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The Center of Skew Polynomial Ring over a Couple of Quaternions

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Abstract: Let, R be a ring and σ is an endomorphism for R. With multiplication rule $xa = \sigma(a)x$ for all $a \in R$, the set of polynomials $a_0+a_1x+, ..., +a_nx^n$ where $a_i \in R$, forms a ring. The ring is called skew polynomial ring. This ring is non-commutative due to the multiplication rule.

INTRODUCTION

Definitions and notations: In mathematics, the quaternions are a real scalar linear combination and three orthogonal imajinary units (denoted by i, j and k) with real coefficients that can be written as:

$$H = \{q_0+q_1i+q_2j+q_3k \mid q_0, q_1, q_2, q_3 \in R\}$$

Where:

$$i^2 = j^2 = k^2 = -1;\ ij = -ji = k;\ jk = -kj = i;$$
$$ki = -i;\ ki = -ik = j,\ ijk = -1$$

For more properties of quaternions, the reader is referred to Shomake and Quaternions. In this study, it is necessary to know the definition of skew polynomial ring and the center of ring. It can be found by Amir (2016, 2017, 2018) and Amir *et al*. (2017).

Definition 1.1; McConnel and Robson (1987): Let R be a ring, σ be an endomorphism for R and δ be a σ-derivation. The skew polynomial ring over R is a ring that consists of all polynomials over R with an indeterminate x denoted by:

$$R[x; \sigma, \delta] = \{f(x) = a_0+a_1x+, ..., +a_nX^n \mid a_i \in R$$

With multiplication rule, for all, $a \in R$, then:

$$xa = \sigma(a)+\delta(a)$$

For cases where $\delta = 0$, the notations $R[x; \sigma, \delta]$ can be written as $R[x; \sigma.]$ Moreover, the structure of skew polynomial ring $R[x; \sigma]$ is different with $R[x; \sigma, \delta]$.

Definition 1.2; McConnel and Robson (1987): Let, R be a ring, the center of R denoted by Z(R) is defined as:

$$Z(R) = \{r \in R \mid rx = xr, \forall x \in R\}$$

THE MAIN RESULTS

In this part, we give the forms of center of skew polynomial ring over a couple of quaternions. Let:

$$\begin{aligned}&\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)\in H\times H,\\ &\sigma: H\times H\rightarrow H\times H\end{aligned}$$

Where:

$$\begin{aligned}&\sigma\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)=\\ &\left(a_0-a_1i-a_2j+a_3k, b_0+b_2i+b_3j+b_1k\right)\end{aligned}$$

It is easy to show that σ is an endomorphism on H×H. So, we have H×H [x; σ] is a skew polynomial ring. Center of ring H×H [x; σ] is stated in the following theorem:

Theorem 2.1:

$$\begin{aligned}&Z\left(H\times H[x;\sigma]\right)=\{\sum\nolimits_{n=0}^{t}\left(a_{6n},b_{6n}\right)x^{6n}+\\ &\left(a_{6n+1}k,b_{6n+1}\left(1+i+j+k\right)\right)x^{6n+1}+\\ &\left(a_{6n+2}k,b_{6n+2}\left(1-i-j-k\right)\right)x^{6n+2}+\\ &\left(a_{6n+3}k,b6_{n+3}\right)x^{6n+3}+\\ &\left(a_{6n+4},b_{6n+4}\left(1+i+j+k\right)\right)x^{6n+4}+\\ &\left(a_{6n+5}k,b_{6n+5}\left(1-i-j-k\right)\right)x^{6n+5}\mid\\ &a_i,b_i\in R\}\end{aligned}$$

Proof: We show that:

$$Z\left(H\times H[x;\sigma]\right)=P$$

Where:

$$\begin{aligned}&P=\{\sum\nolimits_{n=0}^{t}\left(a_{6n},b_{6n}\right)x^{6n}+\left(a_{6n+1}k,b_{6n+1}\left(1+i+j+k\right)\right)x^{6n+1}+\\ &\left(a_{6n+2},b_{6n+2}\left(1-i-j-k\right)\right)x^{6n+2}+\left(a_{6n+3}k,b_{6n+3}\right)x^{6n+3}+\\ &\left(a_{6n+4},b_{6n+4}\left(1+i+j+k\right)\right)x^{6n+4}+\\ &\left(a_{6n+5}k,b_{6n+5}\left(1-i-j-k\right)\right)x^{6n+5}\mid a_i,b_i\in R\}\end{aligned}$$

