

A Linear Programming Model to Compute the Optimum Yield of Sweet Potato at Various Locations

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Abstract: Soil and weather conditions play a major role in the growth and yield of sweet potato. They influence the development of source and sink parameters. The source parameters are important in the production and partitioning of dry matter. They also act as a sink. Assuming that the development of plant parameters as the translation of growing environment, a Linear Programming (LP) model for computing optimum yield was formulated. An object function was formulated as a yield maximizing problem. The plant parameters like the number of Branches (BR), the number of Leaves (LF), the Vine Length (VL), the number of internodes per vine (IND), the Vine Fresh Weight (VFT), the petiole length (PLLN), the Tuber Girth (TGR) and the Leaf Area Index (LAI) were considered as the constraints (Resources). An object function and constraints were formulated in terms of the length of tubers (ln_TBR) and the number of tubers (n_TBR), which are the two major yield determinants of sweet potato. Two separate problems, based on the published data were formulated for Jabalpur and Bihar plateau in India. Solutions for the optimum yield in these locations and the influence of each resource on the tuber yield were found.

Key words: Linear programming, sweet potato, optimum yield

INTRODUCTION

Sweet potato is a food crop grown under different environments. It is grown between latitude 40°N and 40°S of the equator and between sea level and 2300 m altitude^[1]. Like any other crop its growth and development are influenced very much by the environment. Since every location is having its characteristic weather, soil and other environmental conditions existing over the years, their potential to produce sweet potato tubers also vary. Differential reaction of genotypes in different environments was observed by Rajeshkumar *et al.*,^[2]. Potential yield of a crop is usually estimated under the most ideal conditions where all the resources are abundant. Simulation models can be formulated using the experimental data, to find the values of plant parameters that give the maximum yield. Linear programming (LP)^[3] is a tool that finds the maximum value of a function, given the constraints on the independent variables of that function. Using LP as a tool, studies were carried out by Ismail and Jusoff^[4] to highlight the potential of Geospatial Information Technologies (GIT) for Malaysian agriculture in this millennium. Study conducted by Cara and Jayet^[5] using LP, highlighted the importance of allowing afforestation in the set aside land to achieve a cost effective reduction in net emission of greenhouse gases.

In this study, an LP based model is formulated to estimate the optimum sweet potato yield at two locations in India. Physical parameters like weather, soil and other environmental conditions are different in each of these sites. Hence a maximum and minimum limit for the plant Parameters, Vine Length (VL), Number of Branches (BR), Number of Leaves (Lf), Leaf Area Index (Lai), Number of Internodes per Vine (Ind), Petiole Length (PLLN), vine Fresh Weight (VFT) and Tuber Girth (TGR) are assumed to exist, beyond which the components will not grow. The constraints included in this study are the plant parameters. This model is tested using the datasets published from the experiments conducted at Jabalpur and Bihar plateau in India and is found to give results that agree with the experimental data.

MATERIALS AND METHODS

Model formulation: The number of tubers (n-TBR) is the most important component in determining the yield of sweet potato^[6-8]. Importance of the length of tubers (ln_TBR) in determining the tuber yield was stressed by Amarchandra *et al.*,^[6] and Amarchandra and Tiwari^[9]. Therefore, these two components are taken as the independent variables in this problem. The LP problem formulated for estimating the optimum yield consists of the following components.

Object function: An object function was formulated as a yield maximizing problem and is given as:

$$\text{Maximize } Z = c1 \cdot \ln_TBR + c2 \cdot n_TBR \quad (1)$$

where,

Z is Tuber yield (TYLD) (g/plant),
c1 and c2 are arbitrary coefficients.

Constraints: The main constraint in maximizing tuber yield is weather. Soil nutrients, water etc. also affect the tuber yield. These environmental factors influence the development of plant parameters like VL, LF, BR and LAI, which affect the tuber yield. Hence these plant parameters are taken as the factors influencing tuber yield, in place of the environmental parameters. Each of these constraints is represented in terms of the variables of the object function. The amount of constraint required to produce one unit of the object function variable is represented as the coefficients of the variables in the constraint function. The constraints/resources included in the problem are briefly described below.

