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Sensorless Nonlinear Control of Permanent Magnet Synchronous Motor Using the Extended Kalman Filter

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Abstract: This study deals with the Nonlinear Control (NLC) applied to the speed regulation of Permanent Magnet Synchronous Motor (PMSM). The control problem of a PMSM is solved by the application of a nonlinear input-output linearization technique. A state feedback input-output linearizing control is designed for a third order model of a PMSM which included both electrical and mechanical dynamics. Speed and torque control of permanent magnet synchronous motors are usually attained by the application of position and speed sensors. However, speed and position sensors require the additional mounting space, reduce the reliability in harsh environments and increase the cost of motor. Many studies have been performed for the elimination of speed and position sensors. Our proposed control strategy is based on an accurate Extended Kalman Filter (EKF) which estimates speed, position, load and rotor-fixed current i_d, i_q. The investigations show that the EKF is capable of tracking the actual rotor speed and position provided that the elements of the covariance matrices are properly selected. Moreover, the performance of the EKF is satisfactory even in the presence of noise or when there are variations in permanent magnet synchronous motor load.

Key words: PMSM, EKF, NLC, SNLC, simulink

INTRODUCTION

The permanent magnet synchronous motor offer many advantages over induction motor, such as overall efficiency, effective use of reluctance torque, smaller losses and compact motor size[1]. The nonlinear control considered here uses the input-output feedback linearization control, based on differential geometry techniques. These techniques allow us to decompose the model of the motor in two independent mono-variable linear sub-systems. The dynamics of each sub-system is obtained by an optimal pole placement[2].

This study presents a vector control and input-output feedback linearization control applied to the speed regulation of a PMSM and to ensure a maximal torque operation.

Various control algorithms have been proposed for the elimination of speed and position sensors: Estimators using state equations, artificial intelligence, direct control of torque and flux, Model Reference Adaptive System (*MRAS*) and so on. This study proposes the SNLC strategy based on the EKF of a PMSM.

Kalman filter is a special kind of observer which provides optimal filtering of the noises in measurement and inside the system if the covariances of these noises are known[1]. If rotor speed and rotor position (as extended states) are included in the dynamic model of PMSM, the extended Kalman filter EKF can be used to relinearize the nonlinear state model for each new state estimate as it becomes available. Consequently, the EKF is considered to be the best solution for the speed and position estimation.

NONLINEAR MODEL OF THE PMSM

With the simplifying assumptions relating to the PMSM, the model of the motor expressed in the Park reference frame is given in the suitable state from[3]:

$$\begin{cases} \dot{X} = F(X) + G \cdot U \\ Y = H(X) \end{cases}$$

$$Y(X) = \begin{bmatrix} y_1(X) \\ y_2(X) \end{bmatrix} = \begin{bmatrix} h_1(X) \\ h_2(X) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_d \\ \Omega \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} I_d \\ I_q \\ \Omega \end{bmatrix}; U = \begin{bmatrix} V_d \\ V_q \end{bmatrix}; G = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$$

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$$F(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ f_3(X) \end{bmatrix} = \begin{bmatrix} a_1 \cdot x_1 + a_2 \cdot x_2 \cdot x_3 \\ b_1 \cdot x_2 + b_2 \cdot x_1 \cdot x_3 + b_3 \cdot x_3 \\ c_1 \cdot x_3 + c_2 \cdot x_1 \cdot x_2 + c_3 x_2 - \frac{C_r}{J} \end{bmatrix}$$

and

$$a_1 = -\frac{R_s}{L_d};\ a_2 = \frac{p \cdot L_q}{L_d};\ b_1 = -\frac{R_s}{L_q};\ b_2 = -\frac{p \cdot L_d}{L_q}$$

$$b_3 = -\frac{p \cdot \phi_f}{L_q}; c_1 = -\frac{f}{J}; c_2 = \frac{p \cdot (L_d - L_q)}{J}; c_3 = \frac{p \cdot \phi_f}{J}$$

in $f_1(X)$ the load torque C_r is removed from the state equations and will be considered as a perturbation.

Linearizing control: The linearization condition permits to verify if the non linear system admits an input-output linearization. This condition is the order of the relative degree of the system. To obtain the non linear control law, we calculate the output relative degree (i.e. the number of possibilities which is necessary to derive the output in order to obtain the input V).

