

Performance Evaluation of Wavelets for Image Compression

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Abstract: The objective of this study is to evaluate a set of wavelets for image compression. Image compression using wavelet transforms results in an improved compression ratio. Wavelet transformation is the technique that provides both spatial and frequency domain information. These properties of wavelet transform greatly help in identification and selection of significant and non-significant coefficients amongst the wavelet coefficients. DWT (Discrete Wavelet Transform) represents image as a sum of wavelet function (wavelets) on different resolution levels. So, the basis of wavelet transform can be composed of function that satisfies requirements of multiresolution analysis. Depending on the application, different aspects of wavelets can be emphasized. There exists a large selection of wavelet families, depending on the choice of wavelet function. The choice of wavelet function for image compression depends on the image application and the content of image. A review of the fundamentals of image compression based on wavelet is given here. This study also discussed important features of wavelet transform in compression of images. In this study we have evaluate and compare seven different wavelet families i.e., Haar, Daubechies, Symlets, Coiflets, Biorthogonal, Reverse Biorthogonal and discrete approximation of Meyer on variety of test images set. We have also analyzed effects of wavelet functions belonging to each of these wavelet families on image quality at a compression ratio of 10:1 and 100:1 on the variety of test images set at decomposition level 5. Image quality is measured, objectively using peak signal-to-noise ratio and subjectively using visual image quality. Our results provide a reference for application developers to choose an application based wavelet for image compression for their applications.

Key words: Discrete cosine transform, wavelets, wavelet transform, Image compression, compression performance, image quality

INTRODUCTION

The rapid development of high performance computing and communication has opened up tremendous opportunities for various computer-based applications with image and video communication capability. However, the amount of data required to store a digital image is continually increasing and overwhelming the storage devices. The data compression becomes the only solution to overcome this. Image compression is the representation of an image in digital form with as few bits as possible while maintaining an acceptable level of image quality^[1]. A typical still image contains a large amount of spatial redundancy in plain areas where adjacent picture elements i.e. the pixels have almost the same values. It means that the picture elements are highly correlated. The redundancy can be removed to achieve compression of the image data i.e., the fundamental components of compression are redundancy

and irrelevancy reduction. The basic measure of the performance of a compression algorithm is the compression ratio, which is defined by the ratio between original data size and compressed data size. Higher compression ratios will produce lower image quality and the vice versa is also true.

Current standards for compression of images use DCT^[2-4], which represent an image as a superposition of cosine functions with different discrete frequencies^[5]. The transformed signal is a function of two spatial dimensions and its components are called DCT coefficients or spatial frequencies. DCT coefficients measure the contribution of the cosine functions at different discrete frequencies. DCT provides excellent energy compaction and a number of fast algorithms exist for calculating the DCT. Most existing compression systems use square DCT blocks of regular size^[2-4]. The image is divided into blocks of samples and each block is transformed independently to give coefficients.

To achieve the compression, DCT coefficients should be quantized. The quantization results in loss of information, but also in compression. Increasing the quantizer scale leads to coarser quantization, gives high compression and poor decoded image quality. The use of uniformly sized blocks simplified the compression system, but it does not take into account the irregular shapes within real images. The block-based segmentation of source image is a fundamental limitation of the DCT-based compression system^[6,7]. The degradation is known as the “blocking effect” and depends on block size. A larger block leads to more efficient coding, but requires more computational power. Image distortion is less annoying for small than for large DCT blocks, but coding efficiency tends to suffer. Therefore, most existing systems use blocks of 8X8 or 16X16 pixels as a compromise between coding efficiency and image quality.