The proof will be divided into two parts, i.e., P⊆Z (H×H [x; σ]) and Z(H×H [x; σ])⊆P. First, we show that P⊆Z (H×H [x; σ]):

$$\begin{aligned}&\text{Let } p\left(x\right)\in P\\ &\text{Let } p\left(x\right)=\sum\nolimits_{n=0}^{t}\left(a_{6n}, b_{6n}\right)x^{6n}+\left(a_{6n+1}k, b_{6n+1}\left(1+i+j+k\right)\right)x^{6n+1}+\\ &\left(a_{6n+2}, b_{6n+2}\left(1-i-j-k\right)\right)x^{6n+2}+\left(a_{6n+3}k, b_{6n+3}\right)x^{6n+3}+\\ &\left(a_{6n+4}, b_{6n+4}\left(1+i+j+k\right)\right)x^{6n+4}+\left(a_{6n+5}k, b_{6n+5}\left(1-i-j-k\right)\right)x^{6n+5}\end{aligned}$$

We will show that p(x) q(x) = q(x) p(x), ∀q(x)∈H×H [x; σ]. Without loss of its generality, the proof will be done for two cases q(x). They are q(x) = (a, b)∈H×H and q(x) = (1, 1)x.

Case q(x) = (a, b). Let:

$$a=a_0+a_1i+a_2j+a_3k \text{ and } b=b_0+b_1i+b_2j+b_3k$$

Then:

$$\begin{aligned}&p\left(x\right)q\left(x\right)=\sum\nolimits_{n=0}^{t}\left(a_{6n}, b_{6n}\right)x^{6n}+\left(a_{6n+1}k, b_{6n+1}\left(1+i+j+k\right)\right)x^{6n+1}+\\ &\left(a_{6n+2}, b_{6n+2}\left(1-i-j-k\right)\right)x^{6n+2}+\\ &\left(a_{6n+3}k, b_{6n+3}\right)x^{6n+3}+\\ &\left(a_{6n+4}, b_{6n+4}\left(1+i+j+k\right)\right)x^{6n+4}+\\ &\left(a_{6n+5}k, b_{6n+5}\left(1-i-j-k\right)\right)x^{6n+5}\\ &\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)=\\ &\sum\nolimits_{n=0}^{t}\Big[\left(a_{6n}, b_{6n}\right)\\ &\sigma^{6n}\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n}+\\ &\left(a_{6n+1}k, b_{6n+1}\left(1+i+j+k\right)\right)\\ &\sigma^{6n+1}\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n+1}+\\ &\left(a_{6n+2}, b_{6n+2}\left(1-i-j-k\right)\right)\\ &\sigma^{6n+2}\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n+2}+\\ &\left(a_{6n+3}k, b_{6n+3}\right)\sigma^{6n+3}\\ &\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n+3}+\\ &\left(a_{6n+4}, b_{6n+4}\left(1+i+j+k\right)\right)\\ &\sigma^{6n+4}\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n+4}+\\ &\left(a_{6n+5}k, b_{6n+5}\left(1-i-j-k\right)\right)\Big)\\ &\sigma^{6n+5}\left(a_0+a_1i+a_2j+a_3k, b_0+b_1i+b_2j+b_3k\right)x^{6n+5}\Big]\end{aligned}$$