Number of branches: In sweet potato, branching takes place almost throughout the entire vegetation period. This process is influenced very much by soil factors. Irrigation and soil application of nitrogen are found to increase the number of branches^[10]. Climatic factors also play a major role in the development of this parameter. Photoperiod have profound influence in branching. Long photoperiod (18 h) reduces the number of branches, while it increases the branch length when compared to plants exposed to 12.5 h photoperiod^[11]. A short photoperiod of 8 h increases the number of branches while decreasing the branch length^[12].

Number of leaves: Number of leaves retained per plant increases with irrigation and N application^[10]. Ageing and mutual shading promote shedding of older leaves^[13].

Leaf area: Leaf area is a major determinant of the final yield. It contributes through the process of photosynthesis which prepares the food required for the growth of the plant. Leaf area is usually measured as LAI. Climatic factors like rainfall play a very crucial role in the pattern of leaf area development. Enyi^[14] reported that vine growth has a favourable impact on the development of leaf area. A study by Mukhopadhyaya *et al.*^[15] revealed that potassium nutrition has profound influence on LAI.

Vine length: Vine length increases with irrigation and soil nitrogen application^[10]. Enyi^[14] reported a favourable effect of precipitation on vine growth. Significant influence of environment component on VL and LF was observed by Rajeshkumar *et al.*^[2] also.

Petiole length: Petiole length is an important parameter in determining the orientation of leaves. Proper leaf orientation is needed for efficient trapping of the solar energy. It ultimately decides the production of food and thus the tuber yield also. Influence of environment on this parameter was observed by Amarchandra and Tiwari^[9].

Number of internodes per vine: Amarchandra and Tiwari^[9] have reported that the vine length is increased by increasing both the internodal length and the number of internodes. Hence this attribute is also influenced by the environment.

Tuber girth: Influence of environment on tuber girth was observed by Rajeshkumar *et al.*,^[2] This is an important sink parameter having profound influence on the ultimate tuber yield.

Vine fresh weight: Dynamic balance between roots and vines is important for deciding the final yield^[16]. Influence of soil application of N and K on vine yield was observed by Mukhopadhyay *et al.*,^[15]

Using the published data, we formulate a LP problem and apply it for two locations, Jabalpur and Bihar in India and are given in the following sections.

Jabalpur problem: The LP problem for Jabalpur was formed using the data on biometric parameters of sweet potato collected from the experiments conducted by Amarchandra *et al.*,^[6] and Amarchandra and Tiwari^[9] (Table 1 and 2). Amarchandra *et al.*,^[6] conducted the experiment at the Department of Vegetable crops and Floriculture, Jawaharlal Nehru Krishi Viswavidyalaya, Jabalpur (JNKVVJ), India during the rabi (winter) season of 1980-81. Ten sweet potato varieties were planted in randomized block design with four replications at a spacing of 45 X 30 cm. Data on vine length, internodal length, number of internodes per vine, branches per plant, leaves per plant, petiole length, dry weight of shoot per plant, number of tuberous roots per plant, length and girth of tuberous roots and tuber yield per plot were recorded to study the growth pattern. Amarchandra and Tiwari^[9] conducted the experiment at the vegetable farm of the department of vegetable crops and floriculture, JNKVV for three years (1978-81). Twelve sweet potato varieties were planted in replicated field trials at a spacing of 45 X 30 cm. Data on vine length, number of internodes per vine, branches per plant, leaf area, petiole length, number of tuberous roots per plant, length and girth of tuberous roots, weight of tuberous roots and tuber yield per plot were recorded. The correlation coefficients (R) of plant parameters like VL, internode length, IND, BR, PLLN,

Table 1: Biometric parameters of sweet potato at Jabalpur (1980-81) from Amarchandra *et al.*,^[6]

