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Stability Testing of 1-D and 2-D Digital Filters Using the Method of Evaluation of Complex Integrals

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Abstract: The stability testing of 2-Dimensional recursive digital filters had been a very hot problem in 1970's and 80's. The right solution is not available though some researchers have been working on this problem even now. We have proposed in this paper a new test procedure (may be an alternative to Jury-Marden algorithm) to test the stability of 1-D recursive digital filters. We also gave for a restricted order 2-D digital filters, a method to test the stability of the given transfer function which is devoid of nonessential singularities of the second kind. In both the 1-D and 2-D cases the available methods of evaluating the complex integrals for variance is used to conclude about the stability of the filter.

Key words: Stability testing, I D and 2 D digital filters, Parseval's integrals

INTRODUCTION

Several methods do exist to determine whether a particular recursive digital filter, 1-dimensional (1-D) or 2-dimensional (2-D), is Bounded Input Bounded output stable or not [1]. In the case of 1-D digital filters the most efficient method is Jury-Marden table method, lucidly explained in[2]. For 2-D quarter plane filters many methods like root mapping and numerical methods are available in the literature which appear in a monogram [1].

In all the methods to test the 2-D filter stability, the absence of nonessential singularities of the second kind is assumed and the denominator polynomial of the 2-D transfer function of the filter is tested.

Some of the latest methods of testing for stability are by Bistritz[3].

We first introduce the concept of 1-D stability testing in this section and the notation used in this paper.

Theorem 1: A 1-D digital recursive filter whose transfer function H(z) is given with B(z) being its denominator polynomial, is stable if[2]

$$B(z) \neq |z| \geq 1 \qquad (1)$$

So the technique of testing for (1) can be either by directly finding the zeros of the polynomial B(z) or using Jury-Marden algorithm[2]. The latter is more efficient and accurate particularly for polynomials of very high degree.

If h(n) is the impulse response of the 1-D filter, it can be shown using Parseval's Integeral of (2)

$$\sum_{n=0}^{\infty} h^2(n) = \frac{1}{2\pi j} \oint_{|z|=1} H(z)\ H(z^{-1}) \frac{dz}{z} \qquad (2)$$

$$= \frac{1}{2p} \int_0^{2\pi} H(e^{jw})\, H(e^{-jw})\ dw$$

$$= \frac{1}{2p} \int_0^{2\pi} \left| H(e^{jw}) \right|^2 dw \qquad (3)$$

using $z = e^{jw}$.

For a stable H(z) the RHS of (3) is always positive and finite. If in (2), on the right hand side the intergration is done in the clockwise direction it can be shown that with

$$z = e^{jw}.$$

$$\sum_{n=0}^{\infty} h^2(n) = -\frac{1}{2p} \int_0^{2\pi} \left| H(e^{jw}) \right|^2 dw$$

This is equivalent to saying that in H(z), z is replaced by 1/z and we substitute

$$z = e^{-jw}$$

in the Parseval's integral we do get the same value for

$$\sum_{n=0}^{\infty} h^2(n)$$

but with a negative sign. This means that if H(z) is stable transfer function with all its poles inside the unit circle, the Parseval's identity of (2) will result in a positive value for

$$\Sigma_{n=0}^{\infty} h^2(n)$$

In the second case of all the poles of H(z) lying outside the unit circle the same but negative vlaue will result on integration for

$$\Sigma_{n=0}^{\infty} h^2(n)$$

We know that in the second case the 1-D filter is unstable, vide, Theorem 1. If some of the poles of H(z) are outside the unit circle and some are inside the unit circle, we may get positive value or negative value for

$$\Sigma_{n=0}^{\infty} h^2(n)$$

If it is negative we can definitely say the filter is unstable. In case it is positive we cannot take that the filter is stable. Further investigation has to be done to determine about the stability of H(z) using conditions (12). In this paper we review the methods given by Hwang[4,5] for the evaluation of Parseval's integerals (complex integrals) in the 1-D and 2-D cases in section II. We deal with the stability testing of 1-D recursive digital filter in section III. In section IV we deal with stability testing of 2-D filters, restricting ourselves to the filters of order 2. We conclude in section V.

