:

$$\left.\begin{aligned} \sum_{j=0}^{k} \alpha_j^* y_{n+j} &= \sum_{j=0}^{k} \beta_j^* f_{n+j} \\ \sum_{j=0}^{k} \alpha_j y_{n+j} &= \sum_{j=0}^{k} \beta_j^* f_{n+j} \end{aligned}\right\} \quad (6)$$

There are different ways or modes in which the Eq. 6 can be applied. First, utilize the predictor to provide the initial guess $y^{[0]}_{n+k}$ then permit the looping to continue till convergence is attained (in practical applications some criterion comparable $||y^{[v+1]}_{n+k}+y^{[v]}_{n+k}||<\epsilon$. Where, ε is of the order of round-off error is met). This is called the mode of correcting to convergence. In this mode, the predictor represents a very auxiliary role and the local truncation error and stability characteristics of the predictor-corrector pair are those of the corrector exclusively. Nevertheless, this mode is unattempting in practical applications because one cannot assure ahead the looping numbers of the corrector and thus the numbers of function evaluations will be needed at each step.

A practically more satisfactory process is to express ahead the numbers of looping of the corrector are to be allowed at each step. Ordinarily this number is small, commonly 1 or 2. The local truncation error and stability characteristics of the predictor and corrector method in such a bounded mode depend on both the predictor and the corrector. A reliab mnemonic for depicting modes of this form can be built by applying P and C to indicate single application program of the predictor or corrector respectively and P to represent single evaluation of the function f given x and y. Presuppose the predictor is employ to appraise $y^{[0]}_{n+k}$, appraise $f^{[0]}_{n+k} = f(x_{n+k}, y_{n+k})$ and then use Eq. 2 at one time to get $y^{[1]}_{n+k}$. The mode is then named as PEC. When the looping is done a second time to incur $y^{[2]}_{n+k}$ which apparently implies the advance evaluation $f^{[1]}_{n+k} = f(x_{n+k}, y^{[1]}_{n+k})$ then the mode is depicted as PECEC or $P(EC)^2$. There is one father conclusion to make. At the final of the $P(EC)^2$ step obtain a value $y^{[2]}_{n+k}$ for y_{n+k} and a value $f^{[1]}_{n+k}$ for $f(x_{n+k}, y_{n+k})$. There is a choice to modify the value of f by making a farther evaluation $f^{[2]}_{n+k} = f(x_{n+k}, y^{[2]}_{n+k})$ the mode will then be reported as $P(EC)^2$. The two categories of modes $P(EC)^m$ and $p(EC)^mE$ can be spelt as a single mode $p(EC)^mE^{1-t}$ where m is positive integer and t = 0 or 1 and specified by $p(EC)^mE^{1-t}$ P:

$$y^{[0]}_{n+k} + \sum_{j=0}^{k-1} \alpha_j^* y^{[m]}_{n+j} = h\sum_{j=0}^{k-1} \beta_j^* f^{[m-t]}_{n+j}$$

$$\left.\begin{aligned} & f^{[v]}_{n+k} = f\left(x_{n+k}, y^{[v]}_{n+k}\right) \\ & (EC)^m . y^{[v+1]}_{n+k} + \sum_{j=0}^{k-1} \alpha_j y^{[m]}_{n+j} = \\ & h\,\beta_k f^{[v]}_{n+k} + h\sum_{j=0}^{k-1} \beta_j f^{[m-t]}_{n+j} \end{aligned}\right\} v = 0,1,\ldots,m-1$$

$$E^{1-t}\, f^{[m]}_{n+k} = f\left(x_{n+k}, y^{[m]}_{n+k}\right) \quad (7)$$

if t = 0. Instead, the predictor and corrector may be composed as:

$$\rho^*(E)y_n = h\sigma^*(E)f_n, \rho(E)y_n = h\sigma(E)f_n$$

respectively, where ρ^*, ρ and σ possess degree K and σ^* owns degree K-1 at most. With this notational system, the mode $P(EC)^m E^{1-t}$ may be redefined as P:

