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Studying the Variation of Normalized Crosswind Concentration using Different Dispersion Parameters

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Key words: Dispersion parameters, normalized crosswind concentration, plume rise, non-Gaussian plume model, integrated, calculated

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Abstract: Advection-diffusion equation in two dimensions was estimated using the separation technique. This solution was calculated using different dispersion parameters (Brigg's and power law) to get the non-Gaussian crosswind integrated concentrations. The results of predicted model were compared with measuring observed data of Sulfur Hexafluoride SF_6 on Copenhagen in Denmark.

INTRODUCTION

Due to physical complexity for dispersion of pollutants in the atmosphere creates a permanent source of challenging problems. Air pollutant emitted from different sources affect directly or indirectly man and his environment. Air pollutants are transported, dispersed or deposited by meteorological and topographical conditions. The atmospheric advection-diffusion equation (Seinfeld, 1986) had long been used to describe the transport of pollutant in a turbulent atmosphere. Its analytical solution was of fundamental importance in understanding and describing physical phenomena (Pasquill and Smith, 1983). The analytical solution has many advantages over the numerical solution, since, all parameters appear explicitly in the solution, so, their effect can be easily investigated (Nieuwstadt, 1980). The analytical solution was used to examine the accuracy and performance of the numerical solutions (Runca and Sardei, 1975; Liu and Seinfeld, 1975; Runca, 1982). An analytical solution had received much attention and had been studied extensively in the Gaussian plume model. Goulart *et al.* (2017) have proposed simple fractional differential equation models for the steady state spatial distribution of concentration of a non-reactive pollutant in Planetary Boundary Layer (PBL). They found that fractional derivatives models perform better than the traditional Gaussian model.

Marrouf *et al*. (2015) have formulated a mathematical model for dispersion of air pollutants in moderated winds by taking into account the diffusion in vertical height direction and advection along the mean wind by considering the eddy diffusivity and wind speed are assumed to be constant.

In this model, we assumed that wind speed and turbulence diffusion coefficients were invariant with height. The non-Gaussian plume model using different dispersion schemes, wind speed in power law and plume rise were used to get the crosswind integrated concentrations. One used the statistical technique to compare between the observed and all predicted concentrations obtained from different dispersion schemes to know the best predicted model. In this research, we estimated advection-diffusion equation in two dimensions by using separation method and different shaped of dispersion parameters (Brigg's and power law methods). The predicted model was compared with measuring observed data of Sulfur Hexafluoride SF_6 on Copenhagen in Denmark.

MATERIALS AND METHODS

Theoretical aspects: The dispersion of pollutants in the atmosphere is governed by the basic atmospheric diffusion equation. Under the assumption of incompressible flow, atmospheric diffusion equation based on the gradient transport theory can be written in the rectangular coordinate system as (Hanna *et al*., 1982):

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right)+S+R \quad (1)$$

Where:

C = The mean concentration of a pollutant (Bq/m^3), ($\mu g/m^3$) and (ppm)

S and R = The source and removal terms, respectively, (u, v, w) and (k_x, k_y, k_z) are the components of wind and diffusivity vectors in x, y and w directions, respectively in a Eulerian frame of reference

The following assumptions are made in order to simplify Eq. 1:

- Steady-state conditions are considered, i.e., $\partial C/\partial t = 0$
- As the vertical velocity is much smaller than the horizontal one inthex-direction, the term w ($\partial C/\partial z$) is neglected
- The x-axis is oriented in the direction of mean wind u = U and U much greater than the wind speed v in y-direction the term v ($\partial C/\partial y$) is neglected
- Source and removal (physical/chemical) pollutants are ignored, so that, S = 0 and R = 0

With the above assumptions (Eq. 1) reduces to:

$$u\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) \quad (2)$$

The advection term in x direction is larger than the diffusion in x direction then we will neglect the diffusion term in x direction. Equation becomes:

$$u\frac{\partial C}{\partial x}=\frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right) \quad (3)$$