Variety	Vine length (cm)	Inter nodes/ vine	Inter node length (cm)	Branches /plant	Petiole length (cm)	Leaves /plant	LAI	Tubers /plant	Girth of tubers (cm)	Length of tubers (cm)	Tuber yield (g/plant)
jib 7	169.56	33.21	5.14	13.77	20.57	254.05	8.82	4.66	6.19	15.76	381.37
jib 31	121.1	23.62	5.17	16.55	19.57	216.97	7.01	4.10	6.58	14.01	378.28
jib 69	51.27	19.59	2.54	14.7	15.17	139.37	7.40	4.57	4.06	12.15	271.38
jib 114	65.52	21.02	2.88	15.35	17.57	150.80	4.16	2.38	3.52	12.99	184.72
jib 116	39.61	17.38	2.28	14.52	14.13	130.50	4.63	4.66	3.35	12.69	229.70
jib 152	58.47	17.63	3.58	11.10	10.5	91.32	3.32	1.97	3.37	13.57	195.26
jawahar sakarkand145	115.65	27.94	4.1	20.45	19.2	225.00	7.64	5.35	3.08	14.06	394.72
pusa safed	162.4	28.33	5.47	16.20	18.82	216.07	10.22	3.90	7.04	15.42	379.80
pusa lal	36.02	15.39	2.28	15.87	13.17	151.27	4.24	2.39	4.35	12.31	229.70
local	158.24	27.94	5.89	17.35	21.05	228.97	7.49	1.28	3.53	12.38	94.15

Table 2: Biometric parameters of sweet potato at Jabalpur (1978-81) from Amarchandra and Tiwari^[6]

	Vine length (cm)	Internodes /vine	Branches /plant	Petiole length (cm)	Length of tubers (cm)	Girth of tubers (cm)	Tubers /plant	Weight of tubers(gm)	Tuber yield variety (g/plant)
pusa safed	111.83	36.88	6.79	12.96	14.00	14.5	4.20	119.917	472.508
pusalal	53.37	37.75	8.57	10.26	13.33	13.15	2.74	88.797	238.5
b4306	122.67	37.38	6.47	13.49	14.83	9.05	3.60	150.526	541.333
ec3905	167.01	44.9	7.58	13.06	16.67	15.5	2.85	66.098	200.542
b16	101.41	41.66	9.24	9.21	13.67	9	3.22	58.468	199.942
coll no.7	78.35	37.46	8.54	11.39	13.43	13.35	2.97	119.06	374.342
jib 114	75.27	32.2	7.71	9.34	14.77	12.30	3.60	109.258	418.525
jib 116	56.33	39.86	7.19	11.19	15.23	9.800	4.10	72.777	301.25
jib 145	105.09	39.55	6.12	11.4	15.33	9.900	4.44	76.638	342.783
jib 152	88.12	34.3	7.26	9.42	15.00	10.50	3.32	81.949	274.169
jib 177	85.92	32.85	7.05	15.36	13.10	10.25	4.53	64.432	302.083
jib 189	56.84	34.81	5.90	12.13	14.60	9.150	4.47	61.36	254.083

Table 3: Correlation coefficient R for various parameters on TYLD

No.	Plant parameter	Correlation coefficient (R) (absolute values)			
		1980-81 season	1978-81 season	Mean	Standard deviation
1.	VL	0.39	0.10	0.245	0.2
2.	Internode length	0.27	-	-	-
3.	IND	0.47	0.37	0.423	0.067
4.	BR	0.28	0.38	0.3295	0.065
5.	PLLN	0.34	0.27	0.308	0.049
6.	Mean weight of tuber	-	0.90	-	-
7.	TGR	0.639	0.004	0.321	0.449
8.	LF	0.50	-	-	-
9.	LAI	0.60	-	-	-

mean weight of tuber, TGR, LF and LAI with TYLD were computed and are given in Table 3 using the values taken from Table 1 and 2 for both the seasons. These R values give a good estimate of the influence of these parameters on TYLD. The data analysis module of MS EXCEL 2003 was used for calculating the R values.

