

## Texture Image Segmentation Based on Quotient Space Granularity Synthesis

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**Abstract:** Theory of quotient space has been a potential direction of artificial intelligence in recent years. In this paper the granularity synthesis method of quotient space is applied to analyze and processing of texture images and a new image segmentation algorithm of image segmentation is given. Firstly texture features of coarseness and directionality are extracted to form the different granularity spaces respectively. Then the quotient space synthesis method is adopted to segment the texture image. Experimental results demonstrate that the algorithm of granularity synthesis is useful and valid. Furthermore the paper analyzes the relations between synthesis technology and image fusion, and generalizes some key questions when using quotient space granularity synthesis in image segmentation.

**Key words:** quotient space, synthesis technology, directionality, coarseness

### Introduction

Image segmentation is an important and basic problem of image analysis and understanding. The texture property plays an important role in the fields of pattern recognition, image retrieval and computer vision. In recent years, texture image segmentation problems have been studied comprehensively and a large amount of algorithms have been proposed according to the new mathematics theories (Zhang 2001, Chang 1999), eg., texture image segmentation algorithms based on wavelet transform (Mausumi Acharyya, 2000), fuzzy neural network (Guo-xiang Liu, Shunichiro Oe, 2000) and information fusion theory (Jasvinder Singh, Kristin J. Dana, 2004) respectively. Generally, texture image segmentation algorithms include two important steps, that is, texture feature extraction and pixels division based on feature vector. In practical however few algorithms available are effective. Furthermore most of them are related to complicated mathematics theories, which are especially difficult to deal with for multi-dimension vectors.

It is often quite easy to acquire different aspects and hierarchic partial image information and then form the whole conception through cooperating work of brain and vision. That ability is an important feature of human creativeness, of which working process may be described by the granulation synthesis technology of quotient space theory. Synthesis technology is the key part of quotient space theory and can be widely applied to lots of fields, eg., time programming, data mining, image processing etc. This paper focuses on the application of synthesis technology in image texture segmentation, which intends to demonstrate the practical value of quotient space theory in image processing, and meanwhile tries to give a new solution for dealing with complex problems.

**Quotient space basic theory:** Quotient space is a basic set notion. The basic and well known characteristic in problem solving is the ability to conceptualize the world at different granularities and alter from one abstraction level to another easily. The ability of problem solving is an important human intelligence. Quotient space theory has been described in reference (Mausumi Acharyya, 2000), which mainly includes: the relations among different granularities, quotient space formation, synthesis and deduction etc. The basic content is quotient space description about complex problems, granularity computation, and hierarchical and synthesis technology.

Suppose that triplet  $(X, f, T)$  describes a problem space or a simple space  $(X, f)$ . Where  $X$  denotes the universe,  $T$  is the structure of universe  $X$ , and  $f$  indicates the attributes (or features) of universe  $X$ .

Assume  $X$  represents the universe of the finest grain-size. When we view the same universe  $X$  from a coarser grain, we have a coarse-grained universe denoted by  $[X]$ . Then we have a new problem space  $([X], [f], [T])$ . The coarser universe  $[X]$  can be defined by an equivalence relation  $R$  on  $X$ . That is, an element in  $[X]$  is equivalent to a set of elements, an equivalence class, in  $X$ . So  $[X]$  consists of the whole equivalence classes obtained by  $R$ . From  $f$  and  $T$ , we can define the corresponding  $[f]$  and  $[T]$ . Then we have a new space  $([X], [f], [T])$  called a quotient space of  $(X, f, T)$ .

Assume  $R$  is the whole equivalent relations on  $X$ . Define a "coarse-fine", eg., " $<$ ", relation on  $R$  as follows: Assume  $R_1, R_2 \in R$  and  $R_2 < R_1 \Rightarrow x, y \in X$ , if  $xR_1y$ , then  $xR_2y$ , where  $xRy$  indicates  $x$  and  $y$  are  $R$  equivalent. It means that the universe  $X_1$  corresponding to  $R_1$  is finer than the universe  $X_2$  corresponding to  $R_2$ . Then  $R$  composes a complete semi-order lattice under relation " $<$ ".