The eddy diffusivity is k_z expressed as functions of downwind distance x as:

$$k_z = k(x) \quad (4)$$

Also, after integrating Eq. 3 with respect to y from (-∞ to ∞), Eq. 2 becomes:

$$u\frac{\partial C_y}{\partial x}=k(x)\frac{\partial^2 C_y}{\partial z^2} \quad (5)$$

Equation 3 is solved together with the following boundary conditions. The flux at the ground and top of the mixing layer "h" can be given by:

$$K_z\frac{\partial C}{\partial z}=0 \quad \text{at } z=0,h \quad (6)$$

where, "h" is a mixing height. A continuous point source with strength Q is assumed to be located at the point (0, y_s, z), i.e:

$$uC = Q\delta(z-h_s) \quad \text{as} \quad x=0 \quad (7)$$

Where:

δ = Dirac's delta function

h_s = A stack height and

"u" = A constant wind speed

Using the method of eigenfunction expansion, one can represent the general solution C_y (X, Z) as:

$$C_y(x,z)=\sum_{n=0}^{\infty}C_n(x,z)=\sum_{n=0}^{\infty}X_n(x).Z_n(z) \quad (8)$$

in which the functions X_n (x) or Z_n (z) form a complete of eigenfunction which implies that they are orthogonal, i.e:

$$\int Z_n(z)Z_m(z)dz=\begin{cases}0, & n\neq m\\ 1, & n=m\end{cases} \quad (9)$$

After introducing Eq. 8 in Eq. 5 one gets:

$$u\frac{\partial}{\partial x}\sum_{n=0}^{\infty}X_n(x).Z_n(z)=k(x)\frac{\partial^2}{\partial z^2}\sum_{n=0}^{\infty}X_n(x).Z_n(z) \quad (10)$$

$$\sum_{n=0}^{\infty}\left[uZ_n(z)\frac{\partial X_n(x)}{\partial x}-k(x)X_n(x)\frac{\partial^2 Z_n(z)}{\partial z^2}\right]=0 \quad (11)$$

Which simplified for each n, one can get:

$$uZ_n(z)\frac{dX_n(x)}{dx} = k(x)X_n(x)\frac{d^2Z_n(z)}{dz^2} \quad (12)$$

Using the method of separation of variables and divided Eq. 12 by k(x) X_n (x) Z_n (z) one gets:

$$\frac{u}{k(x)}\frac{1}{X_n(x)}\frac{\partial X_n(x)}{\partial x} = \frac{1}{Z_n(z)}\frac{\partial^2 Z_n(z)}{\partial z^2} = -\beta_n{}^2 \quad (13)$$

where, β_n^2 is a constant seperation of variable. Equation 13 can be divided into two equations in the form:

$$\frac{u}{k(x)}\frac{1}{X_n(x)}\frac{dX_n(x)}{dx} = -\beta_n^2 \quad (14)$$

$$\frac{1}{Z_n(z)}\frac{d^2Z_n(z)}{dz^2} = -\beta_n^2 \quad (15)$$

Equation 14 can be taken in the form:

$$\frac{dX_n(x)}{X_n(x)} = \frac{-\beta_n^2}{u}k(x)dx \quad (16)$$

By integrating Eq. 16 with respect to "x", one gets:

$$X_n(x) = \alpha_n \exp\left[\frac{-\beta_n^2}{u}\int_0^x k(\bar{x})d\bar{x}\right] \quad (17)$$

where, a_n is constant. Equation 15 becomes in the form:

$$\frac{\partial^2 Z_n(z)}{\partial z^2} + \beta_n^2 Z_n(z) = 0 \quad (18)$$

Which the solution is given in the form:

$$Z_n(z) = A_n \sin(\beta_n z) + B_n \cos(\beta_n z) \quad (19)$$

where, A_n and B_n are constants, since, the function Zn (z) forms a complete of Eigenfunction. Substituting from Eq. 17 and 19 in Eq. 8 one gets:

$$C_n(x, z) = \alpha_n \exp\left[\frac{-\beta_n{}^2}{u}\int_0^x k(\bar{x})d\bar{x}\right]\left(A_n \sin(\beta_n z) + B_n \cos(\beta_n z)\right) \quad (20)$$