The plant parameters IND, BR and PLLN given in Table 3 were taken as the constraints/resources in producing TYLD in this location, because they showed reasonably high R values (absolute values) consistently in both the seasons (Higher mean and lower standard deviation).

Though LF and LAI data were available for one season only, these were also selected as constraints, as they showed high R values (absolute values) with TYLD and also they make direct influence on TYLD production.

The object function for this problem was formulated using the data on Tubers/plant, Length of tubers (cm) and Tuber yield (g/plant), which were selected from Table 1 and 2. Using the statistical software SYSTAT 9, a multiple regression equation Eq. 2 with Tuber yield (g/plant) as dependent variable and n-TBR and ln-TBR as independent variables were fitted to this data as:

$$Z = 8.619\ln\text{-TBR} + 50.755n\text{-TBR} \quad (2)$$

Equation 2 shows that 8.619 g of tuber is produced by the sweet potato tuber having 1 cm length and 50.755 g of tuber is produced by a single sweet potato tuber at the optimum condition.

The constraints/resources influence the final tuber yield through their effect on ln-TBR and n-TBR, which are the independent variables of the object function. Though each of the constraint influences the development of both ln-TBR and n-TBR in different degrees, both these terms can not form part of the same constraint equation. This is because the compartmentalization of the constraint to each of these variables is theoretically not known. Hence for each constraint, the independent variable of the object function, having higher R value (absolute value) with the constraint was selected from Table 4 and formed the corresponding equation. These R values were computed using the data analysis module of MS EXCEL 2003.

Table 4: Correlation coefficients (R) (absolute values) of various constraints with ln_TBR and n_TBR

No.	Constraint	ln TBR	n TBR
1.	IND	0.64	0.11
2.	BR	0.41	0.04
3.	PLLN	0.11	0.14
4.	LF	0.58	0.28
5.	LAI	0.61	0.46

The constraint/resource IND was represented in the problem in terms of ln-TBR as:

$$c \times \ln\text{-TBR} \leq 31, (3)$$

where c is a constant.

For estimating the value of c, values of IND and the corresponding values of ln_TBR were selected from Table 1 and 2. Using these values the constant c was computed as:

$$c = \sum_{i=1, t=1}^{i=nt, t=2} \frac{\text{IND}_{i,t}}{\ln_TBR_{i,t}}, (4)$$

where,

IND_{i,t} = Value of IND in the ith row of the table t,

ln_TBR_{i,t} = Value of ln_TBR in the ith row of the table t,

nt = Number of rows in table t.

The value of c in Eq. 3 was computed to be 2.21 and this value represents nothing but the amount of IND required for producing 1 cm of ln-TBR and the maximum value of IND possible in this region was estimated as its average value for all the varieties in that location. Average value was selected because the entire problem represents the average behaviour of sweet potato varieties tried in that location. Average value of IND in the region was estimated to be 31.

Similarly Eq. 5, 6, 7 and 8 were formulated to represent the constraints/resources BR, PLLN, LF and LAI respectively.

$$0.77 \ln_TBR \leq 12 (5)$$

$$3.65 n_TBR \leq 15 (6)$$

$$13.42 \ln_TBR \leq 181 (7)$$

$$0.485 \ln_TBR \leq 6.5 (8)$$

Equation 5, 7 and 8 show that for producing 1 cm of ln_TBR, 0.77, 13.42 and 0.485 of BR, LF and LAI respectively were needed whereas Eq. 6, states that for producing one unit of n_TBR, 3.65 PLLN were required.