Wavelets provide good compression ratios^[8,9], especially for high resolution images. Wavelets perform much better than competing technologies like JPEG^[10], both in terms of signal-to-noise ratio and visual quality. Unlike JPEG, it shows no blocking effect but allow for a graceful degradation of the whole image quality, while preserving the important details of the image. The next version of the JPEG standard i.e. JPEG 2000 will incorporate wavelet based compression techniques. In a wavelet compression system, the entire image is transformed and compressed as a single data object rather than block by block as in a DCT-based compression system. It allows a uniform distribution of compression error across the entire image. It can provide better image quality than DCT, especially on a higher compression ratio^[11]. However, the implementation of the DCT is less expensive than that of the DWT. For example, the most efficient algorithm for 2-D 8X8 DCT requires only 54 multiplications^[12], while the complexity of calculating the DWT depends on the length of wavelet filters. A wavelet image compression system can be consists of wavelet function, quantizer and an encoder. In our study, we used various wavelets for image compression on image test set and then evaluate and compare the wavelets. According to this analysis, we show the choice of the wavelet for image compression taking into account objective image quality measures.

OVERVIEW OF TRANSFORM BASED IMAGE COMPRESSION

A number of methods have been presented over the years to perform image compression. They all have one common goal, to alter the representation of information contained in an image, so that it can be represented

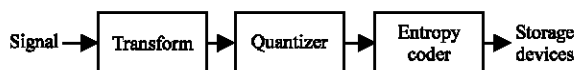


Fig. 1: Generalized Image compression system using transforms

sufficiently well with less information. Regardless of the details of each image compression method, the methods can be classified into two general categories:

- Lossless compression
- Lossy compression

For methods in the first category, guarantees that the original signal can be reconstructed without any errors i.e., it provide a perfect reproduction of the original image. Conversely in lossy method, some information from the original image is lost, even if only a small amount, but can obtain higher compression ratios.

Current methods for lossless image compression, such as that used in the GIF image standard, typically use some form of Huffman or arithmetic coder^[13] or an integer-to-integer wavelet transform^[14,15]. Unfortunately, even the best current lossless algorithms provide relatively small compression performance compared to the best lossy methods. To achieve a high compression performance, a lossy method must be used.

The generalized lossy image compression methods use a transform-based scheme is shown in Fig. 1. In the first step, the signal is processed with an invertible transform, such as Discrete Cosine Transform (DCT) or Wavelet Transform (WT). This step is intended to “decorrelate” the input signal by transforming to a representation in which the set of data values is sparser, thereby compaction of the information content of the signal into a smaller number of coefficients. The choice of transform used depends on a number of factors, in particular, computational complexity and coding gain. Today, the most effective and popular way to achieve good compression of images are based on Discrete Cosine Transform (DCT) and Wavelet Transform (WT)^[16].

The transform coefficients, which may typically be thought of as infinite precision real numbers, are then quantized. This step is not reversible and represents the lossy stage in the process. A good quantizer tries to assign more bits for coefficients with more information content or perceptual significance and fewer bits for coefficients with less information content, based on a given fixed bit budget. The choice of a quantizer depends on the transform that is selected. While transforms and quantizers can be “mixed and matched” to a certain degree, some quantization methods perform better with

particular transform methods^[17,11]. Also, perceptual weighting of coefficients in different subbands can be used to improve subjective image quality^[18]. Quantization methods used with wavelet transforms fall into two general categories: embedded and non-embedded^[19,20]. Scalar and vector quantizers are common examples of non-embedded quantizers. They determine bit allocations based on a specified bit budget, allocating bits across a set of quantizers corresponding to the image subbands^[21].

The last step is entropy coding, which removes redundancy from the output of the quantizer. This process removes redundancy in the form of repeated bit patterns in the output of the quantizer. Frequently occurring symbols are replaced with

shorter bit patterns while infrequently occurring symbols are replaced with longer bit patterns, resulting in a smaller bit stream overall. The most common entropy coding techniques are Run-length Encoding (RLE), Huffman coding, arithmetic coding^[13] and Lempel-Ziv (LZ) algorithms. The arithmetic coder is more effective than others^[13], this allows arithmetic codes to outperform Huffman codes and consequently arithmetic codes are more commonly used in wavelet-based algorithms^[19,20,22].

