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Optimization of Barbell Trajectory During The Snatch Lift Technique By Using Optimal Control Theory

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Abstract: Mathematical modeling and optimizing of snatch lift technique based on dynamic synthesis, is the aim of this study. The barbell trajectory is proposed as the performance index, evaluated experimentally by several researchers who have introduced some optimum trajectories according to the percentage of their owners' successes. Since none of the mechanical parameters were considered into the optimum trajectories, we develop a mechanical approach to fulfill this lack. Therefore, we use a biomechanical model comprised of five links to obtain the optimum trajectory of snatch weightlifting by means of minimizing a criterion function. To achieve this goal, we should solve the differential equations representing the model motion simultaneously with equations representing the performance criterion; therefore we use the optimal control approach via Pontryagin Maximum Principle (PMP) formulation. The performance criterion is defined as minimizing the sum of actuating torques of all joints during the whole snatch. The barbell trajectory of our optimized model is approximately similar to one which could be observed in experimental results. By comparing the results of this theoretical model with experimental observations of other researchers, it could be concluded that we have introduced a good predictive model. Using the biomechanical characteristics of any specific weightlifter as the input data to this model and comparing the results with the same weightlifter's experimental data can help the coaches to improve the performance of weightlifters.

Keywords: Sport biomechanics, weightlifting, pontryagin maximum principle, motion analysis.

INTRODUCTION

Barbell trajectory and other dynamic characteristics of motion, like velocity of barbell during weightlifting, were the common subjects which have been investigated by several researchers[1-8] over the years. The importance of optimizing the barbell trajectory is in agreement with the most above researchers like Garhammer[3] who believed that an error existed on trajectory is due to incorrect motion on participated joints. Most of the above researchers studied the differences between the characteristics of motion of elite weightlifters, and categorized several of these lifting motion patterns as optimized one, such as the study was coordinated by Baumann et al.[7]. These patterns were selected because of the percentage of success of their owners, and none of mechanical parameters were considered. In recent years, some researchers used actuating torque as a mechanical parameter to introduce optimized patterns for lifting tasks. Park et al.[9] investigated the differences in motion patterns for goal-directed lifting activities which are due to biomechanical constraints or physiological responses and believed that the redundancy of degrees of freedom makes it possible to have an optimum motion pattern. But, there was no attempt to use this method for weightlifting which is more complicated than simple lifting task.

On the other hand, using optimal control theory to optimize the gait patterns[10] and the capability of this method for sport activities[11, 12] encourage us to extend this method to the whole motion of snatch lift. We formulate a mathematical model based on dynamic principles to predict the barbell trajectory which is minimized the specific criterion.

MATERIALS AND METHODS

The first step to build a biomechanical model of the weightlifter is to translate the physical property of human into the mathematical one, meaning that we convert the whole body to appropriate model of links

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with proper length, mass and moment of inertia. For this purpose, we can use the anthropometric models developed by several researchers. One of these models was introduced by Chaffin and Anderson[13]. Now we have a multi segment model representing the whole body, and help us to describe its motion using dynamics approach. In this model, the body segments are converted to solid links and body joints converted to simple revolute joints. The second step is simplifying this model to a two dimensional sagittal plane model for weightlifting or general lifting activities. This is a common assumption used by several researchers[9, 14, 15]. The third step is defining a kinematics model which represents the number of links and hence the number of degrees of freedom (DOF) that is the main factor affecting the complication of model, and therefore has a direct effect on time and cost of computing and solving the problem. The best model is the one that minimizes the complication and simultaneously offers a good approximation of the whole motion. Several researchers[9, 14, 15] used models with five DOF to analyze lifting tasks, therefore we used the same five link model.