The inequalities in the Eq. 3 and Eq. 5- 8 can be changed to equalities by using slack variables as:

$$2.21 \ln\text{-TBR} + \text{jsx1} = 31 (9)$$

$$0.77 \ln_TBR + \text{jsx2} = 12 (10)$$

$$3.65 ns_TBR + \text{jsx3} = 15 (11)$$

$$13.42 \ln_TBR + \text{jsx4} = 181 (12)$$

$$0.485 \ln_TBR + \text{jsx5} = 6.5 (13)$$

where, jsx1, jsx2, jsx3, jsx4 and jsx5 are slack variables.

All the constraints were considered as <= function because they represent the plant vegetative parameters and they were severely restricted by external and internal growth conditions and hence they can never achieve their full potential growth under field conditions. None of them are unrestricted or abundantly available.

Bihar plateau problem: Based on the data on biometric parameters on sweet potato, published by Rajeshkumar *et al.*,^[2] Table 5 from the experiment conducted in the plateau region of Bihar during the kharif(rainy) season of 1988 to 1992, an LP problem was formulated to find out the optimum yield in that region. Twelve sweet potato varieties were planted in complete randomized block design having three replications at spacing of 60 X 20 cm. Recommended package of practices were followed every year. Data on vine length, branches per plant, number of leaves per plant, fresh weight of vines, number of tubers per plant, length and girth of tuberous roots and tuber yield per plot were recorded. The correlation coefficients (R) of various plant parameters like VL, BR, LF, VFT and TGR with TYLD at this region (Table 6) were computed using the values taken from Table 5. The data analysis module of MS EXCEL 2003 was used for computing the R values. These R values gave a good estimate of the influence of these parameters on TYLD in this region.

The plant parameters VL, BR, VFT and TGR were selected as constraints in producing TYLD in this location, because they showed reasonably high R values (absolute values) with TYLD.

An object function for this problem was formulated using the data on Tubers/plant, Length of tubers (cm) and Tuber yield (g/plant), which were selected from Table 5. Using the statistical software SYSTAT 9, a multiple regression equation Eq. 14 with Tuber yield (g/plant) as dependent variable and n_TBR and ln_TBR as independent variables were fitted to this data as:

$$Z1 = 5.056 \ln_TBR + 26.052 n_TBR (14)$$

Equation 14 shows that 5.056 g of tuber is produced by the sweet potato tuber having 1 cm length and 26.052 g of tuber is produced by a single sweet potato tuber at the optimum condition.

Similar to the Jabalpur problem, constraints/resources identified from Table 6 were formulated as inequalities with independent variable either ln_TBR or n_TBR and are given in Eq. 15-18. Correlation coefficients (R) of various constraints with ln_TBR and n_TBR are given in Table 7.

Table 5: Biometric parameters of sweet potato at Bihar plateau from Rajeshkumar *et al.*^[2]

Variety	Vine length (cm)	Branches /plant	Leaves /plant	Vine fresh weight (g)	Tubers /plant	Length of tubers (cm)	Girth of tubers (cm)	Tuber yield (g/plant)
Kalmegh	200.66	5.63	142.49	456.97	3.28	14.46	4.40	156.4
h-85-16	99.59	9.74	138.06	316.45	3.00	12.21	4.76	154.53
c-71	274.45	3.67	87.36	579.78	2.46	14.74	4.34	107.6
s-30	213.88	8.33	161.95	496.68	2.30	13.3	4.15	142.1
80/168	118.78	6.21	134.61	364.53	2.10	13.86	4.52	127.33
cross-4	235.67	4.26	60.34	549.21	2.65	11.26	4.38	113.13
x-24	147.19	7.48	98.26	654.73	2.40	11.46	3.98	93.46
v-35	202.80	6.57	221.22	491.80	2.34	10.19	4.17	105.33
76-op/21	116.70	6.00	147.36	387.54	2.37	12.44	4.16	164.58
76-op/217	187.60	8.34	220.45	429.33	1.89	10.23	4.06	109.00
x-69	225.14	5.62	156.14	446.16	2.04	10.71	4.21	122.16
local	94.20	9.50	134.66	368.77	2.32	12.67	3.65	113.80