WAVELET TRANSFORM

The simplest way of performing image compression is through the use of transform coding techniques^[8,7,20]. Wavelet Transform (WT) represents an image as a sum of wavelet functions with different locations and scales^[23]. Any decomposition of an image into wavelets involves a pair of waveforms to represent the high frequencies corresponding to the detailed parts of an image (wavelet function) and for the low frequencies or smooth parts of an image (scaling function)^[24]. Figure 2 shows two waveforms of a family discovered in the late 1980s by Daubechies; the left one to represent smooth parts of the image and the right one can be used to represent detailed parts of the image. The two waveforms are translated and scaled on the time axis to produce a set of wavelet functions at different locations and on different scales^[23].

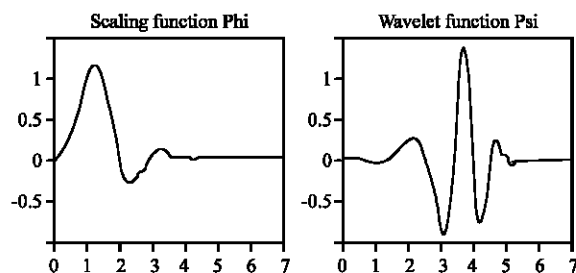


Fig. 2: Scaling and Wavelet functions.

Each wavelet contains the same number of cycles, such that, as the frequency reduces, the wavelet gets longer. High frequencies are transformed with short functions (low scale) and Low frequencies are transformed with long functions (high scale). During computation, the analyzing wavelet is shifted over the full domain of the analyzed function. The result of WT is a set of wavelet coefficients, which measure the contribution of the wavelets at these locations and scales.

DISCRETE WAVELET TRANSFORM

The transform based coding techniques work by statistically decor-relating the information contained in the image so that the redundant data can be discarded^[25]. Therefore a “dense” signal is converted to a “sparse” signal and most of the information is concentrated on a few significant coefficients. The greatest problem associated with the transform coding techniques such as DCT based image compression^[7] is the presence of visually annoying “blocking artifact” in the compressed image. This has caused an inclination towards the use of Discrete Wavelet Transform (DWT) for all image and video compression standards. DWT offers adaptive spatial-frequency resolution (better spatial resolution at high frequencies and better frequency resolution at low frequencies)^[26]. In present scene, much of the research works in image compression have been done on the Discrete Wavelet Transform. DWT now becomes a standard tool in image compression applications because of their data reduction capabilities^[27,28,23]. The basis of Discrete Cosine Transform (DCT) is cosine functions^[10], while the basis of Discrete Wavelet Transform (DWT) is wavelet function that satisfies requirement of multi-resolution analysis^[29]. Discrete wavelet transform have certain properties that makes it better choice for image compression. It is especially suitable for images having higher resolution. DWT represents image on different resolution level i.e., it possesses the property of Multi-resolution^[30]. Since, DWT can provide higher compression ratios with better image quality due to higher decorrelation property. Therefore, DWT has potentiality for good representation of image with fewer coefficients^[31].

WAVELETS FOR IMAGE COMPRESSION

The choice of wavelet function is crucial for performance in image compression. There are a number of basis that decides the choice of wavelet for image compression. Since the wavelet produces all wavelet functions used in the transformation through translation