Figure 1 shows the schematic diagram of this model at initial time which is made by five links represent shin, thigh, trunk, upper arm and forearm, respectively named L1 to L5. Also, five body joints: ankle, knee, hip, shoulder and elbow are represented by O1 to O5 respectively. D1 and H1 describe the position of barbell related to reference coordinate system of $\mathbf{X}_0\mathbf{Y}_0\mathbf{Z}_0$ locked at ankle joint. The model motion can be described by the five relative joint coordinates which are defined by:

$$q_i = (\mathbf{X}_i, \mathbf{X}_{i-1})_{\mathbf{Z}_0} \qquad i = 1, \ldots, 5 \quad (\mathbf{Z}_0 = \mathbf{X}_0 \times \mathbf{Y}_0) \quad (1)$$

Let us add the following complementary notations:

$\mathbf{q} = (q_1, \ldots, q_5)^{\mathrm{T}}$, vector of joint coordinates

$\dot{\mathbf{q}} = (\dot{q}_1, \ldots, \dot{q}_5)^{\mathrm{T}}$, vector of joint velocities

$\ddot{\mathbf{q}} = (\ddot{q}_1, \ldots, \ddot{q}_5)^{\mathrm{T}}$, vector of joint accelerations

where $\dot{q}_i = \mathrm{d}q_i / \mathrm{d}t$, $\ddot{q}_i = \mathrm{d}^2 q_i / \mathrm{d}t^2$

According to Fig. 1, we define the dimension and inertia characteristics of the model by:

$\mathbf{O}_i\,\mathbf{O}_{i+1} = r_i\,\mathbf{X}_i$, $i = 1, \ldots, 5$; r_i, length of link L_i

$\mathbf{O}_i\,\mathbf{G}_i = a_i\,\mathbf{X}_i$, $i = 1, \ldots, 5$; G_i, center of gravity of link L_i

m_i , mass of link L_i

I_i^{zz} , moment of inertia of L_i with respect to the joint axis $(O_i; \mathbf{Z_0})$

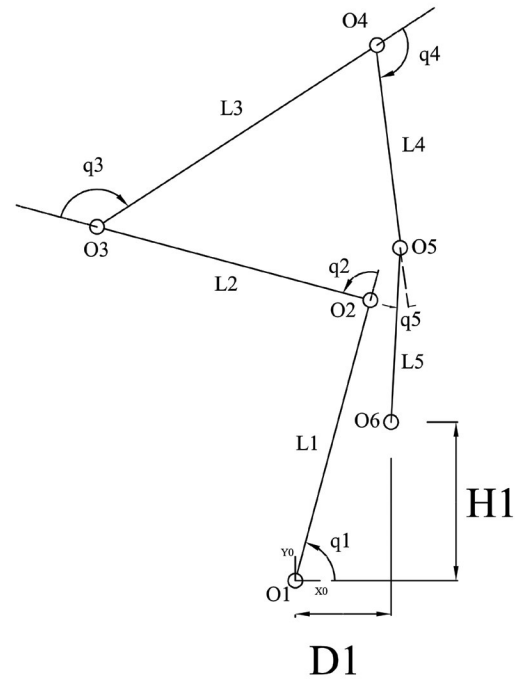

Fig. 1: Proposed model at initial position

Numerical values of these dimensional parameters are calculated based on total body mass and total length of weightlifter using the formula suggested in[13].

We want to solve the dynamics equations of motion which are in the form of differential equations and simultaneously minimize the actuating torques we need to produce the snatch motion. This form of problem called optimal control problem. There are some direct and indirect methods for solving this problem which are well described by Chettibi[16].

Since our intended purpose is using Pontryagin Maximum Principle (PMP) to solve our dynamic optimization problem, we use the formulation of dynamic model in the state space form. As indicated by Rostami[10], the Hamiltonian dynamic model not only fulfills this requirement but, as well, strengthens the robustness of algorithms which are used to solve the optimization problem.

Firstly, the Lagrangian of the model is introduced by:

$$L(\mathbf{q}, \dot{\mathbf{q}}) = T(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}) \quad (2)$$

where V is the gravity potential and T is the kinetic energy defined by:

$$T(\mathbf{q}, \dot{\mathbf{q}}) = 1/2\,\dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q})\,\dot{\mathbf{q}} \quad (3)$$

$\mathbf{M}$ is the $(n \times n)$ mass matrix of the kinematics chain. Equations of motion may be derived by Lagrange's formula:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=Q_i^d+Q_i^a \quad , i=1,\ldots,n \qquad (4)$$

where Q_i^a represents the joint actuating torque exerted by L_{i-1} on L_i at O_i and Q_i^d is joint dissipative torque.

