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Numerical Method for Solving Delay Integro-Differential Equations

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Abstract: In this study, numerical solution of linear delay Volterra integro-differential equations is presented. In the solution, Galerkin's method with Chabyshev polynomials is used. Numerical results are given to illustrate the efficiency and accuracy of the proposed method.

Key words: Delay Volterra integro, differential equation, Galerkin's method, Chebyshev polynomials, approximation, solution

INTRODUCTION

Volterra delay-integro-differential equations arise widely in scientific fields such as biology, ecology, medicine and physics (Bocharov and Rihan, 2000; Brunner and Houwen, 1986; Jerri, 1999). This class of equations plays an important role in modeling diverse problems of engineering and natural science and hence has come to intrigue researchers in numerical computation and analysis. In this study, we consider the following Volterra delay-integro-differential equation:

$$y' = (x) = f(x)+\int_a^x k(x, t)\, y(t-\tau)dt \qquad (1)$$

Where:
f and k = Assumed to be sufficiently smooth with respect to their arguments
τ = A positive number
y = The unknown function to be determined

Hawary and Shami (2013) propose a numerical technique which is based on a mixed of exotic C1-spline collocation method (Hawary and Shami, 2012) and El-Gendi method (Hawary and Mahmoud, 2003) to solve Volterra delay-integro-differential equations. Rihan *et al*. (2009) presented a new technique for numerical treatments of Volterra delay integro-differential equations that have many applications in biological and physical sciences. The technique is based on the mono-implicit Runge-Kutta method described by Rehan *et al*. (2009a) for treating the differential part and the collocation method using Boole's quadrature rule for treating the integral part. Liu *et al*. (2015) concentrated on the differential transform method to solve some delay differential equations based on the method of steps for dilay differential equations and using the computer algebra system Mathematica. Abazari and Kilicman (2014) applied the differential transform method to solve the nonlinear integro-differential equation with proportional delays. Tunc (2016) considered a certain non-linear Volterra integro-differential equations with delay. He studied stability and boundedness of solutions.

MATERIALS AND METHODS

Method of solution: The Chebyshev polynomials of the first kind is defined as (Burden and Faires, 2011):

$$T_i(x) = \cos(i \cos^{-1}(x))$$

which is equivalent to the recurrence relation:

$$\left.\begin{aligned} &T_0(x) = 1\\ &T_1(x) = x\\ &T_i(x) = 2xT_{i-1}(x)\text{-}T_{i-2}(x),\ n \geq 2\end{aligned}\right\}$$

Rewriting the Eq. 1 in the operator form as:

$$L[y(x)] = f(x) \qquad (2)$$

Where:

$$L[y(x)] = y'(x)\text{-}\int_a^x k(x, y)\, y(t-\tau)\, dt \qquad (3)$$

Approximating the unknown function y(x) by $y_n(x)$:

$$y_n(x) = \sum_{i=0}^{n} c_i T_i(x) \qquad (4)$$

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where, c_i and $T_i, i=\overline{0,n}$ are the unknown coefficients and Chebyshev polynomials, respectively. Substituting, Eq. 4 into 2 yields:

$$L[y_n(x)] = f(x) \tag{5}$$

Where:

$$\begin{aligned} L[y_n(x)] &= \sum_{i=0}^{n} c_i T_i'(x) - \int_a^x k(x,y) \sum_{i=0}^{n} c_i T_i(t-\tau)dt \\ &= \sum_{i=0}^{n} c_i \left[T_i'(x) - \int_a^x k(x,y) T_i(t-\tau)dt \right] \\ &= \sum_{i=0}^{n} c_i L[T_i(x)] \end{aligned} \tag{6}$$