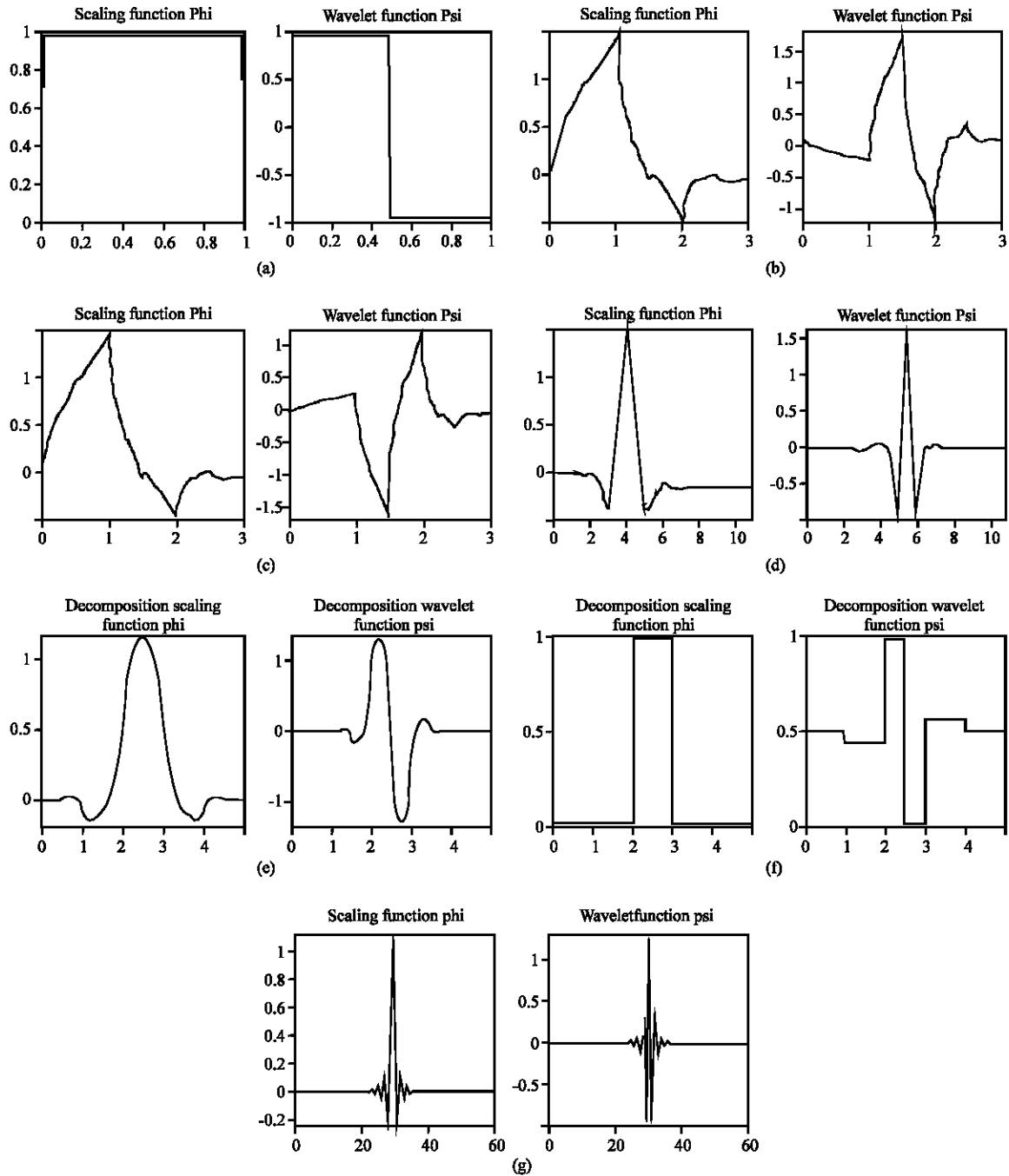


Fig. 3: Different wavelets families used in our experiment: (a) Haar (b) db_2 (c) sym_2 (d) Coif_2 (e) bior_1.3 (f) rbio_1.3 (g) dmey

and scaling, it determines the characteristics of the resulting wavelet transform. Therefore, the details of the particular application should be taken into account and the appropriate wavelet should be chosen in order to use the wavelet transform effectively for image compression. The compression performance for images with different