Differentiating Eq. 20 and from the boundary condition Eq. 6 one gets:

$$A_n\beta_n = 0 \quad (21)$$

But $\beta_n \neq 0$ then, A_n = 0, so, Z_n (z) = B_n Cos (β_n z) and Eq. 20 becomes:

$$C_n(x, z) = \alpha_n B_n \cos(\beta_n z)\exp\left[\frac{-\beta_n{}^2}{u}\int_0^x k(\bar{x})d\bar{x}\right] \quad (22)$$

Differentiating Eq. 22 and from the boundary condition Eq. 6, one obtains:

$$\alpha_n B_n \beta_n \sin(\beta_n h) = 0 \quad \Rightarrow \beta_n = \frac{n\pi}{h}, \quad h \neq 0 \quad (23)$$

In Eq. 23 if one takes n = 0 then and Eq. 18 takes the form:

$$\frac{\partial^2 Z_0(z)}{\partial z^2} = 0 \quad (24)$$

$$Z_0(z) = Lz + M \quad (25)$$

where, L and M are constants. From the boundary condition Eq. 6, Eq. 19 in term of n = 0 leads to:

$$\frac{\partial Z_0(z)}{\partial z} = 0 \quad \text{at} \quad z = 0 \quad (26)$$

Comparing (Eq. 25) with (Eq. 26) one gets:

$$L = 0 \quad \Rightarrow \quad Z_0 = M \quad (27)$$

Equation 8 can be written in another form as follows:

$$C_y(x, z) = X_0(x)Z_0(Z) + \sum_{n=1}^{\infty} X_n(x)Z_n(Z) \quad (28)$$

$$X_0(x) = \alpha_0 \exp\left[\frac{-\beta_0{}^2}{u}\int_0^x k(\bar{x})d\bar{x}\right] = \alpha_0 \quad (29)$$

Substituting from Eq. 22 in Eq. 28, one can get:

$$C_y(x, z) = \xi_0 + \sum_{n=1}^{\infty} \xi_n \cos(\beta_n z)\exp\left[\frac{-\beta_n{}^2}{u}\int_0^x k(\bar{x})d\bar{x}\right] \quad (30)$$

where, ξ_0 = a_0 M, ξ_n = a_n B_n using the boundary condition Eq. 7 at x = 0 in Eq. 30:

$$\xi_0 + \sum_{n=1}^{\infty} \xi_n \cos\left(\frac{n\pi}{h}z\right) = \frac{Q}{u}\delta(z-h_s), \quad 0 \leq h_s \leq a \quad (31)$$

Integrating Eq. 31 with respect to z from 0-h then:

$$\xi_0 = \frac{Q}{uh} \quad (32)$$

Substituting from Eq. 32 in Eq. 31, one gets:

$$\frac{Q}{uh} + \sum_{n=1}^{\infty} \xi_n \cos\left(\frac{n\pi}{h}z\right) = \frac{Q}{u}\delta(z-h_s) \quad (33)$$

Multiplying Eq. 33 by cos $\left(\frac{n\pi}{h}z\right)$ and integrating it with respect to z from 0 to h. then:

$$\xi_n = \frac{2Q}{uh}\cos\left(\frac{n\pi}{h}h_s\right) \quad (34)$$

Substituting by ξ_0, ξ_n and β_n in Eq. 30 then the general solution of Eq. 8 takes the form:

$$C_y(x,z) = \frac{Q}{uh}\left[\begin{array}{l} 1+2\sum_{n=1}^{\infty}\cos\left(\frac{n\pi}{h}h_s\right)\cos\left(\frac{n\pi}{h}z\right) \\ \exp\left[-\frac{n^2\pi^2}{h^2u}\int_0^x k(\bar{x})d\bar{x}\right] \end{array}\right] \quad (35)$$