Thus, the preceding problem representation forms a quotient space model of problem solving.

The model of synthesis technology of quotient space (Zhang Bo and Zhang Ling, 1992, Ling Zhang, Bo Zhang, 2003)

We can observe a texture image from different sides, suppose there are two sides  $(X_1, f_1, T_1 \in \mathcal{E})$  and  $(X_2, f_2, T_2)$ , which form two different granules space of the same problem space  $(X, f, T)$ . Assume that  $(X_1, f_1, T_1 \in \mathcal{E})$  and  $(X_2, f_2, T_2)$  are two coarse problem spaces of  $(X, f, T)$ , the synthesis space  $(X_3, f_3, T_3)$  of space  $(X_1, f_1, T_1 \in \mathcal{E})$  and space  $(X_2, f_2, T_2)$ . Space  $(X_3, f_3, T_3)$  is a new granular space of problem and must meet:

- 1  $X_1$  and  $X_2$  are the quotient spaces of  $X_3$
- 2  $T_1$  and  $T_2$  are quotient topologies of  $X_1$  and  $X_2$ ,  $T_1$  and  $T_2$  are projections of  $T_3$  on  $X_1$  and  $X_2$  respectively,
- 3  $f_1$  and  $f_2$  are projections of  $f_3$  on  $X_1$  and  $X_2$ , respectively.

The space  $(X_3, f_3, T_3)$  accords with some optimal criteria.

Universe  $X$  synthesis: Assume that  $R_1$  and  $R_2$  are equivalent relations on  $X$ .  $R_3$  is the least upper bound and the synthesis of  $R_1$  and  $R_2$ . The corresponding with  $R_3$  universe  $X_3$  is the synthesis space of  $X_1$  and  $X_2$ . So the universe synthesis is described as follows:

$$X_3 = \{ X_1 \cap X_2 \}$$

Topology construction synthesis: Assume the synthesis of  $T_1$  and  $T_2$  is  $T_3$ , which is the least upper bound of  $T_1$  and  $T_2$  on all topology structure universes  $X$  composing a complete semi-order lattice.

Definite:  $B = \{ w | w = u_i \cap v_j, u_i \in T_1, v_j \in T_2 \}$ , and  $B$  is topology base, so  $T_3$  is formed with topology of  $B$ .

Attribute function synthesis: Assume  $(X_1, f_1, T_1 \in \mathcal{E})$  and  $(X_2, f_2, T_2)$  are  $(X, f, T)$  quotient spaces,  $(X_3, f_3, T_3)$  is the synthesis space of  $(X_1, f_1, T_1 \in \mathcal{E})$  and  $(X_2, f_2, T_2)$ , and attribute  $f_3$  must meets some criteria as follows:

- 1  $P_i f_3 = f_i$ , here  $i = 1, 2$  and  $p_i: (X_3, f_3, T_3) \rightarrow (X_i, f_i, T_i)$
- 2 Assume  $D(f, f_1, f_2)$  is the optimal criteria as follows:

$$D(f_3, f_1, f_2) = \min D(f, f_1, f_2) \text{ or } D(f_3, f_1, f_2) = \max D(f, f_1, f_2)$$

Here the max or min is obtained from all the attribute functions  $f$  on  $X_3$  which meet 1.

The optimal criteria of synthesis relate to all fields, and it is difficult to have common criteria (Zhang Bo and Zhang Ling, 1992).

**Algorithm of quotient space of synthesis:** Firstly, according to the model of quotient space synthesis algorithm, it is necessary to extract the texture region features from human vision, and then classify the pixels into different sets to obtain image coarser segmentation as different image granularity spaces. Secondly, we use the method of granularity synthesis to complete the image segmentation.

