

Zhang *et al.*'s Partially Blind Signature Revisited

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Abstract: Partially blind signature is an important cryptographic primitive in privacy oriented e-commerce applications. In INDOCRYPT 2003, Zhang *et al.* proposed a new partially blind signature scheme more efficient than previous schemes. It was later showed linkable by Chow *et al.* and then respectively improved by both Chow *et al.* and Zhang *et al.* at the cost of poorer efficiency. However, in this short study we show that the cryptanalysis of Chow *et al.* is wrong. So the original partially blind signature scheme remains secure and the corresponding improvements are needless.

Key words: Zhang *et al.*, partially blind signature

INTRODUCTION

Blind signatures were first introduced by Chaum^[1] and play a central role in cryptographic protocols such as e-cash or e-voting that require user anonymity. Partially blind signatures were introduced by Abe and Fujisaki^[2] to allow the signer to explicitly include some agreed information in the blind signature. The partially blind signature is a regular digital signature, but it needs to satisfy two additional requirements (partial blindness):

- The signer assures that an issued signature contains the information that it desires and none can remove the embedded information from the signature.
- For the same embedded information, the signer cannot link a signature to the instance of the signing protocol that produces the corresponding blind signature.

In INDOCRYPT 2003, Zhang *et al.*^[3] proposed a partially blind signature which is very efficient in terms of computation and communication and can provide batch verification. Unfortunately, it was later showed linkable by Chow *et al.*^[4] and improved. And the original authors themselves, Zhang *et al.* also accepted this disadvantage and revised their original scheme. Of course, both improved schemes are less efficient than the original scheme. Especially, Chow *et al.*'s scheme lost a lot of efficiency since they applied the conventional paradigm mentioned in^[2], i.e., converting a identity-based blind signature to a partially blind signature scheme.

REVIEW OF ZHANG *et al.*'s PARTIALLY BLIND SIGNATURE SCHEME

First, let us recall the corresponding mathematical tools used in the scheme. Let G_1 be a cyclic additive group generated by P whose order is a prime and G_2 be a cyclic multiplicative group with the same order. Let $e: G_1 \times G_1 \rightarrow G_2$ be a bilinear pairing with the following properties:

- Bilinearity: $e(aP, bQ) = e(P, Q)^{ab}$ for all $P, Q \in G_1$ and $a, b \in \mathbb{Z}_q^*$
- Non-degeneracy: There exists $P, Q \in G_1$ such that $e(P, Q) \in G_2$.
- Computability: There is an efficient algorithm to compute $e(P, Q)$ for all $P, Q \in G_1$.

Zhang *et al.*'s scheme^[3] is given as follows. The system parameters are $\text{params} = (G_1, G_2, e, q, P, H, H_0)$, where $G_1 = \langle P \rangle$, G_2, e, q are defined as above and $H: \{0,1\}^* \rightarrow \mathbb{Z}_q, H: \{0,1\}^* \rightarrow G_1$ are hash functions.

KeyGen: The signers picks random $x \in \mathbb{Z}_q^*$ and compute $P_{\text{pub}} = xP$. The public key is P_{pub} and the secret key is x .

Issue: Suppose the requester now wants to get the signature of message m and the requester has already negotiated with the signer on the agreed information c to be attached to the message. The interaction between the requester and the signer is as follows:

Blinding: The user randomly chooses a number $r \in \mathbb{Z}_q^*$, $U = rH_0(m||c)$ computes and sends U to the signer.

Signing: The signer sends back V , where $V = (H(c)+x)^{-1}U$.

Unblinding: The user computes $S = r^{-1}V$. Then (S,m,c) is the partially blind signature of the message m and public information c .

Verification: A verifier can accept this partially blind signature if and only if.

$$e(H(c)P, P_{pub}, S) = e(P, H_0(m || c))$$

**REVIEW OF CHOW *et al.*'S
CRYPTANALYSIS AND OUR DISCUSSION**

The cryptanalysis given by Chow *et al* is very simple as follows. Given a valid signature (S,m,c) and the view (U,V) , any party can accept the partially blind signature (S,m,c) as the one produced by the instance of the Issue protocol (U,V) if and only if $e(S,U) = e(V,H_0(m || c))$. However, the following theorem is enough to show their conclusion is wrong.

Theorem 1. Given any view (U,V) and any valid signature (S',m',c) , the above mentioned equation $e(S',U) = e(V,H_0(m' || c))$ always holds.

Proof. Let the signature and message corresponding to (U,V) is (S',m',c) , then there exists $r \in Z_q^*$ such that:

$$U = rH_0(m || c), V = \frac{1}{x + H(c)}U, S = \frac{1}{x + H(c)}H_0(m || c)$$

So, for some valid signature and message (S',m',c) satisfying

$$S' = \frac{1}{x + H(c)}H_0(m' || c), \text{ we have:}$$

and

$$\begin{aligned} e(V, H_0(m' || c)) &= e\left(\frac{1}{x + H(c)}U, H_0(m' || c)\right) \\ &= e\left(\frac{1}{x + H(c)}rH_0(m || c), H_0(m' || c)\right) \\ &= e(H_0(m' || c), H_0(m || c))^{\frac{r}{x + H(c)}} \end{aligned}$$

Now we have:

$$e(S',U) = e(V, H_0(m' || c))$$

So, for any given valid signature (S',m',c) and the view of the execution of the issuing protocol (U,V) corresponding to any signature pair (S',m',c) , the following equation always holds: $e(S',U) = e(V, H_0(m || c))$. So we can not determine whether a signature (S,m,c) is issued by the view (U,V) through checking the above equation.

CONCLUSION

We point out the error in the cryptanalysis of the art-and-state partially blind signature based pairing in^[4]. So we conclude that the original scheme remains secure and of course much more efficient than the so-called improved schemes in^[4,3].

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