Equation 35 for n = 1 becomes:

$$\frac{c_y(x,z)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos(\frac{\pi}{h}h_s)COS(\frac{\pi}{h}Z)\exp \\ \left[-\frac{\pi^2}{h^2u}\int_0^x k(\bar{x})d\bar{x}\right] \end{array}\right] \quad (36)$$

Since, K (X) = $\frac{\sigma_z^2}{2x}u$ substituting in Eq. 36, taking z = 0 at ground, the crosswind normalized concentration takes the form:

$$\frac{cy(x,0)}{Q} = \frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{uh^2}\int_0^x \frac{\sigma_z^2 u}{2x}dx\right] \quad (37)$$

Where:

c_y/Q = A crosswind normalized integrated concentration
Q = An emission rate u is the wind speed in x-direction
σ_z = The standard deviation in z-direction
h_s = The stack height
h = The mixing height

We used two shapes of dispersion parameters as follows:

Brigg's method

Firstly: In extremely and moderately unstable condition, taking $\sigma_z = 0.24x\,(1+0.001x)^{1/2}$ in Brigg's (1969) urban condition (Hanna *et al*., 1982) by substituting in above equation, one gets:

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos\left(\frac{\pi h_s}{h}.\right)\exp \\ \left(\frac{-\pi^2}{uh^2}\int_0^x \frac{\left((0.24x)(1+0.001x)^{1/2}\right)^{2u}}{2x}dx\right) \end{array}\right]$$

Then, this equation becomes:

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos\left(\frac{\pi h_s}{h}.\right)\exp \\ \left(\frac{-\pi^2}{h^2}\frac{(0.24)^2}{2}\int_0^x \left(x+0.001x^2\right)dx\right) \end{array}\right]$$

By integrating:

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos\left(\frac{\pi h_s}{h}.\right)\exp\left(\frac{-\pi^2}{h^2}\right) \\ \frac{(0.24)^2}{2}\left[\frac{x^2}{2}+\frac{0.001x^3}{3}\right] \end{array}\right] \quad (38)$$

Secondly: In slightly unstable condition in Brigg's urban condition (Hanna *et al*., 1982):

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos\left(\frac{\pi h_s}{h}\right).\exp \\ \left(\frac{-\pi^2}{h^2}\int_0^x \frac{(0.20x)^2}{2x}dx\right) \end{array}\right]$$

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[\begin{array}{l} 1+2\cos\left(\frac{\pi h_s}{h}\right).\exp \\ \left(\frac{-\pi^2}{h^2}(0.20)^2\int_0^x \frac{(x)^2}{2x}dx\right) \end{array}\right]$$

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\right.$$
$$\left.\left[\frac{-\pi^2}{h^2}(0.020)\int_0^x xdx\right]\right]$$

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\right.$$
$$\left.\left[\frac{-\pi^2}{h^2}\left(0.01x^2\right)\right]\right] \quad (39)$$

Third: In neutral condition in Brigg's urban condition (Hanna *et al*., 1982) by substituting in Eq. 37, one gets:

$$\frac{c_y(x,0)}{Q} = \frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\right.$$
$$\left.\left[\frac{-\pi^2}{h^2}\int_0^x \frac{\left((1+0.003x)^{\frac{-1}{2}}\right)^2}{2x}dx\right]\right]$$

$$\frac{c_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\left(\frac{-\pi^2}{h^2}\frac{(0.14)^2}{2}\int_0^x\frac{x}{(1+0.0003x)}dx\right)\right]$$

By integrating one gets:

$$\frac{c_y(x,0)}{Q}=\frac{1}{uh}\left[\begin{matrix}1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\left(\frac{-\pi^2}{h^2}\right)\frac{(0.14)^2}{2}\\ \left[\frac{x}{0.0003}-\frac{1}{(0.0003)^2}\ln|(1+0.0003x)\right.\end{matrix}\right] \quad (40)$$