The segmentation algorithm framework is shown in Fig. 1

The proposed algorithm is described as follows:

- 1 Extracting image features. Extract image texture feature of regional attribute respectively, eg., directionality and coarseness and contrast feature (see section 5.1 and 5.2).
- 2 Forming different granularity spaces. According to the feature vectors of directionality that have been extracted, we structure the equivalence relations  $R$  to classify the pixels of image, then select a clustering method, eg., k-means clustering (D.A. Clausi, 2002), and classify the image pixels and mark each pixel. Respectively, the set of  $X_1$  denotes the universes of image after pixels classified, and sum up the number of pixels numbers in each universe. In the end, the coarse granularity space is formed.
- 3 Similar to step2, structure the equivalence relation  $R$  according to the feature vectors of coarseness, then select the k-means clustering to classify the image elements and mark each pixel. Respectively, the set of  $X_2$  denotes the universes of image after the elements being classified, and sum up the number of elements in each universe.
- 4 Synthesis the different granules. Compare each region in  $X_1$  and  $X_2$  according to equality  $X_3 = \{ X_1 \cap X_2 \}$  respectively. If the pixels in different regions are found with the same position sign, the elements belong to the same set, and then remark these pixels. If the pixels in two regions have different position marks, elements are marked with new sets  $C_k$ , and then sum up the number of elements in sets of  $C_k$ .
- 5 According to the synthesis principle of 2.2, in step4, perhaps some regions of set  $C_k$  may not meet human vision. So we compare the number of elements of subset in new sets  $C_k$  with the minimum of elements of subset in  $X_1$  and  $X_2$ . If the number of elements in subset of  $C_k$  is not more than the minimum number of elements of subset in  $X_1$  and  $X_2$ , it is demanded to deal with the subset of  $C_k$ , and then remark it with  $C_k$ .
- 6 Classify the elements in set  $C_k$  again. According to the position of elements in region  $C_k$ , compute the feature average of neighboring regions with the elements in  $C_k$ , then compute the distance of the feature average of the elements. When the distance is minimum, classified each of the pixel in region  $C_k$  into regions. Repeatedly, compute each element in sub regions  $C_k$ , and then renew the set  $X_3$ , until the ultimate result is achieved.

**Experiment and results analysis:** We select an image consisting of five Brodatz textures as shown in Fig. 2(a) to do experiments. Firstly, we extract two important texture features of coarseness and directionality, and then form two different granularity spaces respectively.

Form granularity space based on texture features: Rosenfeld designed the algorithm of extracting coarseness feature,

in which it is important to design the size of neighboring region and compute the distance of average between different neighbor regions. Improved algorithm is more effective to differentiate texture and good rotation changelessness feature.

The algorithm of extracting feature coarseness (Sun Xing-hua, 2002, D. Chappard, 2003)

1 Compute average of multi-size neighborhood of each pixel, neighbor size  $(2k+1) \times (2k+1)$ , where  $k \geq 1$ .  
The neighbor average of pixel  $(x, y)$

$$A_k(x, y) = \frac{\sum_{i=x-k}^{x+k} \sum_{j=y-k}^{y+k} f(i, j)}{(2k+1)^2}$$

Where  $f(i, j)$  denotes the gray-level of pixel  $(i, j)$ .

2 Compute magnitude of the gradient  $|\Delta G|$

$$|\Delta G| = (|\Delta_H| + |\Delta_V|) / 2$$

Here,  $\Delta_H$  and  $\Delta_V$  denote the horizontal difference and vertical difference respectively.

If  $|\Delta G| \geq L$   $L$  denotes threshold, then the pixel is named valid pixel. According to the principle of the maximum average difference, compute the average difference among neighborhood of valid pixels, and then set neighborhood average of the invalid pixel as zero.