spectral activity will be decided by the wavelet function from the wavelet family. Daubechies wavelet function will give satisfying results for images with moderate spectral activity^[32]. In our experiment seven wavelet functions of wavelet families are examined namely: Haar, Daubechies, Symlets, bior, rbior, Coiflet and Meyer. Figure 3 illustrates

wavelet functions used in our experiment. Haar wavelet is one of the oldest and simplest wavelet. Therefore, any discussion of wavelets starts with the Haar wavelet. Haar wavelet is discontinuous and resembles a step function. Daubechies wavelets are the most popular wavelets. They represent the foundations of wavelet signal processing and are used in numerous applications. The Haar, Daubechies, bior, rbior and Coiflets are compactly supported orthogonal wavelets^[33].

Biorthogonal wavelets, exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition and the other for reconstruction instead of the same single one, interesting properties can be derived. A major disadvantage of these wavelets is their asymmetry, which can cause artifacts at borders of the wavelet subbands. These wavelets along with Meyer wavelets are capable of perfect reconstruction. The Meyer wavelets are symmetric in shape. The wavelets are chosen based on their shape and their ability to compress the image in a particular application.

PERFORMANCE EVALUATION METHODOLOGY

The performance of image compression techniques are mainly evaluated by the two measures: Compression Ratio (CR) and the magnitude of error introduced by the encoding. The compression ratio is defined as:

$$C. R. = \frac{\text{The number of bits in the original image}}{\text{The number of bits in the compressed image}}$$

For error evaluation, two error metrics are used to compare the various image compression techniques: Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). PSNR is used to measure the difference between two images. In order to quantitatively evaluate the quality of the compressed image the Peak Signal-to-Noise Ratios (PSNR) of the images are computed. PSNR provides a measurement of the amount of distortion in a signal^[34], with a higher value indicating less distortion. For n-bits per pixel image, PSNR is defined as:

$$PSNR = 20 \log_{10} \frac{2^n - 1}{RMSE}$$

Where, RMSE is the root mean square difference between two images. The Mean Square Error (MSE) is defined as follows^[35]:

$$MSE = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |y(m, n) - x(m, n)|^2$$

Where x(m, n), y(m,n) are respectively the original and recovered pixel values at the Mth row and Nth column for M X N size image.

The PSNR is given in decibel units (dB), which measure the ratio of the peak signal and the difference between two images. An increase of 20 dB corresponds to a ten-fold decrease in the rms difference between two images. There are many versions of signal-to-noise ratios, but the PSNR is very common in image processing, probably because it gives better-sounding numbers than other measures

A lower value for MSE means lesser error and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher. Here, the 'signal' is the original image and the 'noise' is the error in reconstruction. Therefore, a compression scheme having a lower MSE (and a high PSNR) recognize that it is a better one. For an 8-bit grayscale image, the peak signal value is 255. Therefore, the PSNR of 8-bit grayscale image and its reconstructed image is calculated as^[36],

$$PSNR = 10 \log_{10} \left(\frac{255^2}{MSE} \right)$$

EXPERIMENTAL RESULTS AND DISCUSSIONS

In our experiment, we have examined the seven types of wavelet families: Haar Wavelet (Haar), Daubechies Wavelet (DB_2), Coiflet Wavelet (COIF_2), Biorthogonal Wavelet (BIOR_1.3), Reverse Biorthogonal Wavelet (RBIOR_1.3), SYMLET (SYM_2) and DMEY. We have analyzed four different test images of different size: Venicem (264×141), Birds (512×512), Doggy (369×283), Mountain (640×480) on seven above defined wavelet families. In our experiment, we have compressed test images of different size on to different wavelets. Results are measured in terms of Peak Signal to Noise Ratio (PSNR), Compression Ratio (CR) and Quality of compressed image. Table 1-4 shown below provides the experimental results of PSNR in terms of decibels for the four test images compressed with wavelet functions corresponding to each of the seven wavelet families.

