Asian Journal of Information Technology 4 (12): 1141-1145 , 2005

An Efficient Algorithm for the Global Optimization Using Order Transformation

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Abstract: In this study present an efficient algorithm of global search to optimizing in *n* dimension space. This method is suitable for functions that have many extremes. Present algorithm can determine a narrow space around the global optimum in very restricted time based on a stochastic tests and an adaptive partition of the search space. The present algorithm reduces the order of the optimization problem in hand to order two before it tries to solve it, then the problem will be transformed again into its original order. Thus, the new algorithm has the ability to avoid the local optima and reduces the number of evaluations and improves the speed of the algorithm converge. The proposed algorithm tested to locate the global optimum of different test problems and finally used to identify a physical stress/ strain problem. It was found that the algorithm was effective to locate the global optimum even though the objective function has a large number of optima

Key words: Space partition, best individual, circular design, region potential, order reduction

INTRODUCTION

Global optimization, in general, is the task of finding the lowest extreme point in the rugged landscapes in high dimensions. Many heuristic methods have been designed for this purpose. Among those, the Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithm (GA) and Evolutionary Programming (EP) seem most popular and have been applied to many engineering and scientific problems[1,2].

These techniques are roughly classified into two categories; one which tries to improve single solutions and another which uses the metaphor of evolution theory and considers multiple solutions in parallel to try to improve the population of solutions rather than just a single solution. A common principle of these algorithms, however, is to improve the solution or tries to breed the best member in the population. In fact it is well known that the success of the evolutionary algorithms such as GA and EP depends strongly on their strategy that preserves the best member called the elite[1]. Our algorithm strategy that describes the Circular design[3] is to use the best members in a population candidate solution to select a sub region of the search space that is the most susceptible of containing the global optimum.

When choosing a global optimization technique we should take into account the cost of evaluation. Many problems in the engineering and science disciplines have a high cost of evaluation. Thus, one may be willing to utilize some computation time in order to minimize the number of function evaluations.

In this study we describe the Circular design technique that is based on the use of the best individuals in a given population to detect a narrow space around the global optimum this technique[3]. We apply this technique in two main steps. First, we transform the individual's representation in the optimization problem of order n into virtual individuals that are represented in a two-dimensional space and secondly, we use the circular design of order two to construct the sub-regions and their corresponding populations. Finally, we end up with a narrow sub region in two-dimensional space that will be transformed again to the original problem order. The obtained sub region represents either a start point for the original problem solving or the solution itself.

In global optimization, there exist two methods of testing global optimization techniques: Convergence and Performance on a series of functions. When testing for convergence, optimization techniques are tested on the ability to converge on the optimal solution[4]. The new technique will be test using two benchmark problems and an identification problem.

THE CIRCULAR DESIGN TECHNIQUE

The Circular Design (CD) is a new technique of a global search in rugged landscapes. It is very effective in function with many extremes [3]. This technique follows an adaptive partition of the search space and select at each iteration time t a sub-region to be partitioned again. Each sub region will be represented by their population. The CD generate a complete population based on only

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two candidates A and B (parents); selected from the best individuals of a sub-region; around a central point expected to be the search global optimum. The new population has a property that the point distribution density decreases when we go far from the expected global optimum, in order to give more opportunities to those individuals that are close to the centre (expected global optimum) to be selected for the next population generation[3].

For a two-dimensional problem, we define two parent vectors $X_p^1 = \left[x_p^1, y_p^1\right]^T$ and. $X_p^2 = \left[x_p^2 \quad y_p^2\right]^T$

The central point is then, given by the vector$X_{t,ip}$:

$$X_{t,ip} = \frac{1}{2}\left|X_p^1 + X_p^2\right| \tag{1}$$

Therefore, the new population individuals will be calculated using the following equation system (2):

$$x_{t,ij} = x_{t,ip} + \frac{R}{q}.ud_{1,j}.\cos\left[\frac{2.\pi}{q}.ud_{2,j} + \operatorname{atan}\left(l_{t,i}^1 / l_{t,i}^2\right)\right] \tag{2.a}$$

$$y_{t,ij} = y_{t,ip} + \frac{R}{q}.ud_{1,j}.\sin\left[\frac{2.\pi}{q}.ud_{2,j} + \operatorname{atan}\left(l_{t,i}^1 / l_{t,i}^2\right)\right] \tag{2.b}$$