So:

$$\begin{aligned}&p\left(x\right)q\left(x\right)=\sum\nolimits_{n=0}^{t}[(a_{6n},b_{6n})\\ &\left(a_0+a_1i+a_2j+a_3k,b_0+b_1i+b_2j+b_3k\right)x^{6n}+\\ &(a_{6n+1},b_{6n+1})\,(a_0k-a_1j+a_2i-a_3,\left(b_0-b_1-b_2-b_3\right)+\\ &\left(b_0+b_1+b_2-b_3\right)i+\left(b_0-b_1+b_2+b_3\right)j+\\ &\left(b_0+b_1-b_2+b_3\right))x^{6n+1}+\left(a_{6n+2},b_{6n+2}\right)\\ &(a_0+a_1i+a_2j+a_3k,\left(b_0+b_1+b_2+b_3\right)+\\ &\left(-b_0+b_1-b_2+b_3\right)i+\left(-b_0+b_1+b_2-b_3\right)\\ &j+\left(-b_0-b_1+b_2+b_3\right))x^{6n+2}+\left(a_{6n+3},b_{6n+3}\right)\\ &\left(a_0k-a_1j+a_2i-a_3,b_0+b_2i+b_3j+b_1k\right)x^{6n+3}+\\ &\left(a_{6n+4},b_{6n+4}\right)(a_0+a_1i+a_2j+a_3k,\left(b_0-b_1-b_2-b_3\right)+\\ &\left(b_0+b_1+b_2-b_3\right)i+\left(b_0-b_1+b_2+b_3\right)j+\\ &\left(b_0+b_1-b_2+b_3\right))x^{6n+4}+(a_{6n+5},b_{6n+5})\\ &(a_0k-a_1j+a_2i-a_3,\left(b_0+b_1+b_2+b_3\right)+\\ &\left(-b_0+b_1-b_2+b_3\right)i+\left(-b_0+b_1+b_2-b_3\right)j+\\ &\left(-b_0-b_1+b_2+b_3\right))x^{6n+5}\end{aligned} \tag{1}$$

On the other hand:

$$\begin{aligned}
&q(x)p(x) = (a_0 + a_1 i + a_2 j + a_3 k, b_0 + b_1 i + b_2 j + b_3 k)\\
&[\textstyle\sum_{n=0}^{t}(a_{6n}, b_{6n})x^{6n} + (a_{6n+1}k, b_{6n+1}(1+i+j+k))x^{6n+1} +\\
&(a_{6n+2}, b_{6n+2}(1-i-j-k))x^{6n+2} + (a_{6n+3}k, b_{6n+3})x^{6n+3} +\\
&(a_{6n+4}, b_{6n+4}(1+i+j+k))x^{6n+4} +\\
&(a_{6n+5}k, b_{6n+5}(1-i-j-k))x^{6n+5}] = \textstyle\sum_{n=0}^{t}[(a_{6n}, b_{6n})\\
&(a_0 + a_1 i + a_2 j + a_3 k, b_0 + b_1 i + b_2 j + b_3 k)x^6 n+\\
&(a_{(6n+1)}, b_{(6n+1)})(a_0 k - a_1 j + a_2 i - a_3, (b_0 - b_1 - b_2 - b_3) +\\
&(b_0 + b_1 + b_2 - b_3)i + (b_0 - b_1 + b_2 + b_3)j +\\
&(b_0 + b_1 - b_2 + b_3))x^{6n+1} + (a_{6n+2}, b_{6n+2})\\
&(a_0 + a_1 i + a_2 j + a_3 k, (b_0 + b_1 + b_2 + b_3) +\\
&(-b_0 + b_1 - b_2 + b_3)i + (-b_0 + b_1 + b_2 - b_3)j +\\
&(-b_0 - b_1 + b_2 + b_3))x^{6n+2} + (a_{6n+3}, b_{6n+3})\\
&(a_0 k - a_1 j + a_2 i - a_3, b_0 + b_2 i + b_3 j + b_1 k)x^{6n+3} + (a_{6n+4}, b_{6n+4})\\
&(a_0 + a_1 i + a_2 j + a_3 k, (b_0 - b_1 - b_2 - b_3) + (b_0 + b_1 + b_2 - b_3)i +\\
&(b_0 - b_1 + b_2 + b_3)j + (b_0 + b_1 - b_2 + b_3))x^{6n+4} + (a_{6n+5}, b_{6n+5})\\
&(a_0 k - a_1 j + a_2 i - a_3, (b_0 + b_1 + b_2 + b_3) + (-b_0 + b_1 - b_2 + b_3)i +\\
&(-b_0 + b_1 + b_2 - b_3)j + (-b_0 - b_1 + b_2 + b_3))x^{6n+5}
\end{aligned} \tag{2}$$