$$E^k y_n^{[0]} + \left[\rho^*(E) - E^k\right] y_n^{[m]} = h\sigma^*(E) f_n^{[m-t]}$$

$$\left.\begin{array}{l} E^k f_n^{[v]} = f\left(x_{n+k}, E^k y_n^{[v]}\right) \\ (EC)^m . E^k y_n^{[v+1]} + \left[\rho(E) - E^k\right] y_n^{[m]} = \\ h\,\beta_k E^k f_n^{[v]} + h\begin{bmatrix}\sigma(E) - \\ \beta_k E^k\end{bmatrix} f_n^{[m-t]} \end{array}\right\} v = 0,1,\ldots,m-1 \tag{8}$$

$$E^{(1-t)} E^k f_n^{[m]} = f\left(x_{n+k}, E^k y_n^{[m]}\right)$$

if t = 0 (Lambert, 1973, 1991).

Theorem: Let $\{Y^{[m]}_{n+1}\}$ be a sequence of approximations of Y_{n+1} obtained by a PECE method if:

$$\left|\frac{\partial f}{\partial y}(x_{n+1}, y)\right| \le L$$

(for all Ynear Y_{n+1} including $Y^{[0]}_{[n+1]}$, $Y^{[1]}_{n+1}$...) where L satisfies the condition $L<1/|h\beta_0|$ then the sequence $\{Y^{[m]}_{n+1}\}$ converges to Y_{n+1}.

Proof: The numeric solution satisfies the Equation:

$$y_{n+1} = \sum_{i=0}^{j-1} \alpha_i y_{n+i} + h\,\beta_0 f(x_{n+1}, y_{n+1}) + h\sum_{i=0}^{j-1} \beta_i f_{n+i}$$

The corrector satisfies the equation:

$$y_{n+1}^{(m+1)} = \sum_{i=0}^{j-1} \alpha_i y_{n+i} + h\,\beta_0 f\left(x_{n+1}, y_{n+1}^{(m)}\right) + h\sum_{i=0}^{j-1} \beta_i f_{n+i}$$

Subtracting these two equations to obtain:

$$y_{n+1} - y_{n+1}^{(m+1)} = h\,\beta_0\left[\left|f\left(x_{n+1}, y_{n+1}\right) - f\left(x_{n+1}, y_{n+1}^{(m)}\right)\right|\right]$$

Applying the Lagrange mean value theorem to arrive at:

$$y_{n+1} - y_{n+1}^{(m+1)} = h\,\beta_0\left(y_{n+1} - y_{n+1}^{(m)}\right)\frac{\partial f}{\partial y}(x_{n+1}, y^*)$$

where $y^{(m)}_{n+1} \le y^* \le y_{n+1}$. Thus:

$$\left|y_{n+1} - y_n^{(m+1)}\right| \le \left|h\,\beta_0\right|\left|y_{n+1} - y_{n+1}^{(m)}\right|\left|\frac{\partial f}{\partial y}(x_{n+1}, y)\right|$$

$$\le hL\left|\beta_0\right|\left|y_{n+1} - Y_{n+1}^{(m)}\right| \le \left[hL\left|\beta_0\right|\right]^m\left|y_{n+1} - y_{n+1}^{(0)}\right|$$

Now:

$$\lim_{m\to\infty}\left|\mathbf{y_{n+1} - y_{n+1}^{(m+1)}}\right| \to 0$$

if $hL|\beta_0|<1$ or $L<1/h|\beta_0|$. This means that the conclusion of Theorem 2.1 holds as seen in (Jain *et al*., 2007).

RESULTS AND DISCUSSION

The local truncation error of predictor-corrector mode: Assume the predictor-corrector pair is implemented in the mode of correcting to convergence then the local truncation error is distinctly that of the corrector exclusively. Suppose the pair is employed in $P(EC)^m E^{1-t}$ mode t = 0, then the local truncation error of the corrector will be contaminated by the that of the predictor. This necessitate the investigation of this contamination.