Secondly, the conjugate momenta is defined by:

$$p_i=\partial L/\partial \dot{q}_i \quad , i=1,\ldots,n \quad (5)$$

and the Hamiltonian

$$H(\mathbf{q},\mathbf{p})=\mathbf{p}^T\dot{\mathbf{q}}-L(\mathbf{q},\dot{\mathbf{q}}) \qquad (6)$$

where $\mathbf{p}=(p_1,\ldots,p_n)^T$

By recasting Lagrange's equations in (4) and in Hamiltonian form

$$\begin{cases}\dot{q}_i=\partial H/\partial p_i\\ \dot{p}_i=-\partial H/\partial q_i+Q_i^a+Q_i^d\end{cases} \qquad (7)$$

Now, considering (2) and (3), the expression of $\mathbf{p}$ can be written through (5) by:

$$\mathbf{p}=\mathbf{M}\dot{\mathbf{q}}$$

or, inversely

$$\dot{\mathbf{q}}=\mathbf{M}^{-1}\mathbf{p}$$

Using these expressions in (3) and (6), one obtains:

$$H(\mathbf{p},\mathbf{q})=1/2\mathbf{p}^T\mathbf{M}^{-1}\mathbf{p}+V$$

Then, (7) becomes more explicitly

$$\begin{cases}\dot{q}_i=G_i(\mathbf{q},\mathbf{p})\equiv\sum_{j=1}^n M_{ij}^{-1}p_j\\ \dot{p}_i=-1/2\mathbf{p}^T\mathbf{M}_{,i}^{-1}\mathbf{p}-V_{,i}+Q_i^a+Q_i^d\end{cases} \qquad (8)$$

where $\mathbf{M}_{,i}^{-1}\equiv\partial\mathbf{M}^{-1}/\partial q_i$, $V_{,i}\equiv\partial V/\partial q_i$

The $\mathbf{M}_{,i}^{-1}$ term makes (8) impracticable but, defining the vector $\mathbf{G}=(G_1,\ldots,G_n)^T$ (see (8)) and using the mathematical formula $(\mathbf{M}^{-1})_{,i}=-\mathbf{M}^{-1}\mathbf{M}_{,i}\mathbf{M}^{-1}$, these equations can be recast by:

$$\begin{cases}\dot{q}_i=G_i(\mathbf{q},\mathbf{p})\\ \dot{p}_i=1/2\mathbf{G}^T\mathbf{M}_{,i}\mathbf{G}-V_{,i}+Q_i^a+Q_i^d\end{cases} \qquad (9)$$

By this formulation, Hamiltonian equations are ideally structured for applying the Pontryagin Maximum Principle (PMP).

Now, we define:

$$i\le n \quad \begin{cases}x_i=q_i\\ x_{n+i}=p_i\\ u_i=Q_i^a\end{cases}$$

$\mathbf{x}=(x_1,\ldots,x_{2n})^T$, vector of state variables

$\mathbf{u}=(u_1,\ldots,u_n)^T$, vector of control inputs (joint actuating torques)

$$F_i\equiv G_i=\sum_{j=1}^n M_{ij}^{-1}x_{n+j}$$

$$\mathbf{G}=(G_1,\ldots,G_n)^T$$

$$F_{n+i}=1/2\mathbf{G}^T\mathbf{M}_{,i}\mathbf{G}-V_{,i}+Q_i^d$$

$$\mathbf{F}=(F_1,\ldots,F_{2n})^T$$

The double set of equations (7) can be recast by the differential vector-equation:

$$\dot{\mathbf{x}}(t)=\mathbf{F}(\mathbf{x}(t))+\mathbf{B}\mathbf{u}(t)\equiv\mathrm{F}(\mathbf{x}(t),\mathbf{u}(t)) \qquad (10)$$

where $\mathbf{F}$ is a nonlinear function and $\mathbf{B}$ is the constant $(2n\times n)$ matrix as following:

$$\mathbf{B}=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix},\ \mathbf{B}_1=\mathbf{0}_{n\times n}\ ,\ \mathbf{B}_2=\mathbf{I}_{n\times n} \qquad (11)$$

In this equation, initial and final states will be specified by:

$$k\le n\ ,\ \begin{cases}x_k(t^i)=q_k^i \quad , \quad x_k(t^f)=q_k^f\\ x_{n+k}(t^i)=p_k(t^i)=\sum_{j\le n}M_{kj}(\mathbf{q}^i)\dot{q}_j^i\\ x_{n+k}(t^f)=p_k(t^f)=\sum_{j\le n}M_{kj}(\mathbf{q}^f)\dot{q}_j^f\end{cases} \qquad (12)$$

It will be assumed that initial and final Lagrangian phase-variables $q_k^i,\dot{q}_k^i,q_k^f,\dot{q}_k^f$ have specified values. Consequently, initial and final Hamiltonian states $\mathbf{x}(t^i)$ and $\mathbf{x}(t^f)$ will be entirely specified as well.