From Eq. 1 and 2, we can see that p(x) q(x) = q(x) p(x). Case q(x) = (1, 1) x:

$$\begin{aligned}
&p(x)q(x) = [\textstyle\sum_{n=0}^{t}(a_{6n}, b_{6n})x^{6n} + \left(a_{6n+1}k, b_{6n+1}\begin{pmatrix}1+i+\\ j+k\end{pmatrix}\right)\\
&x^{6n+1} + (a_{6n+2}, b_{6n+2}(1\text{-}i\text{-}j\text{-}k))x^{6n+2} +\\
&(a_{6n+3}k, b_{6n+3})x^{6n+3} + (a_{6n+4}, b_{6n+4}(1+i+j+k))x^{6n+4} +\\
&(a_{6n+5}k, b_{6n+5}(1\text{-}i\text{-}j\text{-}k))x^{6n+5}](1,1)x =\\
&[\textstyle\sum_{n=0}^{t}(a_{6n}, b_{6n})x^{6n+1} + (a_{6n+1}k, b6_{n+1}(1+i+j+k))\\
&x^{6n+2} + a_{6n+2}, b_{6n+2}(1\text{-}i\text{-}j\text{-}k)x^{6n+3} + (a_{6n+3}k, b_{6n+3})x_{6n+4} +\\
&(a_{6n+4}, b_{6n+4}(1+i+j+k))x^{6n+6}]
\end{aligned} \tag{3}$$

On the other hand:

$$\begin{aligned}
&q(x)p(x) = (1,1)x[\sum_{n=0}^{t}(a_{6n}, b_{6n})x^{6n} +\\
&(a_{6n+1}k, b_{6n+1}(1+i+j+k))x^{6n+1} +\\
&(a_{6n+2}, b_{6n+2}(1\text{-}i\text{-}j\text{-}k))x^{6n+1} + (a_{6n+3}k, b_{6n+3})x^{6n+3} +\\
&(a_{6n+4}, b_{6n+4}(1+i+j+k))x^{6n+4} +\\
&(a_{6n+5}k, b_{6n+5}(1\text{-}i\text{-}j\text{-}k))x^{6n+5}] =\\
&\textstyle\sum_{n=0}^{t}(a_{6n}, b_{6n})x^{6n+1} + (a_{6n+1}k, b_{6n+1}(1+i+j+k))x^{6n+2} +\\
&(a_{6n+2}, b_{6n+2}(1\text{-}i\text{-}j\text{-}k))x^{6n+3} + (a_{6n+3}k, b_{6n+3})x^{6n+4} +\\
&(a_{6n+4}, b_{6n+4}(1+i+j+k))x^{6n+5} + (a_{6n+5}k), b_{6n+5}(1\text{-}i\text{-}j\text{-}k)x^{6n+6}
\end{aligned} \tag{4}$$

From Eq. 3 and 4, we can see that p(x) q (x) = q(x) p(x.) From the two cases q(x) above it proves that p(x) q(x) = q(x) p(x), then such that $p \subseteq Z$ (H×H[x; σ]). Second, we show that Z(H×H [x; σ])⊆P. Let, P(x)∈Z (H×H [x; σ]), so, p(x) holds p(x) q(x) = q(x) p(x), ∀q(x) ∈H×H [x; σ] and p(x) $= \sum_{n=0}^{t}(a_n, b_n)x^n$. By choosing q(x) = (j, i) then:

$$\begin{aligned}
&p(x)q(x) = \left[\textstyle\sum_{n=0}^{t}(a_n, b_n)x^n\right](j,i) =\\
&\begin{bmatrix}(a_0,b_0)x^0 + (a_1,b_1)x^1 + (a_2,b_2)x^2 + (a_3,b_3)x^3 +\\ (a_4,b_4)x^4 + (a_5,b_5)x^5 + (a_6,b_6)x^6 + ... + (a_t,b_t)x^t\end{bmatrix}(j,i)\\
&= (a_0,b_0)(\sigma^0(j,i))x^0 + (a_1,b_1)(\sigma^1(j,i))x^1 +\\
&(a_2,b_2)(\sigma^2(j,i))x^2 + (a_3,b_3)(\sigma^3(j,i))x^3 +\\
&(a_4,b_4)(\sigma^4(j,i))x^4 + (a_5,b_5)(\sigma^5(j,i))x^5 +\\
&(a_6,b_6)(\sigma^6(j,i))x^6 + ... + (a_t,b_t)(\sigma^t(j,i))x^t\\
&p(x)q(x) = (a_0,b_0)(j,i)x^0 + (a_1,b_1)(\text{-}j,k)x^1 +\\
&(a_2,b_2)(i,j)x^2 + (a_3,b_3)(\text{-}j,i)x^3 + (a_4,b_4)(j,k)x^4 +\\
&(a_5,b_5)(\text{-}j,j)x^5 + (a_6,b_6)(j,i)x^6 +, ...,\\
&(a_t,b_t)(\sigma^t(j,i))x^t\\
&p(x)q(x) = (a_0,b_0)(j,i)x^0 + (a_1,b_1)(\text{-}j,k)x^1 +\\
&(a_2,b_2)(i,j)x^2 + (a_3,b_3)(\text{-}j,i)x^3 + (a_4,b_4)(j,k)x^4 +\\
&(a_5,b_5)(\text{-}j,j)x^5 + (a_6,b_6)(j,i)x^6 +, ..., (a_t,b_t)(\sigma^t(j,i))x^t
\end{aligned} \tag{5}$$

Furthermore:

$$\begin{aligned}
&q(x)p(x) = (j,i)\textstyle\sum_{n=0}^{t}(P_n, Q_n)x^n] =\\
&(j,i)(a_0,b_0) + (j,i)(a_1 + b_1)x^1 + (j,i)(a_2,b_2)x^2 +\\
&(j,i)(a_3,b_3)x^3 + (j,i)(a_4,b_4)x^4 + (j,i)(a_5,b_5)x^5 +\\
&(j,i)(a_6,b_6)x^6 + ... + (j,i)(a_t,b_t)
\end{aligned} \tag{6}$$

Because p(x) q(x) = q(x) p(x) then (Eq. 5) = (Eq. 6), we get:

$$(a_0,b_0)(j,i) = (j,i)(a_0,b_0)$$

Let:

$$a_0 = a_0^0 + a_0^1 i + a_0^2 j + a_0^3 k$$

And:

$$b_0 = b_0^0 + b_0^1 i + b_0^2 j + b_0^3 k$$

Then:

$$(a_0 j, b_0 i) = (ja_0, ib_0)$$

So, we have:

$$\left(a_0^0+a_0^1i+a_0^2j+a_0^3k\right)j=\left(ja_0^0+a_0^1i+a_0^2j+a_0^3k\right)$$
$$a_0^0j+a_0^1k-a_0^2-a_0^3i=a_0^0j-a_0^1k-a_0^2+a_0^3i$$
$$a_0^1k=-a_0^1k\rightarrow a_0^1=0$$
$$-a_0^3i=a_0^3\rightarrow a_3^3=0$$
$$\therefore a_0=a_0^0+a_0^2j$$

$$\left(b_0^0+b_0^1i+b_0^2j+b_0^3k\right)i=i\left(b_0^0+b_0^1i+b_0^2j+b_0^3k\right)$$
$$b_0^0i-b_0^1-b_0^2k+b_0^3j=b_0^0i-b_0^1+b_0^2k-b_0^3j$$
$$-b_0^2k=b_0^2k\rightarrow b_0^2=0$$
$$b_0^3j=-Q_0^3j\rightarrow b_0^3=0$$
$$\therefore b_0=b_0^0+b_0^1i$$

$$\left(a_1,b_1\right)\left(-j,k\right)=\left(j,i\right)\left(a_1,b_1\right)$$

Let:

$$a_1=a_1^0+a_1^1i+a_1^2j+a_1^3k$$
$$b_1=b_1^0+b_1^1i+b_1^2j+b_1^3k$$

Then:

$$\left(-a_1j,b_1k\right)=\left(ja_1,ib_1\right)$$

So, from this equation, we get:

$$a_1=a_1^1i+a_1^3k$$

And:

$$b_1=b_1^0+b_1^1i+b_1^0j+b_1^1k$$

$$\left(a_2,b_2\right)(j,j)=(j,i)\left(a_2,b_2\right)$$

Let:

$$\left(a_2,b_2\right)(j,j)=(j,i)\left(a_2,b_2\right)$$
$$b_2=b_2^0+b_2^1i+b_2^2j+b_2^3k$$