If the predictor and corrector defined by Eq. 6 possess associated linear difference operators L^* and L, orders P^* and P and error constants C^*_{p+1} and C_{p+1}, respectively. Established the localizing premise $y^{[v]}_{n+j} = y(x_{n+j})$, j = 0, ..., k-1 and suggest by $y^{-[v]}_{n+j}$ estimations to y at x_{n+k} yielded when the localizing premise is effective. Also, presume that $y(x) \in C^{\bar{p}+1}$ where $\bar{p} = \max(p^*, p)$. It then implies that:

$$\left.\begin{array}{l} L^*\left[y(x);h\right] = C^*_{\bar{p}+1} h^{\bar{p}+1} y^{(\bar{p}+1)}(x) + 0(h^{\bar{p}+2}) \\ L^*\left[y(x);h\right] = C^*_{\bar{p}+1} h^{p+1} y^{(p+1)}(x) + 0(h^{p+2}) \end{array}\right\} \tag{9}$$

For the predictor:

$$\sum_{j=0}^{k-1} \alpha_j^* Y(X_{n+j}) = h\sum_{j=0}^{k-1} \beta_j^*\left(X_{n+j}, Y(X_{n+j})\right) + L\left[y(x_n)_j h\right] \tag{10}$$

And:

$$Y_{n+k}^{[0]} + \sum_{j=0}^{k-1} \alpha_j^* Y_{n+j}^{[m]} = h\sum_{j=0}^{k-1} \beta_j^* f\left(X_{n+j} Y_{n+j}^{[m-t]}\right)$$

On subtracting and using localizing premise with Eq. 9 to get:

$$y\left(x_{n+k}\right) - \bar{y}_{n+k}^{[0]} = C^*_{\bar{p}+1}\, h^{\bar{p}+1}\, y^{(\bar{p}+1)}\left(x_n\right) + 0\left(h^{\bar{p}+2}\right) \tag{11}$$

Because the predictor-corrector pair is being employed in $P(EC)^m E^{1-t}$ mode as specified by Eq. 8 the Equations for the corrector, agreeing to Eq. 10 are:

$$\sum_{j=0}^{k-1} \alpha_j y\left(x_{n+j}\right) = h\sum_{j=0}^{k-1} \beta_j f\left(x_{n+j}, y\left(x_{n+j}\right)\right) + L\left[y\left(x_n\right);h\right]$$

And:

$$\overline{y}_{n+k}^{[v+1]} + \sum_{j=0}^{k-1} \alpha_j y_{n+j}^{[m]} = h\,\beta_k f\left(x_{n+k}, \overline{y}_{n+k}^{[v]}\right) + h\sum_{j=0}^{k-1} \alpha_j f\left(x_{n+j}, y_{n+j}^{[m-t]}\right) v = 0,1,\ldots,m-1$$

Again on subtracting and utilizing localizing premise to obtain:

$$\begin{aligned} y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[v+1]} &= h\,\beta_k\left[f\left(x_{n+k}, y\left(x_{n+k}\right)\right) - f\left(x_{n+k}, \overline{y}_{n+k}^{[v]}\right)\right] + \\ L\left[y\left(x_n\right);h\right] &= h\,\beta_k \frac{\overline{\partial f}}{\partial y}\left(x_{n+k}, \eta_v\right)\left[y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[v]}\right] + \\ & C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h^{p+2}\right), v = 0,1,\ldots,m-1 \end{aligned} \tag{12}$$

Thusly what succeeds relies on the relative magnitudes of P^* and P. Firstly,view the case $P^* \geq P$. Replacing Eq. 11 into 12 with v = 0 to arrive at:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[1]} = C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h^{p+2}\right)$$

This reflection for $y(x_{n+k}) - \overline{y}_{n+k}^{[1]}$ can instantly be replaced into Eq. 12 with v = 1 to have:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[2]} = C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h^{p+2}\right)$$