Let us mention that a complementary transformation remains to be achieved in order to make perfect formulation (9, 10 and 12). It consists in rescaling all the variables of the problem to homogenize their order of magnitude. Therefore, we introduce the following reference quantities: $\overline{L},\overline{M},\overline{T},\overline{I}$ and $\overline{Q}$, respectively represent length, mass, time, moment of inertia and torque of reference which can be defined and linked by:

$$\begin{aligned}&\overline{L}=\max(l_1,\ldots,l_n)\\ &\overline{M}=\max(m_1,\ldots,m_n)\\ &\overline{Q}=\max(Q_1^{a,\max},\ldots,Q_n^{a,\max})\\ &\overline{I}=\overline{M}\,\overline{L}^2 \quad , \quad \overline{T}=\sqrt{\overline{I}/\overline{Q}}\end{aligned} \qquad (13)$$

where l_i and m_i are the length and the mass of link L_i, and $Q_i^{a,\max}$ is the maximal value of $\left|Q_i^a\right|$. Also, let τ be the reduced time $\tau=t/\overline{T}$. Then, we define dimensionless state variables x_i:

$$i \le n \quad , \quad \begin{cases} x_i(\tau) = q_i(t) \\ x_{n+i}(\tau) = p_i(t) / \overline{M}\,\overline{L}^2\,\overline{T}^{-1} \end{cases} \qquad (14)$$

together with the normalized actuating torque:

$$u_i(\tau) = Q_i^a(t) / Q_i^{a,\max} \equiv Q_i^a(t) / \nu_i\,\overline{Q} \qquad (15)$$

where ν_i stands for the dimensionless coefficient $Q_i^{a,\max} / \overline{Q}$.

With these new variables, equation (10) remains formally unchanged except the matrix $\mathbf{B}_2$ that becomes $\mathbf{B}_2 = diag(\nu_1, ..., \nu_n)$.

Initial and final constraints specify the conditions of start position and the end of second pulling phase or start of catching phase. Initial conditions are primarily formulated by:

$$\begin{aligned} &\mathbf{O_1O_6^i} \cdot \mathbf{X}_0 - D1 = 0 && a \\ &\mathbf{O_1O_6^i} \cdot \mathbf{Y}_0 - H1 = 0 && b \\ &\mathbf{V}(O_6)^i \cdot \mathbf{X}_0 = 0 && c \\ &\mathbf{V}(O_6)^i \cdot \mathbf{Y}_0 = 0 && d \end{aligned} \qquad (16)$$

The first two equations (16a,b) define the position of the mass center of barbell, and the last two equations (16c,d) indicate that barbell has no initial velocity at the beginning of motion, i.e., "Lift-off" phase.

We formulate quite similar conditions in final time:

$$\begin{aligned} &\mathbf{O_1O_6^f} \cdot \mathbf{X}_0 - D2 = 0 && a \\ &\mathbf{O_1O_6^f} \cdot \mathbf{Y}_0 - H2 = 0 && b \\ &\mathbf{V}(O_6)^f \cdot \mathbf{X}_0 = V_{xd} && c \\ &\mathbf{V}(O_6)^f \cdot \mathbf{Y}_0 = V_{yd} && d \end{aligned} \qquad (17)$$

The two first ones, express the position of mass center of barbell at the end of second pulling phase (or at the beginning of "catch" phase) of the snatch lift, and the last two equations indicate the horizontal and vertical velocities of barbell at this point.