Then:

$$\left(a_2j,b_2j\right)=\left(ja_2,ib_2\right)$$

So, from this equation we get:

$$a_2=a_2^1i+a_2^3k$$

And:

$$b_2=b_2^0+b_2^1i+b_2^1j-b_2^0k$$

$$\left(a_3,b_3\right)\left(-j,i\right)=(j,i)\left(a_3,b_3\right)$$

Let:

$$a_3=a_3^0+a_3^1i+a_3^2j+a_3^3k$$
$$b_3=b_3^0+b_3^1i+b_3^2j+b_3^3k$$

Then:

$$\left(-a_3j,b_3i\right)=\left(ja_3,ib_3\right)$$

So, from this equation we get:

$$a_3=a_3^1i+a_3^3k$$

And:

$$b_3=b_3^0+b_3^1i$$

$$\left(a_4,b_4\right)\left(j,k\right)=\left(j,i\right)\left(a_4,b_4\right)$$

Let:

$$a_4=a_4^0+a_4^1i+a_4^2j+a_4^3k$$
$$b_4=b_4^0+b_4^1i+b_4^2j+b_4^3k$$

Then:

$$\left(a_4j,b_4k\right)=\left(ja_4,ib_4\right)$$

So, from this equation we get:

$$a_4=a_4^0+a_4^2j$$

And:

$$b_4=b_4^0+b_4^1i+b_4^0j+b_4^1k$$
$$\left(a_5,b_5\right)\left(-j,j\right)=\left(j,i\right)\left(a_5,b_5\right)$$

Let:

$$a_5=a_5^0+a_5^1i+a_5^2j+a_5^3k$$
$$b_5=b_5^0+b_5^1i+b_5^2j+b_5^3k$$

Then:

$$\left(-a_5j,b_5j\right)=\left(ja_5,ib_5\right)$$

So, from this equation, we get:

$$a_5=a_5^1i+a_5^3k$$

And:

$$b_5 = b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k$$
$$(a_6, b_6)(j, i) = (j, i)(a_6, b_6)$$

Let:

$$a_6 = a_6^0 + a_6^1 i + a_6^2 j + a_6^3 k$$
$$b_6 = b_6^0 + b_6^1 i + b_6^2 j + b_6^3 k$$

Then:

$$(a_6 j, b_6 i) = (j a_6, i b_6)$$

So, from this equation we get:

$$a_6 = a_6^0 + a_6^2 j$$

And:

$$b_6 = b_6^0 + b_6^1 i$$

Based on the results above, it can be concluded that: For coefficients x^{6n} can be written as:

$$(a_{6n}, b_{6n}) x^{6n} = (a_0^0 + a_0^2 j, b_0^0 + b_0^1 i) x^{6n}$$

For coefficients x^{6n+1} can be written as:

$$(a_{6n+1}, b_{6n+1}) x^{6n+1} = (a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1 k) x^{6n+1}$$

For coefficients x^{6n+2} can be written as:

$$(a_{6n+2}, b_{6n+2}) x^{6n+2} = (a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k) x^{6n+2}$$

For coefficients x^{6n+3} can be written as:

$$(a_{6n+3}, b_{6n+3}) x^{6n+3} = (a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i) x^{6n+3}$$

For coefficients x^{6n+4} can be written as:

$$(a_{6n+4}, b_{6n+4}) x^{6n+4} = (a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j - b_4^1 k) x^{6n+4}$$

For coefficients x^{6n+5} can be written as:

$$(a_{6n+5}, b_{6n+5}) x^{6n+5} = (a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k) x^{6n+5}$$