REVIEW OF HWANG'S METHODS

In this section, we briefly review the methods suggested by Hwang[4,5] to find out the complex integrals giving variance both in 1-D and 2-D cases. He assumes, since he is basically interested in evaluating either

$$\Sigma_{n=0}^{\infty} h^2(n) \text{ or } \Sigma_{m=0}^{\infty} \Sigma_{n=0}^{\infty} h^2(m,n)$$

for stable filters, that the given transfer functions are stable meaning that Theorem 1 is satisfied in 1-D case and in the 2-D case if the transfer function $H(z_1, z_2)$ is defined as

$$H(z_1,z_2) = \frac{\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}\, z_1^{-i}\, z_2^{-j}}{\sum_{i=0}^{M}\sum_{j=0}^{N} b_{ij}\, z_1^{-i}\, z_2^{-j}} = \frac{A(z_1,z_2)}{B(z_1,z_2)} \tag{4}$$

then ,

$$B(z_1,z_2) \neq 0 \qquad |z_1| \geq 1,\ |z_2| \geq 1 \tag{5}$$

assuming that there are no nonessential singularities of the second kind[6].

In the 1-D case Hwang uses the Laurent series expansion and shows that if

$$H(z) = \frac{A(z)}{B(z)},\ \Sigma_{n=0}^{\infty} h^2(n) = r_0$$

where r_0 is obtained by using the decomposition

$$H(z)H(z^{-1}) = \frac{A(z)A(z^{-1})}{B(z)B(z^{-1})} = \frac{P(z)}{B(z)} + \frac{P(z^{-1})}{B(z^{-1})} \tag{6}$$

and dividing P(z) by B(z) once to get $r_0/2$. It should be noted the degree of P(z) has to be necessarily chosen the same as that of B(z). In the case of 2-D recursive filters, the decomposition of the form (6) is not possible as shown in[7]. So Hwang[5] has given the following method to obtain

$$\Sigma_{m=0}^{\infty} \Sigma_{n=0}^{\infty} h^2(m,n).$$

Let

$$H(z_1,z_2) = \frac{\sum_{i=0}^{M}\sum_{j=0}^{N} a_{ij}\, z_1^{-i}\, z_2^{-j}}{\sum_{i=0}^{M}\sum_{j=0}^{N} b_{ij}\, z_1^{-i}\, z_2^{-j}} = \frac{A(z_1,z_2)}{B(z_1,z_2)} \tag{7}$$

Then,

$$H(z_1,z_2)\ H(z_1^{-1},z_2^{-1}) = \frac{A(z_1,z_2)}{B(z_1,z_2)}\ \frac{A(z_1^{-1},z_2^{-1})}{B(z_1^{-1},z_2^{-1})}$$

$$= \frac{p_1(z_1)z_2^{N-1} + \ldots + p_N(z_1)}{b_0(z_1)z_2^{N} + \ldots + b_N(z_1)} + \frac{q_0(z_1^{-1})z_2^{-N} + \ldots + q_N(z_1^{-1})}{b_0(z_1^{-1})z_2^{-N} \ldots + b_N(z_1^{-1})} \tag{8}$$

From (8), equating the like coefficients, a matrix equation is obtained, where the vector matrix

$$[\, q_0(z_1^{-1}),\, q_1(z_1^{-1}) \ldots\ldots q_N(z_1^{-1}),\, p_1(z_1),\, p_2(z_1), \ldots\ p_N(z_1)]^T$$

is an unknown. The solution of the matrix equation gives

$$q_0(z_1^{-1})$$

Then it has been shown that

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = \frac{1}{2\pi j}\oint_{|z_1|=1} \frac{q_0(z_1^{-1})}{b_0(z_1^{-1})}\frac{dz_1}{z_1} \qquad (9)$$

So far nobody has given a simple method to solve the matrix equation for higher order 2-D recursive filters. Also to evaluate the 1-D integral (9), there is no simple method except to use the residue method.