The relative degree of the d-axis current $I_d = y_1(x)$:

$$\dot{y}_1(x) = \dot{h}_1(x) = L_f h_1(x) + L_g h_1(x) \cdot U = f_1 + g_1 V_d \quad (3)$$

The relative degree of $y_1(x)$ is $r_1 = 1$.
The relative degree of the mechanical speed $\Omega = y_2(x)$:

$$\dot{y}_2(x) = \dot{h}_2(x) = L_f h_2(x) + L_g h_2(x) \cdot U = f_3 \quad (4)$$

$$L_g h_2(x) = 0$$

$$\dot{y}_2(x) = L_f h_2(x) = f_3$$

We note that the inputs U do not appear in (4), a second derivative became then necessary:

$$\ddot{y}_2(x) = \ddot{h}_2(x) = L^2{}_f h_2(x) + L_g L_f h_2(x) \cdot U$$

with:

$$L^2{}_f h_2(x) = f_1 \cdot p\frac{(L_d - L_q)}{J} x_2 + f_2\left(\frac{p \cdot (L_d - L_q)}{J} x_1 + p\frac{\phi_f}{J}\right) - f_3 \frac{f}{J}$$

$$L_g L_f h_2(x) = \begin{bmatrix} \frac{1}{L_d} p \frac{(L_d - L_q)}{J} x_2 & \frac{1}{L_q}(\frac{p(L_d - L_q)}{J} x_1 + p\frac{\phi_f}{J}) \end{bmatrix}$$

The relative degree of $y_2(x)$ is $r_2 = 2$.

The relative degree of the system is equal to the system order n ($r = r_1 + r_2 = n = 3$). The system is exactly linearizable. The vector defining the relation between the physical inputs (U) and the derivative outputs (y(x)) is given by the expression:

$$\begin{bmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} I_d \\ \frac{d^2}{dt^2}\Omega \end{bmatrix} = A(X) + D(X)\begin{bmatrix} V_d \\ V_q \end{bmatrix} \quad (5)$$

where:

$$A(X) = \begin{bmatrix} f_1 \\ f_1 \cdot p\frac{(L_d - L_q)}{J} x_2 + f_2\left(\frac{p \cdot (L_d - L_q)}{J} x_1 + p\frac{\phi_f}{J}\right) - f_3 \frac{f}{J} \end{bmatrix}$$

$$D(X) = \begin{bmatrix} g_1 & 0 \\ g_1 \cdot p\frac{(L_d - L_q)}{J} x_2 & g_2 \cdot (\frac{p(L_d - L_q)}{J} x_1 + p\frac{\phi_f}{J}) \end{bmatrix}$$

To linearize the input-output comportments of the motor in closed loop, we apply the following nonlinear state feedback[2]:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = D^{-1}(X)\left[-A(X) + \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right] \quad (6)$$

The decoupling matrix is $d^{-1}(X)$ must be invertible. The application of the linearizing law (6) on the system (5) allows obtaining tow mono-variable, linear and decoupled sub-systems.

$$\begin{bmatrix} \dot{y}_1(x) \\ \ddot{y}_2(x) \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} I_d \\ \frac{d^2}{dt^2}\Omega \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (7)$$

Control algorithm by poles placement: The internal inputs (v^1, v_2) are calculated by imposing static modes ($I_{dref} = I_d$ and $\Omega_{dref} = \Omega$) and an error dynamic[2,4]:

$$v_1 = k_{11}(I_{dref} - I_d) + \frac{d}{dt} I_{dref} \quad (8)$$

$$v_1 = k_{22}(\Omega_{ref} - \Omega) + k_{21}(\frac{d\Omega_{ref}}{dt} - \frac{d\Omega}{dt}) \frac{d^2\Omega_{ref}}{dt^2} \quad (9)$$

$$\frac{d}{dt} e_1 + k_{11} = 0 \quad (10)$$

$$\frac{d^2}{dt^2} e_2 + k_{21} \frac{d}{dt} e_2 + k_{22} e_2 = 0 \quad (11)$$

where:

$$e_1 = I_{dref} - I_d$$
$$e_2 = \Omega_{ref} - \Omega$$

The gains k_{11}, k_{21}, k_{22} are chosen so that the following Hurwitz polynomial equations:

$$P + k_{11} = 0$$
$$P^2 + k_{21}P + k_{22} = 0$$

The control diagram is given by Fig. 1.