Table 6: Correlation coefficients (R) of various plant parameters on TYLD correlation coefficient (R) (absolute values)

No.	Plant parameter	correlation coefficient (R) (absolute values)
1.	VL	0.31
2.	BR	0.12
3.	LF	0.08
4.	VFT	0.6
5.	TGR	0.45

Table 7: Correlation coefficients (R) (absolute values) of various constraints with ln_TBR and n_TBR

No.	Constraint	ln_TBR	n_TBR
1.	VL	0.013	0.03
2.	BR	0.22	0.07
3.	VFT	0.06	0.015
4.	TGR	0.27	0.48

Table 8: Optimum tableau for the Jabalpur problem

Basic	ln-TBRx1	n-TBRx2	jsx1	jsx2	jsx3	jsx4	Jsx5	solution
Z	0.00	0.00	0.00	0.00	13.91	0.00	17.77	324.09
jsx1	0.00	0.00	1.00	0.00	0.00	0.00	-4.58	1.23
jsx2	0.00	0.00	0.00	1.00	0.00	0.00	-1.59	1.68
x2(n_TBR)	0.00	1.00	0.00	0.00	0.27	0.00	0.00	4.11
jsx4	0.00	0.00	0.00	0.00	0.00	1.00	-27.67	1.14
x1(ln_TBR)	1.00	0.00	0.00	0.00	0.00	0.00	2.06	13.4

From ln-TBR and n-TBR, the one having higher R value (absolute value) with the resource was selected for representing each constraint/ resource. The data for developing these inequalities were taken from Table 5.

$$70.823 \text{ n_TBR} \leq 177 \quad (15)$$

$$0.54 \text{ ln_TBR} \leq 8 \quad (16)$$

$$36.96 \text{ ln_TBR} \leq 462 \quad (17)$$

$$1.71 \text{ n_TBR} \leq 6.5 \quad (18)$$

In this problem Eq. 15 represents VL and the value 177 was calculated to be the maximum value in this location. Eq. 16, 17 and 18 represent BR, VFT and TGR and their maximum values were calculated as 8, 462 g and 5 cm respectively.

For the constraint Eqns. 15 and 18, coefficients of n-TBR were estimated using Eq. 4 after substituting IND with VL and TGR respectively and ln_TBR with n_TBR. Coefficients of Eq. 16 and 17 were also calculated using Eq. 8 after substituting IND with BR and VFT respectively.

Equation 15 and 18 show that for producing 1 cm of ln-TBR, 70.823 cm, and 1.71 cm of VL and TGR respectively are needed. From Eqns. 16 and 17, it is understood that for producing one n-TBR, 0.54 and 36.96 g of BR and VFT respectively are required.

The inequalities in the Eqns.15-18 can be changed to equalities by using slack variables as:

$$70.823 \text{ n_TBR} + \text{bsx1} = 177 \quad (19)$$

$$0.54 \text{ ln_TBR} + \text{bsx2} = 8 \quad (20)$$

$$36.96 \text{ ln_TBR} + \text{bsx3} = 462 \quad (21)$$

$$1.71 \text{ n_TBR} + \text{bsx4} = 6.5 \quad (22)$$

where, bsx1, bsx2, bsx3 and bsx4 are slack variables.

RESULTS AND DISCUSSION

Solution for Jabalpur Problem: The Eq.2 with the constraints given in Eq. 3,5-8 was solved using the software TORA 1.03. The optimum tableau obtained after solving the problem is given in Table 8.

From the optimum tableau given in Table 8, it is seen that at the optimum condition, the tuber yield is 324.09 g/plant. The main components of yield, ln-TBR and n-TBR reached values 13.4 and 4.11 respectively at this point.