STABILITY TESTING OF 1-D FILTERS

In this section we propose a method, using the complex integral evaluation, to test the stability of 1-D recursive digital filters. We give a few examples to show that our method always works.

Since we are interested in the stability of the transfer function H(z), for simplicity we assume that *H(z)* is of the form

$$H(z) = \frac{1}{b_0 z^N + b_1 z^{N-1} + \ldots\ldots b_N} = \frac{1}{B(z)}$$

with $b_0 > 0$ We then use the decomposition of the form given in (6) which results in a matrix equation.

$$\left\{\begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_N \\ 0 & b_0 & b_1 & \cdots & b_{N-1} \\ 0 & 0 & b_0 & \cdots & b_{N-2} \\ \ddots & \ddots & \ddots & \ddots & \\ 0 & 0 & 0 & \cdots & b_0 \end{bmatrix} + \begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_N \\ b_1 & \cdots & b_{N-1} & b_N & 0 \\ b_2 & \cdots & b_N & 0 & 0 \\ \therefore & \therefore & \therefore & \therefore & \\ b_N & \cdots & 0 & 0 & 0 \end{bmatrix}\right\} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (10)$$

It has to be noted that to get (10), we don't have to assume the H(z) is a stable transfer function. Or in otherwords polynomial need not satisfy the Theorem 1. It is because even if H(z) is not a stable transfer function it can be expanded by longhand division like

$$H(z) = \sum_{i=0}^{\infty} h_i z^{-i}$$

and

$$H(z)\ \ H(z^{-1}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_i h_j z^{-i} z^j \qquad (11)$$

The constant part of the RHS will give us

$$\sum_{i=0}^{\infty} h_i^2 = \sum_{n=0}^{\infty} h^2(n)$$

So the decomposition of the type (6) resulting in the matrix equation of the form (10) can always be used to evaluate

$$\sum_{i=0}^{\infty} h_i^2$$

as done in[4] like

$$\sum_{n=0}^{\infty} h^2(n) = 2.\frac{p_0}{b_0} = 2.\frac{r_0}{2} = r_0 .$$

In this we have three cases as discussed below.

Case (i) : All the poles of H(z) or zeros of B(z) are inside the unit circle $|z| = 1$.

In this case H(z) is a stable transfer function and the r_0 we get will be finite and positive as discussed in[4].

Example 1: Let

$$H(z) = \frac{1}{z^6 - 0.4z^5 - 0.93z^4 + 0.412z^3 + 0.1172z^2 - 0.0494z + 0.0035}$$

It has been tested that all the poles of H(z) are inside the unit circle. The value of

$r_0 = 4.3368$, which gives $\sum_{n=0}^{\infty} h^2(n)$.

So we simply find out r_0 and if it is positive, we test for the conditions

i. $B(1) > 0$
ii. $(-1)^N B(-1) > 0$ (12)

and if these conditions are satisfied we conclude that the given transfer function is stable. For this example it has been found that $B(1) = 0.1533$ and $(-1)^6\ B(-1) = 0.2281$ and so the filter is stable. Conditions (12) are the same as the ones in Jury-Marden test.

Case (ii) : All the poles are outside the unit circle $|z| = 1$.

Example 2: Consider now the following transfer function

$$H(z) = \frac{1}{z^3 - z^2 - 14z + 24}.$$

The r_0 has been found to be equal to $r_0 = -0.0026$.
As discussed in Section II, this is a case where all the poles are outside the unit circle and so the transfer function *H(z)* is an unstable one. So we get a negative value for

$$\Sigma_{n=0}^{\infty} h^2(n)$$

Case (iii): Some poles are inside the unit circle and some are outside.