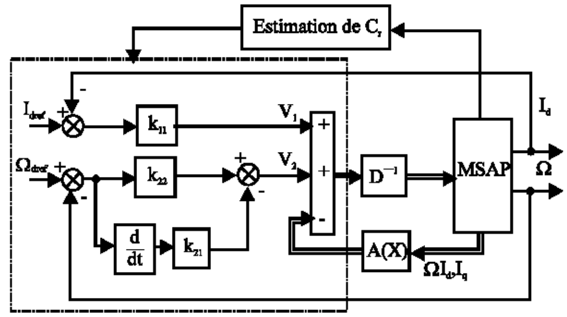

Fig. 1: Bloc diagram of the nonlinear controller

Design of ekf observer: Accurate and robust estimation of motor variables which are not measured is crucial for high performance sensor less drives. A multitude observers have been proposed, but only a few are able to sustain persistent and accurate wide speed range sensor less operation. At very low speed, their performances are poor. One of the raisons is high sensitivity of the observers to unmodeled nonlinearities, disturbance and model parameters detuning.

The Kalman filter provides a solution that directly cares for the effects of disturbance noises including system and measurement noises. The errors in parameters will normally also handled as noise[5].

The dynamic state model for non linear stochastic machine is as follows where all symbols in the formulations denote matrices or vectors[6]:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), t) + w(t) \\ y(t) = h(x(t), t) + v(t) \end{cases} \quad (12)$$

w(t): System noise vector.
v(t): Measurement noise vector
w,v: Are unrelated and zero mean stochastic processes.

A recursive algorithm is presented for the discrete time study. For the given sampling time T_s, both the optimal estimate sequence $x_{k/k}$ and its covariance matrix $P_{k/k}$ generated by the filter go through a tow step loop.

The first step (prediction) performs a prediction of both quantities based on the previous estimates $x_{k-1/k-1}$ and the mean voltage vector actually applied to the system in the period from T_{k-1} to T_k. F is system gradient matrix (Jacobean matrix).

$$F(\tilde{x}(t), t) = \left. \frac{\partial f(x(t), u(t), t)}{\partial x^T(t)} \right|_{x(t)=\tilde{x}(t)} \quad (13)$$

$$x_{k/k-1} = x_{k-1/k-1} + T_s.f(x_{k-1/k-1}, u_{k-1}) \quad (14)$$

$$P_{k/k-1} = P_{k/k-1} + (FP_{k-1/k-1} + P_{k-1/k-1}F^T).T_s + Q \quad (15)$$

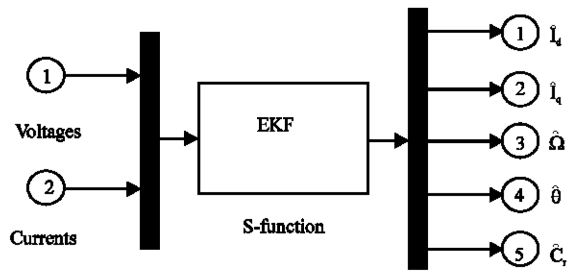

Fig. 2: S-function Representation of the EKF

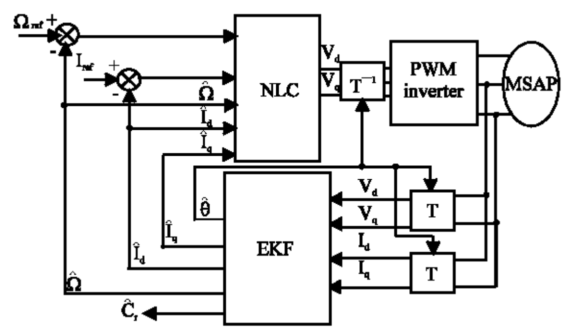

Fig. 3: Speed control of a PMSM using the NLC with an EKF for speed and position estimation

