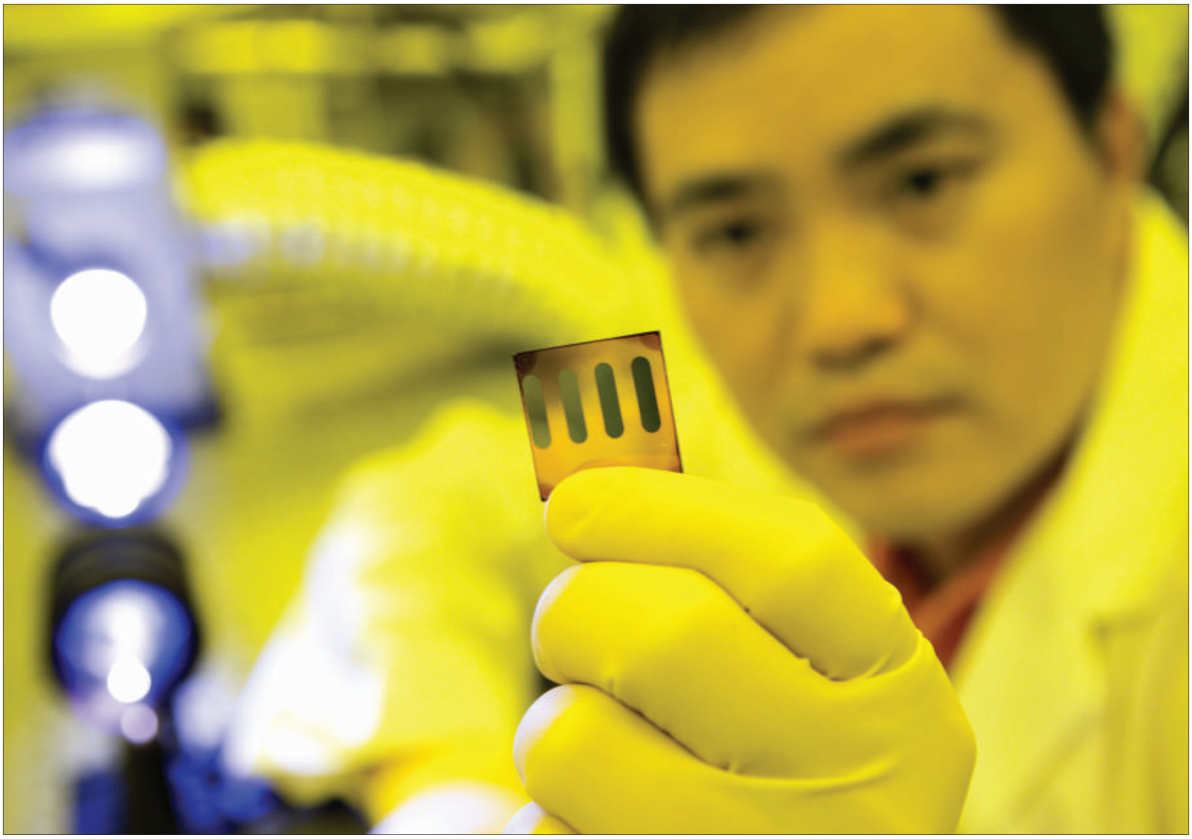

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# A Queue System with Arrival Process Follows Hyper Geometric Distribution

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## ABSTRACT

There are a lot of machines and devices need periodic examination and after the results of the examination is to exclude the non defective machines while the rest need to change one part or more of its damaged components or that close to expiration date, assume that the system has served carry out switching and replacement the damaged parts by new parts. In this research was to obtain the probability distribution of the number of defective machines in the system during the period of time t and which can calculate the average number of defective machines in the system.

**Key words:** Hyper geometric distribution, arrival rate, service rate, poisson distribution, probability distribution

## DESCRIPTION OF THE PROBLEM AND ITS SOLVING

Kella (1989) deals with the M/G/1 queue with server vacations in which the return of server to service depends on the number of customers present in the system where the distribution of arrival customers is Poisson distribution and the queue system contains only one server. Hur *et al.* (2003) consider an M/G/1 system with two policies, N- and T- policy simultaneously to optimize the operating cost of this system. Tadj and Choudhury (2005) deal with the three policies (N, T and D) to get the optimal control of queueing systems. Agnihothri and Kenett (1995) carried out. The impact of defects on a process with rework. Kennedy *et al.* (2002) has done work in "An overview of recent literature on spare parts inventories". Buzacott and Shanthikumar (1992) deal with.