Hence, p(x) can be written by:

$$p(x) = \sum_{n=0}^{t} (a_{6n}, b_{6n}) x^n =$$
$$\sum_{n=0}^{t} [(a_{6n}, b_{6n}) x^{6n} + (a_{6n+1} + b_{6n+1}) x^{6n+1} +$$
$$(a_{6n+2}, b_{6n+2}) x^{6n+2} +$$
$$(a_{6n+3}, b_{6n+3}) x^{6n+3} + (a_{6n+4}, b_{6n+4}) x^{6n+4} +$$
$$(a_{6n+5}, b_{6n+3}) x^{6n+5}] =$$
$$\sum_{n=0}^{t} (a_0^0 + a_0^2 j, b_0^0 + b_0^1 i) x^{6n} +$$
$$(a_1^1 i + a_1^3 k, b_1^0 i + b_1^1 i + b_1^0 j + b_1^1 k) x^{6n+1} +$$
$$(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k) x^{6n+2} +$$
$$(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i) x^{6n+3} +$$
$$(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k) x^{6n+4} +$$
$$(a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k) x^{6n+5}$$

Choose q(x) = (i, j), then:

$$p(x) q(x) = \Big[\sum_{n=0}^{t} (a_0^0 + a_2^2 j, b_0^0 + b_0^1 i) x^{6n} +$$
$$(a_1^1 i + a_1^3 k, b_1^0 i + b_1^1 i + b_1^0 j + b_1^1 k) x^{6n+1} +$$
$$(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k) x^{6n+2} +$$
$$(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i) x^{6n+3} +$$
$$(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k) x^{6n+4} +$$
$$(a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k) (i, j)$$

So:

$$\begin{aligned} p(x) q(x) = {} & \sum_{n=0}^{t} (a_0^0 + a_2^2 j, b_0^0 + b_0^1 i)(i, j) x^{6n} + \\ & (a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1 k)(-i, i) x^{6n+1} + \\ & (a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k)(i, k) x^{6n+2} + \\ & (a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i)(-i, j) x^{6n+3} + \\ & (a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k)(i, i) x^{6n+4} + \\ & (a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k)(-i, k) x^{6n+5} \end{aligned} \quad (7)$$

Furthermore:

$$q(x) p(x) = (i, j) [\sum_{n=0}^{t} (a_0^0 + a_2^2 j, b_0^0 + b_0^1 i) x^{6n} +$$
$$(a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1 k) x^{6n+1} +$$
$$(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k) x^{6n+2} +$$
$$(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i) x^{6n+3} +$$
$$(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k) x^{6n+4} +$$
$$(a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k)] =$$
$$\sum_{n=0}^{t} (i, j)(a_0^0 + a_0^2 + b_0^0 + b_0^1 i) x^{6n} +$$

$$\begin{aligned}
&(i, j)\left(a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1 k\right) x^{6n+1} + \\
&(i, j)\left(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k\right) x^{6n+2} + \\
&(i, j)\left(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i\right) x^{6n+3} \\
&(i, j)\left(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k\right) x^{6n+4} + \\
&(i, j)\left(a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k\right) x^{6n+5}
\end{aligned} \tag{8}$$

Because p(x) q(x) = (x) p(x) then (Eq. 7) = pers (Eq. 8), we get:

$$\left(a_0^0 + a_0^2 j, b_0^0 + b_0^1 i\right)(i, j) = (i, j)\left(a_0^0 + a_0^2 j, b_0^0 + b_0^1 i\right)$$
$$\left(a_0^0 i - a_0^2 k, b_0^0 j + b_0^1 k\right) = \left(a_0^0 i + a_0^2 k, b_0^0\ j - b_0^1 k\right)$$

Then:

$$-a_0^2 k = a_0^2 k \rightarrow a_0^2 = 0$$
$$b_0^1 k = -b_1^3 k \rightarrow b_0^1 = 0$$

$$\left(a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1\right)(-i, i) =$$
$$(i, j)\left(a_1^1 i + a_1^3 k, b_1^0 + b_1^1 i + b_1^0 j + b_1^1 k\right)$$
$$\left(a_1^1 - a_1^3 j, b_1^0 i - b_1^1 - b_1^0 k + b_1^1 j\right) = \left(-a_1^1 - a_1^3 j, b_1^0 j - b_1^1 k - b_1^0 + b_1^1 i\right)$$