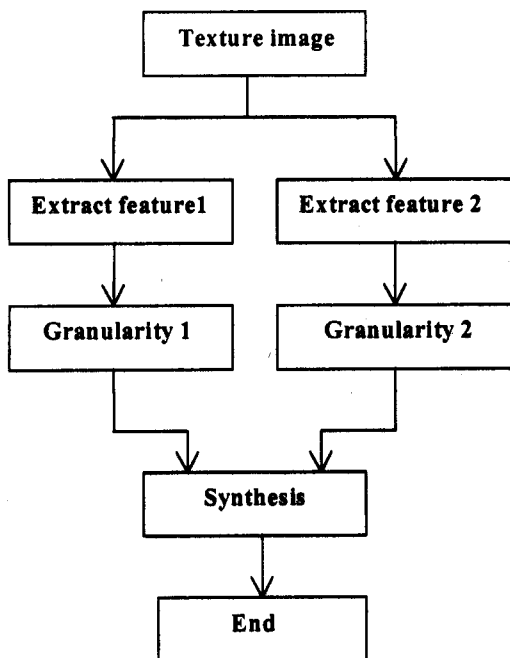


Fig. 1: Algorithm framework

Give the most appropriate size of neighborhood of each valid pixel as follows:

$$S_{best}(x, y) = 2k+1 \text{ Ensure } E_x = \max(E_1, E_2, E_L)$$

Here  $E$  denotes the potential energy of neighborhood average and  $L$  denotes the neighborhood size.

Compute the average of most optimal size as follows:

$$C = \frac{1}{V} \sum_i^m \sum_j^n S_{best}(i, j)$$

Here  $m$  and  $n$  denote the width and height of image respectively and  $V$  denotes the number of the valid pixels.

Use the coarseness vector to mark each pixel of image, select K-mean clustering, and then classify pixels into different classes. In the end, form the new granularity space  $\{X_1, f_1, T_1$

The result is shown in Fig. 2(b).

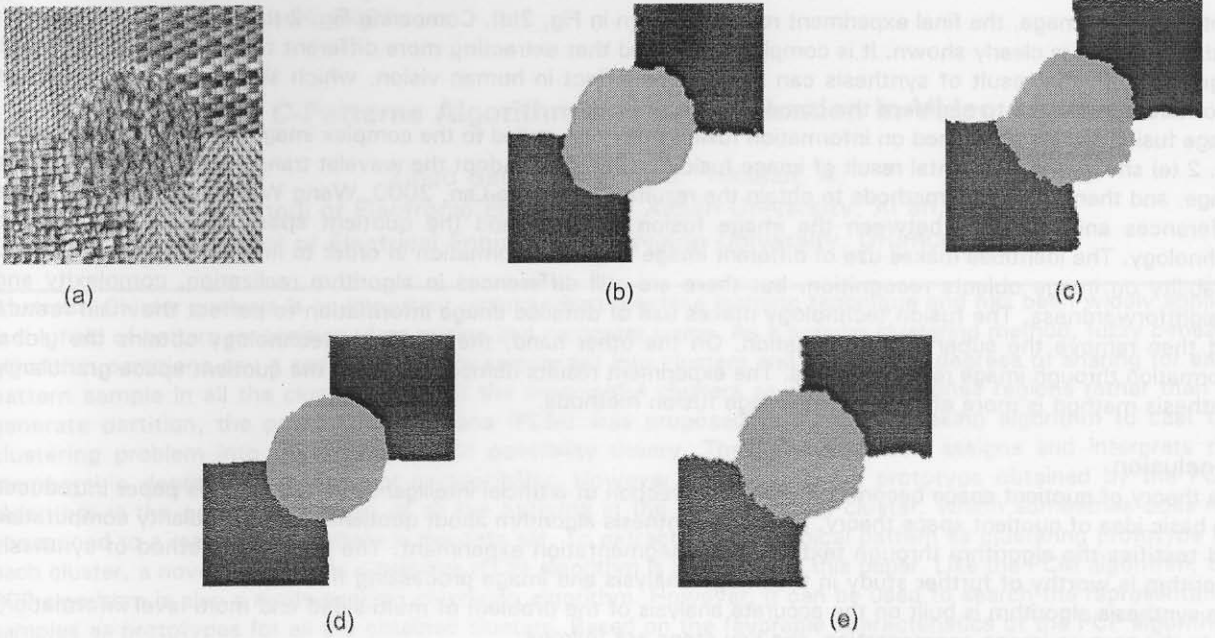


Fig. 2: Experiment results (a) five-texture image, (b) segmentation result based on coarseness feature, (c) segmentation result based on directionality feature, (d) synthesis result, (e) image fusion result

Granularity space formed based on the directionality feature: The directionality is a basic texture feature, which is denoted by the direction neighborhood.