Fourth: In slightly and moderately stable, σ_z = 0.08x $(1+0.00015x)^{-1/2}$ in Brigg's urban condition (Hanna *et al*., 1982) by substituting in Eq. 37, one gets:

$$\frac{c_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos10\left(\frac{\pi h_s}{h}.\exp\left[\left(\frac{-\pi^2}{h^2}\right)\int_0^x\frac{\left(0.08x(0.00015x)^{\frac{-1}{2}}\right)^2}{2x}dx\right]\right)\right]$$

$$\frac{c_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}.\exp\left[\left(\frac{-\pi^2}{h^2}\right)\left((0.08)^2\int_0^x\frac{x}{1+0.00015x}dx\right)\right]\right)\right]$$

By integrating, one gets:

$$\frac{C_y(x,y)}{Q}=\frac{2}{uh}\left(\frac{\pi h_s}{h}.\exp\left[\left(\frac{-\pi^2}{h^2}\right)\right.\right.$$

$$\cdots\cdots \left(\frac{(0.08)^2}{2}\left[\frac{x}{15\times10^{-5}}-\frac{1}{(15\times10^{-5})^2}\ln|(1+15\times10^{-5}x)\right)\right. \quad (41)$$

Power law method: Smith worked out analytical Power-Law formulae for σ_y and σ_z to be used easily than using a graph or a table. He used the Brookhaven National Laboratory (BNL) formulas which are defined by him using wind direction θ recorded over 1 h as follows:

Where:

A = Fluctuations of θ exceed 90° is a very unstable conditions

B_1 = Fluctuations of θ from 40-90° is a moderately unstable

B_2 = Flsuctuations of θ from 15-40° is a slightly unstable

C = Fluctuations of θ>15° with strip chart showing an unbroken solid core in the trace is a neutral

D = Trace in a line, short-term fluctuations of θ<15° is a Moderately stable

Smith summarized the BNL formulas which were based on hourly average measurements of diffusion to about 10 km of a no buoyant plume released from a height of 108 m:

$$\sigma_y = ax^b$$

$$\sigma_z = ax^d$$

where, values of the parameters "a, b, c" and "d" are given in Table 1 and 2.

Firstly: $\sigma_z = 0.33x^{0.86}$ and by substituting in Eq. 37, one gets:

$$\frac{C_y(X,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\int_0^x\frac{\left(0.33x^{0.86}\right)^2}{2x}dx\right]$$

$$\frac{C_y(X,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\frac{(0.33)^2}{2}\int_0^x x^{0.72}dx\right] \quad (42)$$

Secondly, $\sigma_z = 0.41x^{0.91}$ and by substituting in Eq. 37, one gets:

$$\frac{C_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\int_0^x\frac{\left(0.41x^{0.91}\right)^2}{2x}dx\right]$$

After integrating, one gets:

$$\frac{C_y(X,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\frac{(0.41)^2}{2}\left[\frac{x^{1.82}}{1.82}\right]\right] \quad (43)$$

Third: $\sigma_z = 0.22x^{0.78}$ and by substituting in Eq. 37, one gets:

$$\frac{C_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\int_0^x\frac{\left(0.22x^{0.78}\right)^2}{2x}dx\right]$$

After integrating, one gets:

$$\frac{C_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\frac{(0.22)^2}{2}\left[\frac{x^{1.56}}{1.56}\right]\right] \quad (44)$$

Fourth: σ_z = 0.06 $x^{0.71}$ and by substituting in Eq. 37, one gets:

Table 1: Brookhaven national laboratory parameters

Stability classes	Very and moderately unstable (B1)	Slightly unstable (B2)	Neutral (D)	Moderately stable (F)
A	0.36	0.40	0.32	0.31
B	0.86	0.91	0.78	0.71
C	0.33	0.41	0.22	0.06
D	0.86	0.91	0.78	0.71