Constraints (16, 17) can be formally expressed by:

$$k \le 4 \quad , \quad \begin{cases} C_k^i(\mathbf{q}^i, \dot{\mathbf{q}}^i) = 0 \\ C_k^f(\mathbf{q}^f, \dot{\mathbf{q}}^f) = 0 \end{cases} \qquad (18)$$

In order to respect joint stops, to prevent counter-flexion and to moderate total joint coordinate variations, we have to prescribe bounds on the joint coordinates, defined by the box constraints:

$$t \in [t^i, t^f], \quad i \le n, \quad q_i^{\min} \le q_i(t) \le q_i^{\max} \qquad (19)$$

where $q_i^{\min}$ and $q_i^{\max}$ are specified values.

This set of double inequalities can be recast under the standard form of $2n$ simple constraints:

$$\begin{cases} h_i(\mathbf{q}(t)) \le 0 \quad , \text{ where } h_i(\mathbf{q}(t)) = q_i(t) - q_i^{\max} \\ h_{n+i}(\mathbf{q}(t)) \le 0 \quad , \text{where } h_{n+i}(\mathbf{q}(t)) = q_i^{\min} - q_i(t) \end{cases} \qquad (20)$$

In contrast with state or kinematics constrains, we use control constraints term for the inequalities defining limitation on torques acting on the mechanical system. Torques which are produced by actuators (i.e., muscles) have limited values. When they are considered at the joint level, with the notations introduced before, we can write:

$$\forall \; t \in [0, T], \quad |Q_i^a(t)| \le Q_i^{a,\max} \qquad (21)$$

These box constraints allow the set of feasible normalized control-variables u_i shown in (15), to be defined by:

$$\mathcal{U} = \{(u_1, ..., u_i, ..., u_n) \in \Re^n \mid u_i \in [-1,1], \; i \le n\} \qquad (22)$$

It is the time to formulate an optimal control problem. We want to generate an optimal motion by minimizing a performance criterion representing a dynamic cost. Roughly speaking, we have the choice between minimizing actuating torques[17], or energy expenditure[18]. Since our model stands and moves in a vertical plane, it is essentially submitted to gravity. For this reason, we have used the first choice by introducing the integral cost:

$$J(\mathbf{u}) = \int_{t^i}^{t^f} L(\mathbf{x}(t), \mathbf{u}(t))\, dt \qquad (23)$$

where the Lagrangian is the quadratic function of the normalized control variables u_i

$$L(\mathbf{x}, \mathbf{u}) \equiv 1/2 \sum_{i=1}^n \xi_i \nu_i^2 u_i^2 \qquad (24)$$

where ξ_i is weighting factor and $\nu_i u_i$ represents dimensionless joint actuating torque as defined by (15).

Inequality constraints which were defined in (20) can be easily dealt with using computing techniques similar to penalty method developed in the frame of mathematical programming[19]. We have chosen to implement an exact penalty method defined by introducing the positive function:

$$i \le N_h \;,\; h_i^+(\mathbf{x}(t)) = \max(0, h_i(\mathbf{x}(t)) + b_i) \;,\; b_i \ge 0 \qquad (25)$$

Each positive constant b_i in (25) defines an augmented constraint. This penalty technique consists in minimizing h_i^+ functions when the constraint is infringed, in order to bring h_i functions back to zero. This operation is carried out by minimizing the augmented criterion:

$$J_r(\mathbf{u}) = J(\mathbf{u}) + r/2 \int_{t^i}^{t^f} \mathbf{h}^+(\mathbf{x}(t))^T \mathbf{D_h} \mathbf{h}^+(\mathbf{x}(t))\, dt \quad , \quad r > 0 \qquad (26)$$

where $\mathbf{h}^+ = (h_1^+, ..., h_{N_h}^+)^T$ and $\mathbf{D_h} = \text{diag}(\zeta_1, ..., \zeta_{N_h})$ are weighting matrices.

The functional J_r has to be minimized by sufficiently great value of the penalty multiplying factor r. At this point, the minimization problem may be summarized by: finding a phase trajectory $t \to \mathbf{x}(t)$ and a control vector $t \to \mathbf{u}(t)$, minimizing J_r, namely:

$$\min \quad J_r(\mathbf{u}), \quad \mathbf{u} \in \mathcal{U}, \quad r \text{ great} \tag{27}$$

and satisfying the state equation:

$$t \in [t^i, t^f] \quad , \quad \dot{\mathbf{x}}(t) = F(\mathbf{x}(t)) + \mathbf{B}\mathbf{u}(t) \tag{28}$$

together with the boundary conditions:

$$\mathbf{x}(t^i) = \mathbf{x}^i \quad , \quad \mathbf{x}(t^f) = \mathbf{x}^f \tag{29}$$

where $\mathbf{x}(t) \in \Re^{2n}$ and $\mathbf{u}(t) \in \Re^n$.