The comparison of PSNR values of wavelets of each wavelet family for different test images shown in Fig. 4. We also are presenting compression results of test images in terms of visual quality for different wavelet functions for wavelet Families. The results of image are shown in Fig. 5. All of these images shown have been compressed at the compression ratio of 10:1 and 100:1 each at decomposition level of 5. The presented results shown that Peak Signal T O Noise Ratios (PSNR) obtained

Table 1: Image : Venicem (264×141)

Compression ratio	Wavelet	HAAR	DB 2	SYM 2	COIF 2	BIOR 1.3	RBIO 1.3	DMEY
2:1	PSNR(db)	49.37	50.54	50.54	49.98	49.58	50.67	42.26
4:1	PSNR(db)	38.83	39.38	39.38	38.61	38.44	39.45	29.59
10:1	PSNR(db)	30.04	30.10	30.10	29.63	29.52	30.31	20.99
16:1	PSNR(db)	26.91	27.03	27.03	26.41	26.34	27.35	18.87
50:1	PSNR(db)	21.98	22.25	22.25	21.24	21.29	21.98	---
100:1	PSNR(db)	20.22	20.26	20.26	18.75	19.31	20.22	---

Table 2: Image : Birds (512×512)

Compression ratio	Wavelet	HAAR	DB 2	SYM 2	COIF 2	BIOR 1.3	RBIO 1.3	DMEY
2:1	PSNR(db)	55.65	52.50	52.50	52.44	55.08	53.51	47.90
4:1	PSNR(db)	44.41	44.44	44.44	44.48	44.18	44.72	41.78
10:1	PSNR(db)	38.48	39.08	39.08	39.19	38.14	39.31	35.84
16:1	PSNR(db)	36.10	36.87	36.87	37.03	35.68	37.18	33.02
50:1	PSNR(db)	31.11	32.11	32.11	32.25	30.57	32.53	26.35
100:1	PSNR(db)	28.61	29.60	29.60	29.56	28.01	29.91	--

Table 3: Image : Doggy (369×283)

Compression ratio	Wavelet	HAAR	DB 2	SYM 2	COIF 2	BIOR 1.3	RBIO 1.3	DMEY
2:1	PSNR(db)	37.17	36.97	36.97	36.57	36.74	37.89	32.19
4:1	PSNR(db)	29.04	29.12	29.12	28.78	28.64	29.15	24.57
10:1	PSNR(db)	24.10	24.26	24.26	23.99	23.74	24.27	19.69
16:1	PSNR(db)	22.50	22.67	22.67	22.41	22.16	22.72	17.97
50:1	PSNR(db)	19.71	19.96	19.96	19.65	19.36	20.01	---
100:1	PSNR(db)	18.41	18.66	18.66	18.14	17.97	18.67	---

Table 4: Image : Mountain (640×480)

Compression ratio	Wavelet	HAAR	DB 2	SYM 2	COIF 2	BIOR 1.3	RBIO 1.3	DMEY
2:1	PSNR(db)	36.56	36.17	36.17	35.91	36.26	36.12	33.48
4:1	PSNR(db)	27.27	27.12	27.12	26.85	26.85	27.11	24.52
10:1	PSNR(db)	21.98	21.96	21.96	21.70	21.59	22.01	19.37
16:1	PSNR(db)	20.36	20.38	20.38	20.09	19.99	20.45	17.76
50:1	PSNR(db)	17.79	17.86	17.86	17.60	17.46	17.94	15.43
100:1	PSNR(db)	16.65	16.84	16.84	16.56	16.47	16.89	---