Definition: Give a direct-line  $\Gamma$  at random passing the pixel point  $s$ , and then have it rotate taking the pixel point  $s$  as the center. When  $\Gamma$  rotates to random angle  $\phi$ , the direct-line will cover the set of pixels in  $s$ -neighborhood. Let  $\Gamma s = \{s_1, s_2, \dots, s_n\}$ , where the value of  $n$  is related to  $s$ -neighborhood size and figure. Design a difference function  $|\Psi(x_i, i = 1, 2, \dots, n)$ , and use it to compute the pixel gray scale  $g_{s_i}$  difference  $\Psi_\phi(g_{s_i})$  in set  $\Gamma s$ , and then compare the difference  $\Psi_\phi$ . When  $\phi = \theta$ , the value of  $\Psi_\theta(g_{s_i}, i = 1, 2, \dots, n)$  obtains minimum, then mark the  $s$ -neighborhood direction with  $\theta$ .

The directionality feature extraction (Wang Zhen, 2002)

To each pixel, in the different size neighborhoods, compute the pixel gray-scale difference  $\Psi(\Gamma_j s, j = 1, 2, 3, 4)$  in four basic directions  $j \in \{45^\circ, 90^\circ, 135^\circ, 0^\circ\}$ , where the difference function  $|\Psi(\Gamma_j s) = \Psi(g_{s_i}, i = 1, 2, \dots, 2 \times T + 1)$  is computed by the variance of the gray scale  $g_{s_i}$ .

Take the standard direction  $j$ , which has the minimum difference as the main direction of pixel neighborhood. Take each pixel as the center and the rectangle region with size  $(2T + 1)(2T + 1)$  as its neighborhood respectively. When  $T$  changes, sum up the main direction frequencies which are four standard directions, take the frequency as elements to structure a four-dimensions vector which is given as  $fs = [fs_1, fs_2, fs_3, fs_4]^T$ , and then the texture directionality of pixel  $s$  is denoted as  $fs$ .

Select K-mean clustering method and classify image pixels into subsets according to the vector  $fs$ . The texture image is roughly segmented, which is described by granularity space  $(X_2, f_2, T_2)$ . The segmentation result is shown in Fig. 2(c).

Segmentation results by synthesis: Fig. 2(a) consists of five Brodatz texture images. In the section 5.1 and section 5.2, the texture region feature information is respectively extracted from coarseness and directionality according to human vision property. Two different granulate are formed and shown in Fig. 2(b) and Fig. 2(c).

According to the quotient space theory, take the coarseness and direction feature information as basis to design equivalence relation  $R$ . When  $R_1$  and  $R_2$  are fixed, the universe  $X_1$  and  $X_2$  are determined exclusively. Fig. 2(b) and Fig. 2(c) show some regions with clear boundary after having been segmented, which can be explained with the set  $X_1 = \{a_1, a_2, a_3, a_n\}$  and  $X_2 = \{b_1, b_2, b_3, b_n\}$ , where  $a_i$  and  $b_j$  denote the set of pixels in relevant regions respectively. According to section 2, we know the quotient topology structure  $T_1$  and  $T_2$ , which corresponds to the universe  $X_1$  and  $X_2$  respectively, are determined exclusively, and meanwhile the attribute  $f_1$  and  $f_2$  are determined too.