Defining the Pontryagin function:

$$\mathbf{w} \in \Re^{2n} \quad , \quad H(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \mathbf{w}^T (F(\mathbf{x}) + \mathbf{B}\mathbf{u}) - L(\mathbf{x}, \mathbf{u}) \tag{30}$$

The Maximum Principle states that if $t \to (\mathbf{x}(t), \mathbf{u}(t))$ is a solution of (27-29), then there is a co-state function $t \to \mathbf{w}(t)$, $\mathbf{w}(t) \in \Re^{2n}$, satisfying the co-state equation:

$$\dot{\mathbf{w}}(t)^T = -\partial H / \partial \mathbf{x} \tag{31}$$

and the maximally condition:

$$H(\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)) = \max_{\mathbf{v} \in \mathcal{U}} H(\mathbf{x}(t), \mathbf{v}, \mathbf{w}(t)) \tag{32}$$

A prominent interest of the PMP lies in condition (32) which allows the constraints on $\mathbf{u}$ to be exactly satisfied, and yields through (24), (28) and (30) an explicit expression of the optimal control under the form of the saturation function[20]:

$$i \le n \quad , \quad u_i(t) = \text{Sat}[w_{n+i}(t) / \xi_i \nu_i] \tag{33}$$

defined here by:

$$\text{Sat}(x) = \begin{cases} x & \text{if} \quad |x| \le 1 \\ x^{\max} \text{sign}(x) & \text{if} \quad |x| > 1 \end{cases} \tag{34}$$

Substituting the expression (33) for $\mathbf{u}$ in (28) and (31), the unknown functions $\mathbf{x}$ and $\mathbf{w}$ appear as a solution of the differential system of the type:

$$t \in [t^i, t^f] \quad , \quad \begin{cases} \dot{\mathbf{x}}(t) = F_1(\mathbf{x}(t), \mathbf{w}(t)) \\ \dot{\mathbf{w}}(t) = F_2(\mathbf{x}(t), \mathbf{w}(t)) \end{cases} \tag{35}$$

accompanied by the boundary conditions (29).

Typically, we deal with a two-point boundary value problem. The two-point boundary value problem (35, 29) can be solved using computing techniques such as finite difference algorithm or shooting method. We have chosen the latter approach because of its efficiency and the simplicity of the implementation. This technique was described[21] as the so-called transition matrix method and other mathematicians like Kincaid and Cheney[22] defined it as the so-called shooting method.

Because of the non-linearity of equations (35), we have used the multiple shooting method by solving the two-point boundary value problem when considering a short motion step. Then the optimization problem can be solved iteratively by increasing boundary values until the desired final values are reached. In the same way, any optimal solution can be used by a guess solution to solve swiftly a problem relating to the previous one. This method was described as the so-called multiple shooting method[22].

RESULTS AND DISCUSSION

We solved a problem for a weightlifter with 55 kg mass and 1.6 m height who lifts a 90 kg barbell by snatch technique. Other dimensional parameters were calculated base on this information in a manner described before. We selected the summation of actuating torques of all joints as the optimization criterion and solved our problem between two points representing the start of snatch and the start of catch phase respectively. The start of catch phase was so selected that the barbell has a good condition to continue its motion and the weightlifter could move under the bar quickly. Figure 2 shows the barbell trajectory during the snatch lift from the time just prior to when the barbell left the floor ("lift-off") until just after the bar reached at the end of second pull. At this point the barbell continues to move as a "projectile" and let the athlete to complete "move under the bar" to catch it. In the experiments, the typical form of trajectory described by Garhammer[3, 8] showed that when the barbell was lifted from the "lift-off" phase, it moved toward the athlete during the first pull, then away from the athlete and finally toward him again as it began to descend during the catch phase. Figure 2 shows this typical form roughly. One can see the good agreement between optimized trajectory and experimental results shown in Fig. 2 according to the study published by Garhammer[3].