The second step (innovation) corrects the predicted state estimate and its covariance matrix trough a feedback correction scheme that makes use of the actual measured quantities; this is realized by the following recursive relations:

$$x_{k/k} = x_{k/k-1} + K_k(Y_k - Hx_{k/k-1}) \quad (16)$$

$$P_{k/k} = P_{k/k-1} - K_k HP_{k/k-1} \quad (17)$$

Where the filter gain matrix is defined by:

$$k_k = P_{k/k-1}H^T(HP_{k/k-1}H^T + R)^{-1} \quad (18)$$

Fig. 4: Simulation results considering the feedback linearization with load variation

Fig. 5: Simulation results considering the feedback linearization with speed inversion

H is transformation matrix.

$$H(\tilde{x}(t),t) = \left.\frac{\partial h}{\partial x}\right|_{x(t)=\tilde{x}(t)} \quad (19)$$

The proposed *EKF* observer is designed in rotor reference frame (d,q frame).
State vector is chosen to be:

$$X = [I_d \; I_q \; \Omega \; \theta \; C_r]^T$$

Input:

$$U = [u_d \; u_q]^T;$$

Output:

$$Y = [I_d \; I_q]$$

I_d, I_q and u_d, u_q are motor stator currents, voltages in rotor reference frame shown in Fig. 2.

The critical step in the *EKF* is the search for the best covariance matrices Q and R have to be set-up based on the stochastic properties of the corresponding noise. The noise covariance R accounts for the measurement noise introduced by the current sensors and quantization errors of the A/D converters[7]. Increasing R indicates stronger disturbance of the current. The noise is weighted less by the filter, causing also a slower transient performance of system.

The noise covariance Q reflects the system model inaccuracy, the errors of the parameters and the noise introduced by the voltage estimation[8]. Q has to be increased at stronger noise driving the system, entailing a more heavily weighting of the measured current and a faster transient performance.

An initial matrix P_0 represents the matrix of the covariance in knowledge of the initial condition. Varying P_0 affects neither the transient performance nor the steady state condition of the system. In this study, the value of these elements is tuned “manually”, by running several simulations. This is maybe one of the major drawbacks of the kalman filter.

Figure 3 Show the proposed Sensor less SMC using *EKF.* In this study, the outputs of a PWM voltage source inverter are used as the control inputs for the EKF. These signals contain components at high frequencies, which are used as the required noise by the kalman filter. Thus, no additional external signals are then needed.

$$f(x(k),u(k)) = \begin{bmatrix} I_d & I_q & \Omega & \theta & C_r \end{bmatrix}^T =$$

$$\begin{bmatrix} (1 - T_s\frac{R_s}{L_d})I_d + p\Omega T_s\frac{L_q}{L_d}I_q + T_s\frac{1}{L_d}V_d \\ (-p\Omega T_s\frac{L_d}{L_q})I_d + (1 - T_s\frac{R_s}{L_q})I_q - T_s\frac{\phi_{sf}}{L_q}p\Omega + T_s\frac{1}{L_q}V_q \\ pT_s\frac{L_d - L_q}{J}I_qI_d + pT_s\frac{\phi_{sf}}{J}I_q + (1 - T_s\frac{f}{J})\Omega - T_s\frac{1}{J}C_r \\ \Omega \\ 0 \end{bmatrix}$$

where:

$$h = \begin{bmatrix} I_d & I_q \end{bmatrix}^T$$

SIMULATION RESULTS

Extensive simulations have been performed using Matlab/Simulink Software to examine the control algorithm of the SNLC applied for PMSM presented in Fig. 4 and based on 3Kw motor parameters.

Figure 4 shows the responses of speed, position, currents I_d, I_q and load torque with errors between the actual and estimated states for step reference with 75% load at t = 0.1.

These responses illustrate high performance of the proposed EKF observer during transients and steady state.

High accuracy and strong robustness of the nonlinear controller and EKF observer are proved by Fig. 5 which contain the same responses as Fig. 4, when speed reversion around (+100rad/s: -100rad/s) are applied.

From Fig. 5, it can be noticed that the proposed EKF works in very low speed region, where many speed estimators or observers have poor performances

CONCLUSION

In this study, authors determine nonlinear control of a permanent magnet synchronous motor with nonlinear input-output linrearization technique. The dynamics behavior and the control performances obtained are satisfactory. The perturbation is rejected.

The EKF observer is able to increase the performance of the SNLC in terms of low speed behavior, speed reversion and load rejection.

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