With the texture feature extracted and different granularity spaces formed, the complex texture image can be segmented. But the experiment result shows that texture image segmentation is unsatisfactory. It is clear that partial feature information can not express complete information of the image. Adopt the algorithm in section 4 to

synthesize the image, the final experiment result is shown in Fig. 2(d). Comparing Fig. 2 (b) and Fig. 2 (c) with Fig. 2 (d), the effect is clearly shown. It is completely testified that extracting more different texture feature for image segmentation, the result of synthesis can have better effect in human vision, which shows that the synthesis algorithm is effective to segment the complex texture image.

Image fusion technology based on information fusion theory is applied to the complex image segmentation recently. Fig. 2 (e) shows the segmental result of image fusion on Fig. 2(a). Adopt the wavelet transform to decompose the image, and then use fusion methods to obtain the results (Wang Yue-Lan, 2000, Wang Wen-jie, 2001). There are differences and identities between the image fusion methods and the quotient space granularity synthesis technology. The identities makes use of different image features information in order to improve the accuracy and reliability on image objects recognition, but there are still differences in algorithm realization, complexity and straightforwardness. The fusion technology makes use of detailed image information to perfect the main feature and then remove the superfluity information. On the other hand, the synthesis technology obtains the global information through image region features. The experiment results demonstrate that the quotient space granularity synthesis method is more effective than image fusion methods.

## Conclusion

The theory of quotient space becomes a potential direction of artificial intelligence in 1980s. This paper introduces the basic idea of quotient space theory, gives the synthesis algorithm about quotient space granularity computation and testifies the algorithm through texture image segmentation experiment. The proposed method of synthesis algorithm is worthy of further study in the image analysis and image processing field.

The synthesis algorithm is built on the accurate analysis of the problem of multi-sided and multi-level information.

As to the texture image segmentation, the key steps are follows.

Extract the texture region features according to human vision.

Design equivalence relations to divide the universe effectively.

Select valid cluster method to form different granularity spaces.

Where, the synthesis algorithm is the most important step to complete texture image segmentation.

## References

- Clausi, D. A., 2002. K-means Iterative Fisher (KIF) Unsupervised Clustering Algorithm Applied to Image Texture Segmentation. *Pattern Recognition* 35: 1959-1972.
- Chappard, D., I. Degasne, G. Hure ext., 2003. Image Analysis Measurements of Roughness by Texture and Fractal Analysis Correlate with Contact Profilometry. *Biomaterials*, 24: 1399-1407.
- Jasvinder, Singh, Kristin J. Dana, 2004. Clustering and Blending for Texture Synthesis. *Pattern Recognition Letters*, 25: 619-629.
- Kyong, I. Chang, W. Kevin, 1999. Evaluation of Texture Segmentation Algorithms, *Trans. IEEE* pp: 1063-1068.
- Ling Zhang, Bo Zhang, 2003. The Quotient Space Theory of Problem Solving. *RSFDGrC, LNAI2639*, pp: 11-15.
- Mausumi Acharyya, Malay K. Kundu, 2000. Two Texture Segmentation Using M-Band Wavelet Transform. In *Proceeding of the International Conference on Pattern Recognition*.
- Sun Xing-hua, Yang Jing-yu, Guo Li, 2002. Research of image retrieval based on improved coarseness, *Computer Engineering*, pp: 144-246.
- Wang Zhen, Wang Zhi-quan, Mao Yao-bin, 2002. Description Based on Texture Direction and the Clustering and Segmentation to Directional Texture Images, *J. Image and Graphics*, 12: 1279-1280.
- Wang Yue-Lan, Zeng Ying-Sheng. 2000. The Study On the Information Fusion Used for Color Images Segmentation. *Chinese Journal Computers*, 23: 763-767.
- Wang Wen-jie, Tang Ping, Zhu Chong-guang, 2001. A Wavelet Transform-Based Image Fusion Method. *J. Image and Graphics*, 11: 1130-1135.
- Zhang, Bo and Zhang Ling, 1992. *Theory and Application of Problem Solving*, North-Holland, Elsevier Science Publishers B.V.
- Zhang, Yu-jin, 2001. *Image Segmentation*, Science Publishers(Chinese), Beijing China.