Figure 3 is the graph which shows how the vertical barbell velocity changes with the time during snatch lift until the start of catch phase, and how they can be continued due to the projectile motion of barbell. Furthermore, we have observed the experimental diagram[3] that shows the increase of vertical velocity to about 2 m/s and decreasing it as the barbell moves toward the final position as shown in Fig. 3 with scaled time according to our model. Although our model is more simple than an athlete, comparing these two

velocities, one can see the good qualitative agreement between our optimized results with the experimental results which the elite weightlifters can achieved.

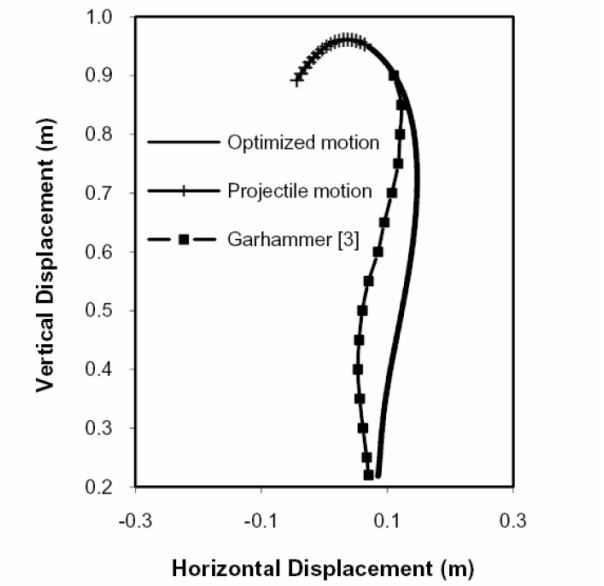

Fig. 2: Optimized and experimental barbell trajectory during snatch lift

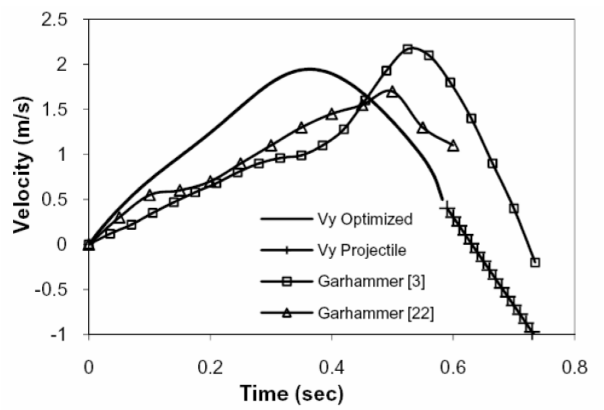

Fig. 3: Optimized and experimental vertical velocity of barbell during snatch lift

In Fig. 4, the motion sequences of our model in the form of joint space between the initial and final posture of snatch lift can be observed. The final position is selected at the start of catch phase in a manner described before. According to this Figure coaches can help their weightlifters at closing the gap between their motion and optimized one. Also this Figure shows us the validity of results which is obtained by solving the optimization problem.

Figures 5 and 6 respectively show how angular velocities of joints and actuating torques at joints were changed between initial and final positions shown in Fig. 4 during the snatch lift. The role of each joint in making a complete snatch lifting motion can be realized by these diagrams considering kinematics or kinetics aspect. For example the importance and descending role of hip joint and also the ascending role of ankle and knee joints during the snatch lift can be easily seen. Also these variables are good parameters to show us the practical differences between an actual snatch motion of weightlifter and the ideal optimized one which he could achieved. We can reduce these differences by advise our weightlifter about the correct velocity he should reached or the strength training he should do to compensate the weakness of particular joint.