Then:

$$a_1^1 = -a_1^1 \quad \rightarrow a_1^1 = 0$$
$$b_1^0 i = b_1^1 i \quad \rightarrow b_1^0 = b_1^1$$

$$\left(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k\right)(i, k) =$$
$$(i, j)\left(a_2^1 i + a_2^3 k, b_2^0 + b_2^1 i + b_2^1 j - b_2^0 k\right)$$
$$\left(-a_2^1 + a_2^3 j, b_2^0 k - b_2^1 j + b_2^1 i + b_2^0\right) = \left(-a_2^1 - a_2^3 j, b_2^0 j - b_2^1 k - b_2^1 - b_2^0 i\right)$$

Then:

$$a_2^3 j = -a_2^3 j \quad \rightarrow a_2^3 = 0$$
$$b_2^0 k = -b_2^1 k \ \rightarrow b_2^0 = -b_2^1$$

$$\left(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i\right)(-i, j) = (i, j)\left(a_3^1 i + a_3^3 k, b_3^0 + b_3^1 i\right)$$
$$\left(a_3^1 - a_3^3 j, b_3^0 j + b_3^1 k\right) = \left(-a_3^1 - a_3^3 j, b_3^0 j - b_3^1 k\right)$$

Then:

$$-a_3^1 = -a_3^1 \rightarrow a_3^1 = 0$$
$$b_3^1 k = -b_3^1 k \rightarrow b_3^1 = 0$$

$$\left(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k\right)(i, i) =$$
$$(i, j)\left(a_4^0 + a_4^2 j, b_4^0 + b_4^1 i + b_4^0 j + b_4^1 k\right)$$
$$\left(a_4^0 i - a_4^2 k, b_4^0 i - b_4^1 - b_4^0 k + b_4^1 j\right) =$$
$$\left(a_4^0 i + a_4^2 k, b_4^0 j - b_4^1 k - b_4^0 + b_4^1 k\right)$$

Then:

$$-a_4^2 k = a_4^2 k \rightarrow a_4^2 = 0$$
$$b_4^0 i = b_4^1 i \rightarrow b_4^0 = b_4^1$$

$$\left(a_5^1 i - a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k\right)(-i, k) =$$
$$(i, j)\left(a_5^1 i + a_5^3 k, b_5^0 + b_5^1 i + b_5^1 j - b_5^0 k\right)$$
$$\left(a_5^1 - a_5^3 j, b_5^0 k - b_5^1 j + b_5^1 i + b_5^0 k\right) = \left(-a_5^1 - a_5^3 j, b_5^0 j - b_5^1 - b_5^0 i\right)$$

Then:

$$a_5^1 = -a_5^1 \rightarrow a_5^1 = 0$$
$$b_5^0 k = -b_5^1 k \rightarrow b_5^0 = -b_5^1$$

So that:

$$\begin{aligned}
p(x) = \sum_{n=0}^{t} &\left(a_{6n}, b_{6n}\right) x^{6n} + \left(a_{6n+1}\ k, b_{6n+1}\left(1+i+j+k\right)\right) x^{6n+1} + \\
&\left(a_{6n+2}, b_{6n+2}\left(1-i-j-k\right)\right) x^{6n+2} + \\
&\left(a_{6n+3} k, b_{6n+3}\right) x^{6n+3} + \\
&\left(a_{6n+4}, b_{6n+4}\left(1+i+j+k\right)\right) x^{6n+4} + \\
&\left(a_{6n+5}\ k, b_{6n+5}\left(1-i-j-k\right)\right) x^{6n+5}
\end{aligned}$$

Where:

$$a_i, b_i \in R$$

Therefore, p(x)∈P, it show that Z(H×H [x; σ])⊆P because P⊆Z (H×H [x; σ]) and Z(H×H [x; σ])⊆P then it proves that Z(H×H [x; σ]) = P.

CONCLUSION

In this study, will describe the center forms of skew polynomial ring over a couple of quaternions.

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