Range within which constraints/resources can be varied at the optimum condition represented in Table 8: The values of PLLN and LAI can be changed to improve the optimum production since the corresponding slack variables jsx3 and jsx5 are zero. A positive slack means that the resource is not used completely, thus is abundant, whereas a zero slack indicates that the entire amount of the resource is consumed by the activities of the model^[3]. This indicates that there is still a scope for improvement in TYLD by changing values of PLLN and LAI. To maintain this optimum condition, all the basic variables should remain non negative despite the changes in any constraint. When the value of PLLN is changed by an amount D1, following changes happen in the optimum tableau rows.

$$\begin{array}{ll}
 Z: & 324.09+ & 13.91 \text{ D1} \\
 jsx1: & 1.23+ & 0 \text{ D1} \\
 jsx2: & 1.68+ & 0 \text{ D1} \\
 x2: & 4.11+ & 0.27 \text{ D1} \\
 jsx4: & 1.14+ & 0 \text{ D1} \\
 x1: & 13.4+ & 0 \text{ D1}
 \end{array} \quad (23)$$

Coefficients of D1 in equations 23 are the value of jsx3, the slack variable of PLLN in the corresponding rows of the optimum tableau. By solving equations in 23, subject to the condition that jsx1, jsx2, x2, jsx4 and x1 remain non negative, the permissible range, for change in PLLN, to maintain this optimum condition was obtained as: $0 < PLLN$

Dual price (Unit worth) of PLLN (DP_{pln}) is the value of jsx3, the corresponding slack in the object function at this optimum condition. In this problem it is 13.9055. This means that with a unit change in PLLN, 13.9055 g change happens in TYLD. Similarly the range of variation for LAI was obtained as: $0 < LAI$

Dual price for LAI (DP_{lai}) was obtained from the optimum tableau as 17.7711.

Other constraints IND, BR and LF are having positive slack. So they are already in abundance. Their dual prices are zero and hence no change in the object function can be brought about by changing them under this optimum condition.

Range within which tuber yield can be varied at the optimum condition represented in Table 8: By changing the values of PLLN and LAI, tuber yield can be changed within some limits without disturbing this optimum condition. To study this change, object function can be written in terms of PLLN and LAI as:

$$\begin{aligned}
 Z &= TYLD_{opt} - (PLLN - PLLN_{opt}) DP_{pln} \\
 &DP_{pln} - (LAI - LAI_{opt}) DP_{lai}
 \end{aligned} \quad (24)$$

Subscript opt represents the values at this optimum condition. By changing the values of PLLN and LAI, tuber yield under this optimum condition can be increased to 648.19 g.

Range within which coefficients of the object function can be varied at the optimum condition represented in Table 8: The coefficients of the object function can also be varied without affecting this optimum condition. When the coefficients change, only the object function variables change. If the coefficient of ln-TBR changes from $c1$ to $c1 + d$ then the following changes happen in the object function.

Table 9: Results of LP model validation for Jabalpur problem for two seasons

No.	Item	1978-81	1980-81
1.	Number of bootstrap samples generated	100	100
2.	Number of bootstrap samples tested	100	100
3.	Number of samples where mean values are not significantly different	100	100

Table 10: Optimum tableau for the Bihar plateau problem

Basic	ln TBRx1	n TBRx2	bsx1	bsx2	bsx3	bsx4	solution
Z1	0.00	0.00	0.37	0.00	0.14	0.00	128.31
1.x2(ln_TBR)	0.00	1.00	0.01	0.00	0.00	0.00	2.5
2.bsx2	0.00	0.00	0.00	1.00	-0.01	0.00	1.25
3.x1(ln_TBR)	1.00	0.00	0.00	0.00	0.03	0.00	12.5
4.bsx4	0.00	0.00	-0.02	0.00	0.00	1.00	0.73

$$\text{Jsx5: } 17.7711 + 2.0619 d \quad (25)$$

By solving Eq. 25 subject to the condition that jsx5 remains non negative, the permissible range for change in $c1$ to maintain this optimum condition was obtained as $0 < c1$. Similarly the range for $c2$ was obtained as $0 < c2$.