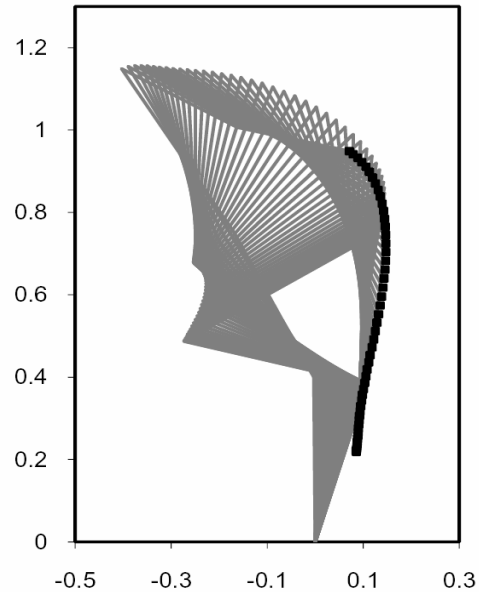

Fig. 4: Optimized model motion during snatch lift until the start of catch phase

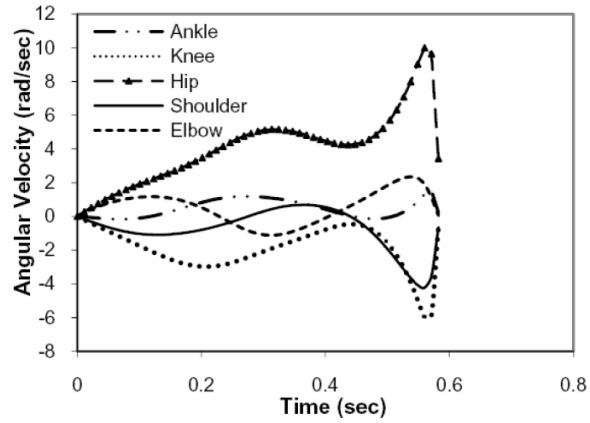

Fig. 5: Optimized joint angular velocities until the start of catch phase

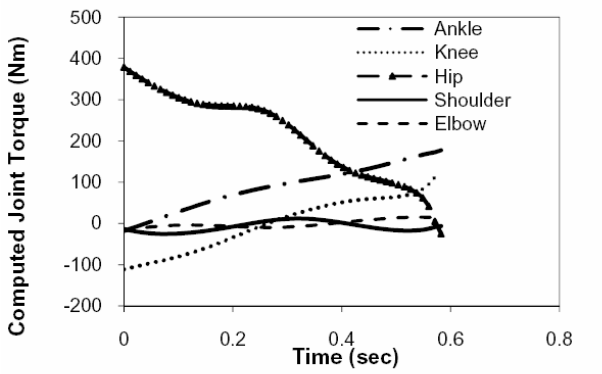

Fig. 6: Optimized actuating joint torques until the start of catch phase

CONCLUSION

Barbell trajectory which is produced by our optimized model shows the typical form we can see in experimental data. Since we obtain this optimized trajectory by using dynamic motion equations, we ensure that this trajectory can be produced by a real weightlifter while other optimizing strategy like geometrical path optimization could not give this possibility to us. On the other hand, we believe that the results show the relative success to predict the optimal motion based on our selective criterion. Therefore, we can conclude that the selecting criterion is in agreement with that is selected by weightlifter. However selecting the best criteria to improve the performance of weightlifters, requires more studies and it could be consists of more than one criterion combined together during the full snatch. We can select several parameters such as actuating torques, time of snatch lift, energy expenditure, injury risk, and critical stress on body joints as the criteria which should be minimized during an optimized snatch lift.

The results of this optimization can help us to train our weightlifters to behave like our optimized kinematics parameters or to make their characteristics like resultant kinetics parameters. Comparing the actual parameters of our case of study with optimized one, can guide us to achieve these useful comments. But the problem we are faced is the lack of sufficient experimental inputs such as the correct values for maximum capacity of torque production of each joint. We believe that the modeling with correct characteristics of each athlete can lead us to get better results tailored for our case of study.

This dynamic model can provide an insight into control and improve the motions during the snatch lift. The ability to introduce and modify the proper criterion which is in agreement with human motion pattern is another point that we notice in solving the problem and we believe that we are successful regarding to this matter.

The determination of optimal motion during the whole motion of snatch lift can help coaches to train weightlifters on a more systematic manner. This model can help them, not only to increase their success but also to reduce their injury risks. Finally, the Pontryagin Maximum Principle seems quite appropriate to deal with an extended problem of this type. Successfully performing this dynamic optimization was very hard due to the extreme non-linearity of the system of differential equations which is resulting from mathematical formulation and we have to use numerical techniques to overcome to this problem. However, the good results that we obtained from optimization problem showed that this method is very reliable. Therefore the fair success of our model encourages us to continue our approach and improve our model by the models which have more degrees of freedom in the near future.

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