Where $ud = \left[ud_{ij}\right]_{q*n}$ is a vector array generated by the Uniform Design. The selection of the sub-region most susceptible to contain the global optimum is based on a decision factor evaluated from the potential of the considered sub-region represented by its population. This potential measure the member ship values of $f(x_{ij})$, with $x_{ij} \in D_i(t)$ and $D_i(t)$ represent the population of the i^{th} sub-region[3,5]..

A member ship value of x_{ij} is measured by a member ship function $\mu_{t,ij}$ that maps sample points x_{ij} in i^{th} sub-region to the unit interval[1,6].

The member ship values can be calculated using Eq. 3.

$$\mu_{t,ij} = \left(f(x_{ij}) - f(x^*)\right)/R(t) \tag{3}$$

Where $f(x^*)$ is the minimum functional values obtained and *R(t)* is the range of all functional values gathered up to and including the iteration *t*. the member ship function measures the location of $f(x_{ij})$on the range scale.

The member ship function μ can be calculated also using a modified Gaussian function.

This information can then be transformed into a potential value and we can define the potential of a sub-region by the relation (4)[5,7].

$$r_{t,i} = \frac{1}{q}\sum_{j=1}^{q}\mu_{t,ij}.\exp\left(1-\mu_{t,ij}\right) \tag{4}$$

q represents the size of the individuals set $D_i(t)$. The potential of a sub-region takes smaller values when the actual region contains the global optimum within its boundaries.

The Circular design forces the sub-region whose central point located at the nearest position to the global optimum to have the smallest decision factor value by ensuring a well point distribution around this optimum. Following this procedure, we ensure that the probability of missing the global optimum when the iteration *t* increases goes to zero[3].

$$\lim_{t\to\infty}(1 - p(x^*)) = 0 \tag{5}$$

ORDER TRANSFORMATION

Order transformation is a new present way to reduce the optimization effort, where optimizing in lower dimensions is much easier. The main idea is to transform the problem of order *n* into a problem of order 2, by transforming the individuals (vectors) representation from order *n* to order 2. Then, The virtual problem will be solved using the Circular design that show large capabilities in solving two dimensional problems and gives best results with minimum evaluations and time consumption, compared to other techniques[3].

Order transformation realization is done through different steps. It is based on the binary representation of the individuals or the construction of the chromosomes. Like in genetic algorithms, chromosomes construction started by a normalized representation of the genomes that construct the individual using different formula like that given by Eq 6.

$$y_i = \frac{x_i - x_{imin}}{x_{imax} - x_{imin}}.y_{max} \tag{6}$$

where: y_{max} is the maximum value of the normalized landscape and it is strictly depends on the length of the genome in the chromosome.

$X_{i\,min}$ $x_{i\,max}$ are the minimum and maximum values of the vector element variation respectively.

The aim from the use of the normalization step is to produce homogenous genomes before the final construction of the chromosomes. At this level, we define the Transformation site that is needed to finish the full description of the order transformation as followed.