Proceeding in this fashion to observe that:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[m]} = C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h^{p+2}\right)$$

Hence, if $p^* \geq p$, the Principal Local Truncation Error (PLTE) of the $P(EC)^m E^{1-t}$ mode is for all $m \geq 1$ incisively that of the corrector exclusively. Forth with study the case p^* = p-1. On replacing Eq. 11 and 12 with v = 0 to immediately find that:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[v]} = \left[\beta_k \frac{\overline{\partial f}}{\partial y} C_p^* y^{(p)}\left(x_n\right) + C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right)\right] h^{p+1} + 0\left(h^{p+2}\right)$$

Therefore, if m = 1 that is assume the mode is $PEC\,E^{1-t}$ the PLTE is not the indistinguishable with that the corrector, merely the order of the PC method is that of the corrector. Moreover, on consecutive replacing into Eq. 12 to observe that for $m \geq 2$:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[m]} = C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h^{p+2}\right)$$

and the PLTE of the PC method turns that of the corrector exclusively. Right away the case p^* = p-2. Putting Eq. 11 into 12 together with v = 0 to obtain:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[v]} = \beta_k \frac{\overline{\partial f}}{\partial y} C_{p-1}^{\ *} h^p y^{(p-1)}\left(x_n\right) + 0\left(h^{p+2}\right) \tag{13}$$

Hence if m = 1 the order of the PC method is entirely p-1. Again, putting Eq. 13 into 11 with v = 1 yields:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[2]} = \left[\left(\beta_k \frac{\overline{\partial f}}{\partial y}\right)^2 C_{p-1}^{\ *} y^{(p-1)}\left(x_n\right) + C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right)\right] h^{p+1} + 0\left(h^{p+2}\right)$$

and thus for m = 2 the order of the PC method is that of the corrector only the two PLTEs are not exactly alike. Farther consecutive replacing into Eq. 11 demonstrate that for $m \geq 3$:

$$y\left(x_{n+k}\right) - \overline{y}_{n+k}^{[m]} = C_{p+1} h^{p+1} y^{(p+1)}\left(x_n\right) + 0\left(h_{p+2}\right)$$

and the PLTE is that of the corrector solely. It is instantly obvious that the order and the PLTE of a PC method rely on the gap within p^* and pand on m the amount of times the corrector is named. In distinction from others:

- If $p^* \geq p$ (or $p^* \geq p$ with $m > p-p^*$) then the PC method and the corrector possess the same order and the same PLTE as the corrector
- If $p^* \leq p$ and $m = p-p^*$ then the PC method possess the same order as the corrector but different PLTE
- If $p^* \leq p$ and $m \leq p-p^*-1$ then the PC method possess the same order equal to p^*+m (thus less than p)

Specifically, it is observe that suppose the predictor has order p-1 and the corrector has order p, the PEC answers to get a method of order p. Moreover, the $p(EC)^m$ or $p(EC)^m E$ scheme has always the same order and the same PLTE as discussed in (Lambert, 1973; 1991; Quarteroni *et al*., 2000).

Estimationof $P(EC)^m$ and $P(EC)^mE$ mode: Following (Faires and Burden, 2012; Lambert, 1973; Lambert, 1991; Oghonyon *et al.*, 2015):

- Predictor-corrector techniques always bring forth two approximations at apiece step thus, they are natural prospects for error-control adaptation
- To illustrate the error-control procedure a variable step-size predictor-corrector pair expending k-step explicit Adams-Bash forth method as predictor while the k-1-step implicit Adams-Moulton method as corrector methods are constructed

Firstly, the k-step predictor Local Truncation Error (LTE) is:

$$\frac{y(t_{i+1}) - W^p_{i+1}}{h} = C_{p+1}\, y^{(5)}\left(\bar{\mu}\right)h^4 \tag{14}$$

Secondly, the k-1-step corrector Local Truncation Error (LTE) is:

$$\frac{y(t_{i+1}) - W^c_{i+1}}{h} = -C_{p+1}\, y^{(5)}\left(\bar{\bar{\mu}}\right)h^4 \tag{15}$$