Design of manufacturing systems using queueing models. Ayanzadeh *et al.* (2009) have work done on "Determining Optimum Queue Length in Computer Networks by Using Mimetic Algorithms". Kharat and Chaudhari (2005) consider "Parameter Selection Schemes for Web Proxy Servers and their Performance Evaluation". Zhou *et al.* (2010) carried out "Performance Study of a Network Coded Non-orthogonal User Cooperation System over Nakagami-m Channels". Wang *et al.* (2010) have done work on "Characterization and Evaluation of End-System Performance Aware Transport Schemes for Fast Long-Distance Optical Networks". Wang *et al.* (2009) deal with "Performance Study of Cooperative Diversity System over Nakagami-m fading Channels".

This study, dealing with queue system which the process of arrival follows Hyper Geometric Distribution and allow to add more than one server and departure process follows Poisson distribution. In Hyper geometric distribution the probability of k defective machines in the sample of size n is given by:

$$p(X=k)=\frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}\ ; \quad k=0,1,2,\dots,\min(M,n) \tag{1}$$

where, M is the number of Machines that need maintenance, N is the number of defective and non defective machines, n is the size of the sample taken from the group size N, k is the number of defective machines in the sample, so n-k is the number of non defective machines and the mathematical expression:

$$\binom{M}{k}$$

is called M combinatory k and it can compute by mathematical program as follow:

$$\binom{M}{k} = \text{Binomial}[M,k].$$

Similarly:

$$\binom{N-M}{n-k} = \text{Binomial}[N\text{-}M,n\text{-}k].$$

$$\binom{N}{n} = \text{Binomial}[N,n].$$

Let P is the percentage of defective machines in the sample, so:

$$P = \frac{M}{N}$$

then, the average of the number of defective machines in the sample of size n is given by:

$$E\ (X) = nP = \frac{nM}{N}$$

Let $\lambda$ is the rate of the number of arrival defective machines per unit time in the queue system that performs maintenance operations, then:

$$\lambda = \frac{nM}{N}$$

per Unit time, so during the interval time t, $M = m\lambda t$, where:

$$m = \frac{N}{n}.$$

Therefore, the Eq. 1 re-written as follows:

$$p\ (X=k)=\frac{\binom{m\lambda t}{k}\binom{N-m\lambda t}{n-k}}{\binom{N}{n}}\ ;\ \ k=0,1,2,\ldots,\min\ (m\lambda t, n)$$

Now, suppose that $\mu t$ is the departure rate of the number of machines that have been repaired during the interval time t and the probability distribution of the number of departure machines follows the Poisson distribution, so the probability of y departure machines during interval time t can be written as follows:

$$p\ (Y=y)=e^{-\mu t}\frac{(\mu t)^y}{y!}\qquad y=0,1,2,\ldots,M. \tag{2}$$

In the queue system can add a new server for each machine comes to maintenance to increase the speed of service, this server is lifted in the case of non-arrival of a new machine to the system to minimize the cost, so the Eq. 2 re-written as follows:

$$p\ (Y=y)=e^{-j\mu t}\frac{(j\mu t)^y}{y!}\qquad y=0,1,2,\ldots,M.$$

where, j is the number of adding servers in the queue system. Under the condition that the number of departure machines is independent of the number of machines coming, so during interval time t the probability distribution of the number of defective machine in the queue system allowing servers to add or withdrawn where ($j \le n$) will take the form:

$$P(0)=1-\sum_{l=1}^{n}P(l)$$

$$P\ (1)=\frac{\binom{m\lambda t}{1}\binom{N-m\lambda t}{n-1}}{\binom{N}{n}}e^{-\mu t}+\sum_{j=2}^{n}\frac{\binom{m\lambda t}{j}\binom{N-m\lambda t}{n-j}}{\binom{N}{n}}\frac{((j-1)\mu t)^{j-1}}{(j-1)!}e^{-(j-1)\mu t}$$

$$P\ (2)=\frac{\binom{m\lambda t}{2}\binom{N-m\lambda t}{n-2}}{\binom{N}{n}}e^{-\mu t}+\sum_{j=3}^{n}\frac{\binom{m\lambda t}{j}\binom{N-m\lambda t}{n-j}}{\binom{N}{n}}\frac{((j-2)\mu t)^{j-2}}{(j-2)!}e^{-(j-2)\mu t}$$

$$P\ (3)=\frac{\binom{m\lambda t}{3}\binom{N-m\lambda t}{n-3}}{\binom{N}{n}}e^{-\mu t}+\sum_{j=4}^{n}\frac{\binom{m\lambda t}{j}\binom{N-m\lambda t}{n-j}}{\binom{N}{n}}\frac{((j-3)\mu t)^{j-3}}{(j-3)!}e^{-(j-3)\mu t}$$

$$\vdots$$

$$P\,(n) = \frac{\binom{m\lambda t}{n}\binom{N - m\lambda t}{0}}{\binom{N}{n}} e^{-\mu t}.$$

On the other hand, The average number of the defective machines in the system can be obtained from the equation:

$$E\,(l) = \sum_{l=1}^{n} l\,P\,(l)$$

**Application with numerical results:** Let N = 50, n = 6, t = 1 h, $\lambda$ = 3 defective machines per one hour, $\mu$ = 2 repaired machine per one hour. Then: m = N/n, M = m$\lambda$t = 25 defective machines.