Definition: The transformation site is a point selected randomly from the chromosome and divides it into two parts. Each part will represent a new genome for a virtual

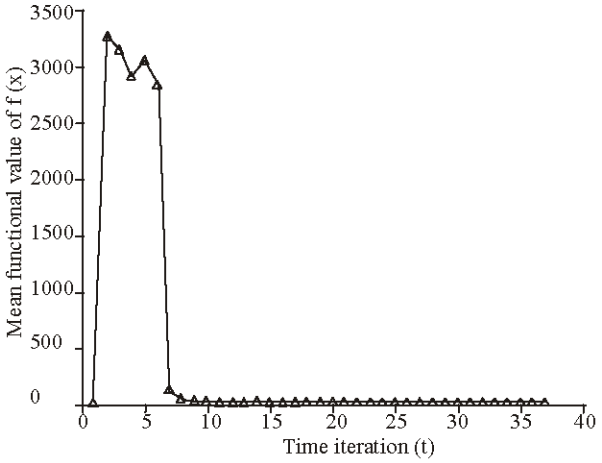

Fig. 1. The mean functional values of the Extended *Kearfott* optimization function

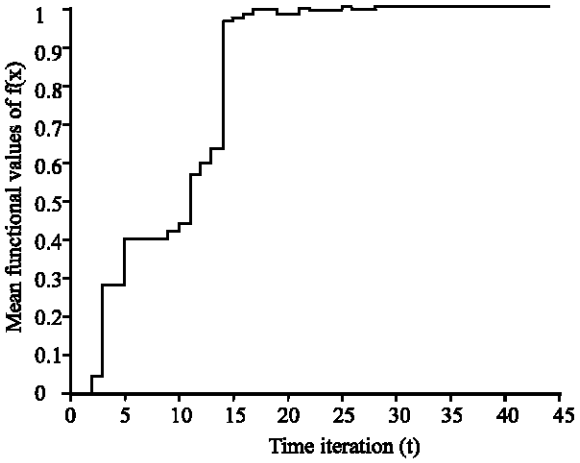

Fig. 2: the adaptive functional values of the Extended *Kearfott* optimization problem

representation of the individuals. The selection of the transformation site can be made randomly or can be fixed depending on the problem in hand.

Using order transformation, we ensure an order transformation of individual's representation in optimization problems of order n to a virtual representation of order two. This operation helps the Circular design to construct the sub-regions and calculate the potential of each sub-region and the decision factor of each one to select the sub-region for the next repartition. At each iteration time *t* we need to calculate the real parent vectors i.e. in real dimension to facilitate the calculation of the potential of each su-region.

NUMERICAL APPLICATIONS

A proof of convergence is theoretically satisfying and cannot be proven, in general, for all the classes of functions. To test the convergence of our technique we choose two benchmark problems and an identification model with non homogenous landscapes.

The extended *kearfott* function: This function has the following general form:

$$f(x_i) = (x_1^2 - x_2)^2 + (x_2^2 - x_3)^2 + (x_3^2 - x_4)^2 + (x_4^2 - x_1)^2$$

I=1..4.

This function has a large nubmer of optima and two global optimums at

$x_i^* = 1$, for *i*=1..4, with *f*(■) = 0.

and

$x_i^* = 0$, for *i*=1..4, with *f*(■) = 0.

We apply the technique of order transformation and we obtain the global optimum situated at x^*= [-0.0098 0.0098 -0.0098 0.0098]. Figures 1 and 2 show the mean functional values and the adaptive functional values respectively of the Extended *Kearfott* optimization problem. the algorithm speed convergence can be guessed from these figures.

The Helical Valley Function: This function has the following general form:

$$f(x_i) = 100.(x_3 + 10.\theta(x_1, x_2))^2 + 100.\left(\sqrt{x_1^2 + x_3^2} - 1\right)^2 + x_3^2$$

$$\theta = \begin{cases} \arctan(x_2 / x_1)/(2.\pi) \\ \arctan(x_2 / x_1)/(2.\pi) + 0.5 \end{cases}$$

This function has one global optimum at the point $(x_1,x_2,x_3) = (1,0,0)$, With $f(x^*) = 0$.

The new technique found the global optimum located at the point given by:
x^*= [1.0068 0.0098 0.0098].