Model validation: The predictions made by this LP model were tested with the field observations. To confirm the results further, bootstrap samples were generated from the field data and paired t test between the observed and predicted values were done.

The predictions were made using Eq. 24 in which $PLLN_{opt}$ and LAI_{opt} were the values of PLLN and LAI at the optimum condition i.e. 15 and 6.5 respectively. PLLN and LAI were the observed values in the field. TYLD at optimum condition was computed using Eq. 2. Results of the test at both the seasons were the same and are summarized in Table 9.

Predictions made by the LP model agree very well with the observed values at 95% confidence interval. This shows that the LP model works well for Jabalpur location and this can be used for predicting the performance of sweet potato in this location.

Solution for bihar plateau problem: The Eq.14 with the constraints given in Eq. 15-18 was solved using the software TORA 1.03. The optimum tableau obtained by solving the problem is given in Table 10.

From the optimum tableau given in Table 10, it is seen that at the optimum condition tuber yield is 128.31 g^{-1} plant. The main components of yield, ln-TBR and n-TBR reach a value of 12.5 and 2.49 respectively at the optimum.

Range within which constraints/resources can be varied at the optimum condition represented in Table 10: The values of VL and VFT can be changed to improve the optimum production since the corresponding slack variables bsx1 and bsx3 are zero. Improvement in TYLD

can be achieved by changing values of VL and VFT. The permissible range for change in VL to maintain this optimum condition is computed from the optimum tableau as: $0 < VL < 207.014$

Dual price (Unit worth) of VL (DP_{vl}) was obtained as 0.3678 from the optimum tableau. Similarly the range of variation for VFT was obtained as: $0 < VFT < 547.62$

Dual price of VFT (DP_{vft}) was obtained from the optimum tableau as 0.1368. The constraints (resources) BR and TGR are having positive slack. Their dual prices are zero and hence no change in the object function can be brought out by changing them under this optimum condition.

Range within which tuber yield can be varied at the optimum condition represented in Table 10: By changing the values of VL and VFT, tuber yield can be changed within some limits without disturbing this optimum condition. To study this change, the object function is written in terms of VL and VFT as:

$$ZI = TYLD_{opt} - (VL - VL_{opt}) DP_{vl} - (VFT - VFT_{opt}) DP_{vft} \quad (26)$$

By changing the values of VL and VFT, the tuber yield under this optimum condition can be increased to 256.611 g and decreased to 105.54 g.

Range within which coefficients of the object function can be varied at the optimum condition represented in table 10: The permissible range for change in c_1 to maintain this optimum condition for yield was computed as $0 < c_1$. Similarly the range for c_2 was calculated as $0 < c_2$.

Model validation: The predictions made by this LP model were tested with the field observations. Here also bootstrap samples were generated from the field data to confirm the results further and the paired t test between the observed and predicted values were done. The predictions were made using Eq. 26 in which VL_{opt} and VFT_{opt} were the values of VL and VFT at the optimum condition with values 177 and 462 respectively. VL and VFT were the observed values in the field. TYLD at optimum condition was computed using Eq. 14. Results of the test are summarized in Table 11.

Predictions made by the LP model agree very well with the observed values at 95% confidence interval. This shows that the LP model works well for Bihar plateau location also and this can be used for predicting the yield of sweet potato in this location.

Table 11: Results of LP model validation for Bihar Problem for two seasons

No.	Item	value
1.	Number of bootstrap samples generated	100
2.	Number of bootstrap samples tested	100
3.	Number of samples where mean values are not significantly different	100

CONCLUSION

A model for predicting maximum yield of sweet potato using LP method was developed and was used for two locations, Jabalpur and Bihar plateau in India. Potential yield of sweet potato at Jabalpur and Bihar plateau were found to be 324.095g and 128.308 g per plant respectively. These can be improved up to 648.19 g and 256.611 g per plant respectively by making changes in the resources. While selecting new varieties for these locations for higher yield, the plant parameter values as obtained in the LP solution can be used.

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