Apply mathematic program on the above equations P($l$) where $l$ = 1, 2,.., n and entering these data inside the equations, the equations P($l$) will take the forms in mathematic program as follow:

$$P(1) = N\left[\frac{\text{Binomial}[25,\,1]*\text{Binomial}[25,\,5]}{\text{Binomial}[50,6]}*E^{-2} + \sum_{j=2}^{6}(\text{Binomial}[25,\,j]*\text{Binomial}[25,6-j]* \right.$$
$$\left. E^{-2\,(j-1)}*(2\,(j-1)^{j-1})/(\text{Binomial}[50,\,6]*(j-1)!)\right]$$

$$P(2) = N\left[\frac{\text{Binomial}[25,\,2]*\text{Binomial}[25,\,4]}{\text{Binomial}[50,\,6]}*E^{-2} + \sum_{j=3}^{6}(\text{Binomial}[25,\,j]*\text{Binomial}[25,\;6-j]* \right.$$
$$\left. E^{-2\,(j-2)}*(2\,(j-2)^{j-2})/(\text{Binomial}[50,\,6]*(j-2)!)\right]$$

$$P(3) = N\left[\frac{\text{Binomial}[25,\,3]*\text{Binomial}[25,\,3]}{\text{Binomial}[50,\,6]}*E^{-2} + \sum_{j=4}^{6}(\text{Binomial}[25,\,j]*\text{Binomial}[25,\;6-j]* \right.$$
$$\left. E^{-2\,(j-3)}*(2\,(j-3)^{j-3})/(\text{Binomial}[50,\,6]*(j-3)!)\right]$$

$$P(4) = N\left[\frac{\text{Binomial}[25,\,4]*\text{Binomial}[25,\,2]}{\text{Binomial}[50,\,6]}*E^{-2} + \sum_{j=5}^{6}(\text{Binomial}[25,\,j]*\text{Binomial}[25,\;6-j]* \right.$$
$$\left. E^{-2\,(j-4)}*(2\,(j-4)^{j-4})/(\text{Binomial}[50,\,6]*(j-4)!)\right]$$

$$P(5) = N\left[\frac{\text{Binomial}[25,\,5]*\text{Binomial}[25,\,1]}{\text{Binomial}[50,\,6]}*E^{-2} + (\text{Binomial}[25,\,6]*\text{Binomial}[25,\;0]* \right.$$
$$\left. E^{-2}*2)/\text{Binomial}[50,\,6]\right]$$

$$P(6) = N\left[\frac{\text{Binomial}[25,\,6]*\text{Binomial}[25,\,0]}{\text{Binomial}[50,\,6]}*E^{-2}\right]$$

After the implementation of these formulas we can obtain the Fig. 1 of P($l$).

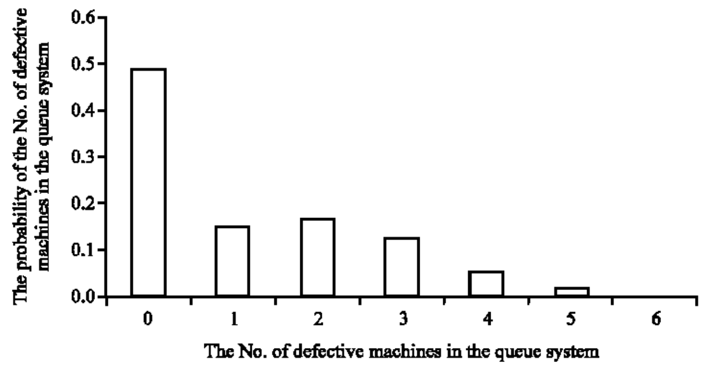


Fig. 1: The probability distribution of the number of defective machines in the queue system

The value of the average of the defective machines in the system in this application will be:

$$E\,(l) = \sum_{l=1}^{n} l\,P\,(l) = 1.158 \approx 2$$

defective machines in the system during one hour. Therefore, the queue system needs 2 servers to carry out maintenance on all the defective machines.

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