Stress / strain identification model: We employ the new technique to determine the parameters of a real physical model problem. The general form of this model is given by Eq (9) and (10). To identify this model, we used the minimization of the squared error function between the measured values and the model output that is given by:

$$\Phi = \sum_{i=1}^{N} (\sigma_i - \sigma_{w_i})^2 \quad (7)$$

The output of this system or the strain is given by Eq. 8.

$$\sigma = f_\alpha \sigma_\alpha + f_\beta \sigma_\beta \quad (8)$$

where:

$$\sigma_{\alpha_i} = k_\alpha \left(1 - e^{-\varepsilon / n_\alpha}\right) \quad (9)$$

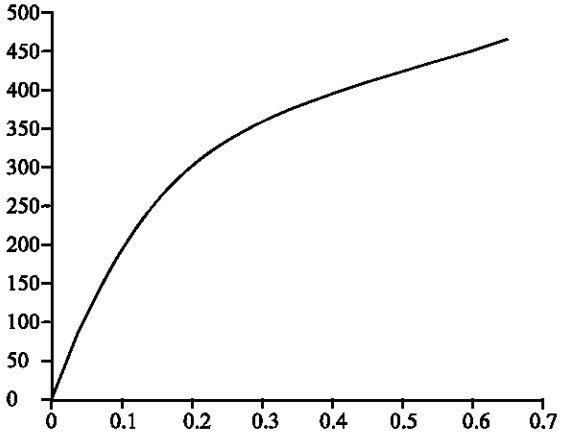

Fig. 3: Measured output of the real system

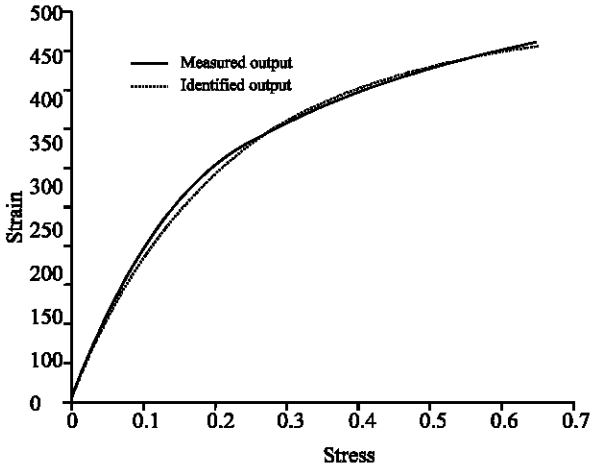

Fig. 5: Measured output and model output

$$\sigma_{\beta_i} = k_\beta \left(1 - e^{-e/n_\beta}\right) \tag{10}$$

and:

f_α and $f_{\beta\_}$ are constants. e is the input or the stress of the metal.

To determine the model parameters, (k_α, k_β, n_α and n_β), we minimize the squared error function described above. Eq. 8,9 and 10. describe the interrelation between the stress and the strain of a metal. The measured values of this interrelation are shown in Fig. 3.

The identification problem will be over the search space indicated in Table 1.

Table 1: Search space for the identification problem

k_α	k_β	n_α	n_β
[0 1000]	[1500 2500]	[0 1]	[0 2]

Table 2: Stress / strain identified parameters

n_α	n_β	k_α	k_β
0.1877	1.9511	466.2757	$1.7346\ 10^3$

The obtained results after the use of the new technique are shown in Fig. 4, with $f_\alpha = 0.9$ and $f_{\beta\_} = \_0.1$. The obtained identified parameters are given in table 2. We can see clearly that the model output matches approximately the measured system output which verifies the efficiency of the present technique; we should notice that the identified parameters are of different scales.

CONCLUSIONS

In this study we presented a new technique that is based on the order transformation and the circular design to construct a final efficient algorithm for the global optimization of a non convex function.

The use of the order transformation shows it efficiency in reducing the optimization efforts by reducing the order of the individual's representation.

The given results show how this technique proceeds toward the global optimum in a minimum run time with best converge. The proof of convergence is very clear in the identification problem, which represent a hard test problem because the identified parameters are of different orders i.e. some of them are of order of 10^3 and others of order of 10^{-3}.

The obtained results shows that the present technique is very efficient and can employed with different class of optimization problems.

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