Table 2: Comparison between the predicated and observed crosswind-integrated normalized concentration at different downwind distance, wind speed and distance for the different runs

					c^y (x, z)/Q (10^{-4}s/m^2)		
Run No.	Stability	h (m)	$U_{(115)}$ (m/sec)	X(m)	Observed	Predicated Brigg's	Predicated power law
1	A	1980	3.40	1900	6.48	3.66	4.31
1	A	1980	3.40	3700	2.31	2.01	4.10
1	C	1920	10.60	2100	5.38	1.35	1.33
2	C	1920	10.60	4200	2.95	1.09	1.09
2	B	1120	5.00	1900	8.20	3.13	4.82
3	B	1120	5.00	3700	6.22	1.80	4.19
3	B	1120	5.00	5400	4.30	1.79	3.55
3	C	390	4.60	4000	11.66	5.57	5.57
3	C	820	6.70	2100	6.72	3.55	3.37
4	C	820	6.70	4200	5.84	2.07	2.05
5	C	820	6.70	6100	4.97	1.83	1.84
5	C	1300	6.70	2000	3.96	1.47	1.43
6	C	1300	13.20	4200	2.22	0.98	0.97
6	C	1300	13.20	5900	1.83	0.73	0.74
6	B	1850	13.20	2000	6.70	1.66	2.05
7	B	1850	7.60	4100	3.25	0.81	1.91
7	B	1850	7.60	5300	2.23	0.72	1.82
7	D	810	9.40	1900	4.16	3.26	1.72
8	D	810	9.40	3600	2.02	2.66	2.41
8	D	810	9.40	5300	1.52	2.15	3.31
8	C	2090	10.50	2100	4.58	1.27	1.26
9	C	2090	10.50	4200	3.11	1.06	1.05
9	C	2090	10.50	6000	2.59	0.85	0.87

$$\frac{C_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{H}\right).\ \exp\frac{-\pi^2}{h^2}\int_0^x\frac{\left(0.06x^{0.71}\right)^2}{2x}dx\right]$$

After integrating, one gets:

$$\frac{C_y(x,0)}{Q}=\frac{1}{uh}\left[1+2\cos\left(\frac{\pi h_s}{h}\right).\exp\frac{-\pi^2}{h^2}\frac{(0.06)^2}{2}\left[\frac{x^{1.42}}{1.42}\right]\right] \quad (45)$$

RESULTS AND DISCUSSION

The used data set was observed from the atmospheric diffusion experiments conducted at the Northern part of Copenhagen, Denmark, under unstable conditions (Gryning and Lyck, 1984; Gryning *et al*., 1987). The tracer Sulfur Hexafluoride (SF_6) was released from a tower at a height of 115 m without buoyancy. The values of different parameters such as stability, wind speed at 10 m (U_{10}), wind speed at 115 m (U_{115}) and downwind distance during the experiment are represented in (Table 2). Comparison between the predicated and observed crosswind-integrated normalized concentration with the emission rate at different downwind distance for the different runs are estimated.

From two Fig. 1 and 2 one concludes that some predicted data is one to one with observed crosswind normalized integrated concentrations and others located inside a factor of two and others inside a factor of four.

Model evaluation statistics: Now, the statistical method is presented and comparison between predicted and observed results as offered by Hanna (1989). The following standard statistical performance measures that characterize the agreement between prediction ($C_p = C_{pred}/Q$) and observations ($C_o = C_{obs}/Q$) (Table 3):

$$\text{Fraction Bais (FB)}=\frac{\left(\overline{C_o}-\overline{C_p}\right)}{\left[0.5\overline{C_o}+\overline{C_p}\right]}$$

$$\text{Normalized, Mean Square Error (NMSE)}=\frac{\overline{\left(C_p-C_o\right)^2}}{\left(\overline{C_p}\overline{C_o}\right)}$$

$$\text{Correlation coefficient (COR)}=\frac{1}{N_m}\sum_{i=1}^{Nm}\left(C_{pi}-\bar{C}_p\right)\times\frac{\left(C_{oi}-\bar{C}_o\right)}{\left(\sigma_p\sigma_o\right)}$$