Where the k-step predictor and k-1-step corrector methods use this presumption such that the estimations $W_0, W_1..., W_i$ are all exact, W^p_{i+1} and W^c_{i+1} represents the predicted and corrected estimations given by the k-step predictor and k-1-step corrected methods. To advance, the presumption that for small values of h is establish to arrive at:

$$y^{(5)}\left(\bar{u}_i\right) \approx y^{(5)}\left(\bar{\bar{u}}_i\right) \tag{16}$$

The potencyof the error-control technique relies now on this presumption. On subtracting Eq. 14 from 15 and merging the local truncation error estimates to get:

$$\frac{W^c_{i+1} - W^p_{i+1}}{h} \approx C_{p+1}\, y^{(5)}\left(\bar{\bar{\mu}}\right)h^4 \tag{17}$$

Therefore, decimating the term involving $y^{(5)}\left(\bar{\bar{\mu}}\right)h^4$ in Eq. 15 obtains eventually the following approximation to the k-1-step corrector LTE:

$$\left|\tau_{i+1}\right|(h) » C_{p+1}\frac{\left|W^c_{i+1} - W^p_{i+1}\right|}{h} < \varepsilon \tag{18}$$

Equation 18 is Adam's estimate for correcting to convergence which is bounded by a prescribed tolerance ε otherwise known as stopping criteria. Moreover, the error estimate Eq. 18 is utilized to determine whether to admit the results of the current step or to redo the step with a smaller step size. The step is admitted based on a test as reported by Eq. 18 as sited in (Uri and Linda, 1998).

The Harmonizing (Uri and Linda, 1998; Ibrahim *et al.*, 2007) varying the step size is very essential for the effective performance of a discretization method. Step size adjustment for k-step predictor and k-1-step corrector block multistep methods applying variable step has been stated previously earlier. On the given step, the user will supply a prescribed tolerance. In the block multistep, variable step-size strategy codes, the block solutions are accepted if the local truncation error, LTE is less than the prescribed tolerance. Suppose the error estimate is greater than the accepted prescribed tolerance, the value of τ_{i+1} is rejected, the step is repeated with halving the current step size or otherwise, the step is multiply by 2.

Furthermore, Eq. 18 insures the convergence criterion of the method during the test evaluation. Finally, a number of approximation presumptions have been constructed in this development, so in practical application, a new step size (qh) is chosen conservatively, often as:

$$qh = \left(\frac{\varepsilon}{2\left|w^c_{i+1} - w^p_{i+1}\right|}\right)^{\frac{1}{4}} \tag{19}$$

Noting that $w^c_{i+1} \neq w^p_{i+1}$. Equation 19 is used in determining a new step size for the method.

CONCLUSION

Designing a variable step size for the successful implementation of $P(EC)^m$ and $P(EC)^mE$ mode have been studied. Block predictor-corrector pair is a compendium of Adams family of the predictor-corrector pair which can be executed in $P(EC)^m$ and $p(EC)^mE$ mode as presented above in (Dormand, 1996; Faires and Burden, 2012; Lambert, 1991; Uri and Linda, 1998). All of these sited above favoured the designing of a variable step size for the successful implementation of block predictor-corrector pair for solving nonstiff ODEs.

Moreover, designing a variable step size for the successful implementation of $P(EC)^m$ and $P(EC)^mE$ mode possesses the same order thus, necessitate that the stepnumber of the predictor to be one step greater than the corrector method. Again, the PLTE of both the predictor-corrector pair are considered in the development for properexecution and evaluation of the max errors.

In addition, the implementation is achieved with the support of the convergence criteria (stopping criteria). This convergence criteria decide whether the result is admitted or not as discussed in (Ibrahim *et al.*, 2007). Finally, the implementation of this method comes with many computational advantages as mention previously in (Dormand, 1996; Faires and Burden, 2012; Gear, 1971; Oghonyon *et al.*, 2015a, b).

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