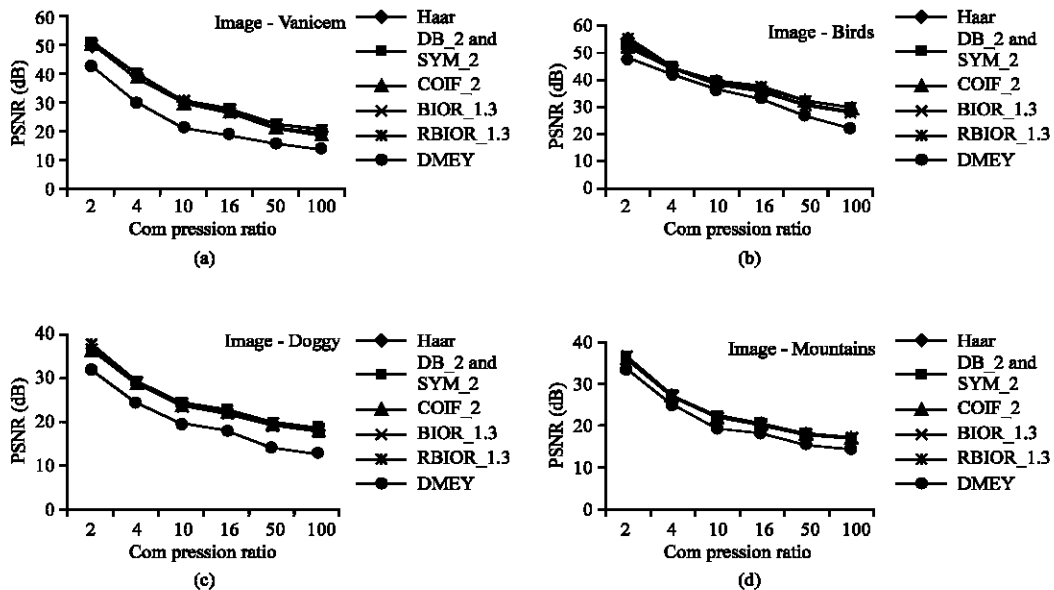


Fig. 4: Comparison of PSNR values for image compression on images (a) Venicem (264×141) (b) Birds (512×512) (c) Doggy (369×283) (d) Mountains (640×480)

for RBIOR_1.3 are highest. It results that RBIOR_1.3 gives best compression performance for our test images of various sizes. At the compression ratio of 2:1, the RBIOR_1.3 wavelet not giving the best compression performance for large size images which is marked in the result tables. The DMEY wavelet gives poor compression performance as compare to all other wavelets we have taken. As well as this wavelet provides compression of the grayscale image of small size at higher compression ratio with very poor image quality. The wavelet DB_2 and SYM_2 provides the same PSNR values for all the test images at all the compression ratio. The variation in PSNR is more at low compression ratio and variation in PSNR less, as the compression ratio increases. In all the study, if the decomposition level was increased the compression performance improves but the quality of image deteriorates.

CONCLUSION

This study focused on the evaluation and comparison of the wavelets using PSNR as image quality measure. In this study, we evaluate results from a comparative study of different wavelet functions on variety of images to facilitate image compression. The effects of Haar, Daubechies, Symlets, Coiflets, Biorthogonal, Reverse Biorthogonal and Discrete approximation of Meyer (demy) wavelet function on different test images have been examined. The compression ratio and visual image quality for all wavelet functions is also presented. The Peak Signal to Noise Ratio (PSNR) is taken as the objective measure for performance evaluation of wavelets using for images compression. We analyzed the results for a wide range of wavelets and found that the RBIOR_1.3 wavelet provides

best compression performance for all variety of images almost at all the compression ratio. Additionally, we also found that the two wavelets DB_2 and SYM_2 gave the same values of PSNR for each of the four test images. As far as the image quality is concerned we got a fair image quality in case of all the wavelets examined, except the DMEY. At the compression ratio of 100:1, wavelet Daubechies_2 showed the best picture quality for the test image bird. So, we can conclude that compression performance depend not only on the size of the image but also on the content of the image. Therefore, we can say that the choice of wavelet in the process of image compression depends on size of the image and content of the image for desired image quality.

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