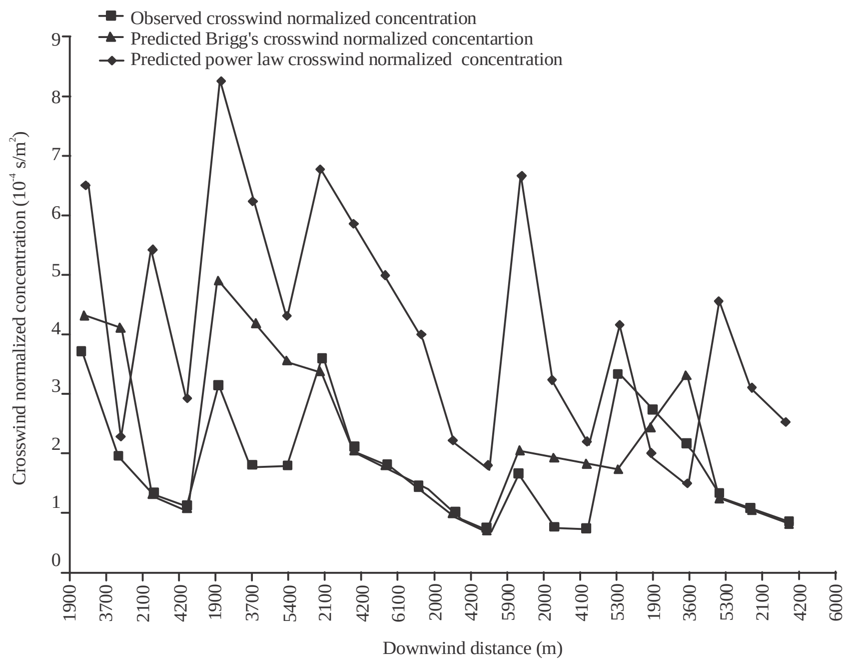

Fig. 1: The variation of the observed and predicted crosswind normalized integrated concentration via. downwind distance

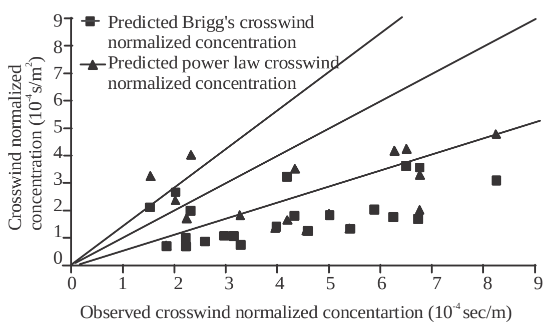

Fig. 2: The observed and predicted crosswind normalized integrated concentration

Table 3: The statistical evaluation of present two models

Models (stable condition)	NMSE	FB	COR	FAC2
Brigg's method	1.040	0.780	0.74	0.49
Power law method	0.710	0.600	0.64	0.63

$$\text{Factor of Two (FAC2)} = 0.5 \leq \frac{C_p}{C_o} \leq 2.0$$

where, σ_p and σ_o are the standard deviations of predicted C_p and observed C_o concentration, respectively. Here, the over bars indicate the average over all measurements. A perfect model would have the following idealized performance NMSE = FB = 0 and COR = FAC2 = 1.0.

From the statistical evaluations, one finds that the two models are inside a factor of two with observed data. Regarding to NMSE and FB for the power law is good with observed data than the Brigg's method but the correlation for the Brigg's method is well than power law.

CONCLUSION

In this study, one has formulated a mathematical model for dispersion of air pollutants in moderated winds. The non-Gaussian crosswind integrated concentrations are obtained by using separation technique to get on the solution of the advection-diffusion equation in two dimensions. The different dispersion parameters Brigg's and power law are used. One used observed normalized concentration data for Sulfur Hexafluoride (SF_6) from the atmospheric diffusion experiments conducted at the Northern part of Copenhagen, Denmark to compare with predicted concentration data using different schemes of dispersion parameters. From the statistical evaluations, one finds that the two models are inside a factor of two with observed data. Regarding to NMSE and FB for the power law is a good with observed data than the Brigg's method but the correlation for the Brigg's method is well than power law.

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