From Eq. 5 and 6 we have:

$$\sum_{i=0}^{n} c_i L[T_i(x)] = f(x) \tag{7}$$

To get the best unknown coefficients $c_i, i=\overline{0,n}$, we minimize the residual term:

$$E(x) = L[y(x)]\text{-}f(x) \tag{8}$$

that means we chose the unknown coefficients to satisfy the relation:

$$\int_a^x \omega_j E(x)dx = 0,\ j = \overline{0,n} \tag{9}$$

where, ω_j (x) is called weighted functions which is defined as:

$$\omega_j(x) = \frac{\partial y_n(x)}{\partial c_j} = T_j(x) \tag{10}$$

Substituting, Eq. 8 into 9 yields:

$$\int_a^x T_j\left[L[y_n(x)]\text{-}f(x)\right]dx = 0 \tag{11}$$

From Eq. 4 and 11, we obtain:

$$\int_a^x T_j(x)\left[\sum_{i=0}^{n} c_i L[T_i(x)]\text{-}f(x)\right]dx = 0 \tag{12}$$

which is equivalent to the following system:

$$\sum_{i=0}^{n} c_i \int_a^x T_j(x)\, L[T_i(x)]dx = \int_a^x T_j(x)\, f(x)dx.\ \ i, j = \overline{0,n} \tag{13}$$

Solving the above system for the coefficients $c_i, i=\overline{0,n}$ and substituting into Eq. 4, we obtain the approximate solution of Eq. 1.

RESULTS AND DISCUSSION

Numerical results: Let us consider the following equation:

$$y'(x) = 1 - \frac{x^5}{4} + \int_0^x xt^2\ y(t\text{-}1)dt \tag{14}$$

which has the following exact solution:

$$y(x) = x+1 \tag{15}$$

Comparing, Eq. 14 with 1, we find that:

$$\left.k(x,\ y) = xt^2,\ f(x) = 1 - \frac{x^5}{4}\right\} \tag{16}$$

First, we calculate:

$$\int_a^x T_j(x) f(x)dx,\ i = 0, 1$$

as:

$$\int_0^x T_0(s)\left[1-\frac{s^5}{4}\right]ds = \int_0^x\left[1-\frac{s^5}{4}\right]ds = \left[s-\frac{s^6}{24}\right]_0^x = x - \frac{x^6}{24} \tag{17}$$

$$\int_0^x T_1(s)\left[1-\frac{s^5}{4}\right]ds = \int_0^x\left[s-\frac{s^5}{4}\right]ds = \left[\frac{s^2}{2}-\frac{s^7}{28}\right]_0^x = \frac{x^2}{2} - \frac{x^7}{28} \tag{18}$$

Second, we calculate:

$$\int_a^x T_j(x) L[T_i(x)]dx,\ i, j = 0, 1$$

as:

$$\int_0^x \varphi_0(s)\ L\,[\varphi_0(s)]ds = -\int_0^x\int_0^x st^2\ dtds = -\frac{x^5}{15} \tag{19}$$

$$\int_0^x \varphi_0(s)\ L[\varphi_1(s)]ds = -\int_0^x\int_0^x st^2\ (t\text{-}1)\ dtds = -\frac{x^6}{24} - \frac{x^5}{15} \tag{20}$$

$$\int_0^x \varphi_1(s)\ L[\varphi_0(s)]ds = -\int_0^x\int_0^x s^2 t^2\ dtds = \text{-}\frac{x^6}{18} \tag{21}$$

$$\int_0^x \varphi_1(s)\, L[\varphi_1(s)]ds = \frac{x^2}{2} - \int_0^x\int_0^x s^2 t^2(t\text{-}1)dtds = \frac{x^2}{2} - \frac{x^7}{28} + \frac{x^6}{18} \quad (22)$$

Substituting, Eq. 17-22 into 13, we obtain the following system:

$$\left.\begin{aligned} -c_0\frac{x^5}{15} + c_1\left(-\frac{x^6}{24} + \frac{x^5}{15}\right) = x - \frac{x^6}{24} \\ -c_0\frac{x^6}{18} + c_1\left(\frac{x^2}{2} - \frac{x^7}{28} + \frac{x^6}{18}\right) = \frac{x^2}{2} - \frac{x^7}{28} \end{aligned}\right\} \quad (23)$$

which has the following solution:

$$c_0 = 1,\ c_1 = 1$$

Substituting, Eq. 24 into Eq. 4 for 2 n = 2 yields, the approximate solution of Eq. 14:

$$y_2(x) = 1 + x$$

which is identical to the exact solution.

CONCLUSION

The linear delay Volterra integro-differential equations is solved numerically by Galerkin's method with Chabyshev polynomials. Numerical results show that our method is very effective and efficient. Moreover, our proposed method provides exact solution for some problems.

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