Example 3: Let us consider the following transfer function,

$$H(z) = \frac{1}{z^3 + 3.7z^2 - 1.18z + 0.08}$$

The r_0 for this case has been found to be $r_0 = 0.0138$.
Now we apply conditions (12) and see whether they are satisfied or not.

i. $B(1) = 3.60 > 0$
ii. $(-1)^3 B(-1) = -3.96 < 0$

We find that the second condition is not satisfied. So we conclude that the transfer function is an unstable one. In fact the poles of H(z) are

$$z_{p_1} = -4,\ z_{p_2} = 0.2 \text{ and } z_{p_3} = 0.1.$$

Example 4: Consider the transfer function.

$$H(z) = \frac{1}{z^{74} - 1.2z^{73} + 0.5z^{72} - 1.5z^{38} + 1.8z^{37} - 0.75z^{36} + 0.6z^2 - 0.72z + 0.2718}$$

Here $r_0 = -5581.0234$. Since it is negative we can straightaway say that the filter is unstable without even testing for condition (12). Now we state the following theorem.

Theorem 2: A 1-D transfer function

$$H(z) = \frac{A(z)}{B(z)}$$

of order N is BIBO stable if

i. $\sum_{n=0}^{\infty} h^2(n) > 0$ and finite
ii. $B(1) > 0$
iii. $(-1)^N B(-1) > 0$.

Thus in this section we have given a method (may be alternative and simpler than Jury-Marden algorithm) to test the stability of 1-D recursive digital filters.

STABILITY TEST FOR 2-D DIGITAL FILTERS

In this section using the method suggested by Hwang[5] to evaluate the complex integral

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = \frac{1}{(2\pi j)^2} \oint_{|z_1|=1} \oint_{|z_2|=1} H(z_1, z_2) H(z_1^{-1}, z_2^{1}) \frac{dz_1}{z_1}\frac{dz_2}{z_2} \quad (13)$$

We arrive at a method for testing the stability of 2-D recursive digital filter having a transfer function $H(z_1, z_2)$.

We assume that $H(z_1, z_2)$ is devoid of nonessential singularities of the second kind. We don't assume that the given transfer function $H(z_1, z_2)$ is a stable one. Inspite of this, we can say the decomposition of the type in (8) is possible and the resulting matrix equation can be solved for $q_0(z_1^{-1})$. We restrict ourselves to second order transfer function $H(z_1, z_2)$ since any higher filter design is done with a cascade of second order transfer functions due to the advantages like less coefficient sensitivity and less quantization error etc. Since we are interested in only testing the stability of $H(z_1, z_2)$ using Parseval's integral, we take $H(z_1, z_2)$ to be of the special form

$$H(z_1, z_2) = \frac{1}{\sum_{i=0}^{N}\sum_{j=0}^{N} b_{ij} z_1^{-i} z_2^{-j}} = \frac{1}{B(z_1, z_2)}$$

Let

$$H(z_1, z_2) = \frac{1}{(jz_1^2 + kz_1 + \ell)z_2^2 + (mz_1^2 + nz_1 + o)z_2 + (pz_1^2 + qz_1 + r)} \quad (14)$$

$$a = jz_1^2 + kz_1 + \ell$$
$$d = mz_1^2 + nz_1 + o$$
$$g = pz_1^2 + qz_1 + r$$
$$a^{-1} = jz_1^{-2} + kz_1^{-1} + \ell$$

$$d^{-1} = mz_1^{-2} + nz_1^{-1} + o$$
$$g^{-1} = pz_1^{-2} + qz_1^{-1} + r$$

Using the Eq. (8), we get a matrix equation.

$$\begin{bmatrix} a & d & g & \vdots & d^{-1} & g^{-1} \\ 0 & a & d & \vdots & g^{-1} & 0 \\ 0 & 0 & a & \vdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ d & g & 0 & \vdots & a^{-1} & d^{-1} \\ g & 0 & 0 & \vdots & 0 & a^{-1} \end{bmatrix} \begin{bmatrix} q_0(z_1^{-1}) \\ q_1(z_1^{-1}) \\ q_2(z_1^{-1}) \\ \cdots \\ p_1(z_1) \\ p_2(z_2) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

We solve (13) for $q_0(z_1^{-1})$ and write

$$\frac{q_0(z_1^{-1})}{b_0(z_1^{-1})} = \frac{(aa^{-1} - gg^{-1})}{aa^{-1}(aa^{-1} - gg^{-1}) - d(-dg^{-1}a^{-1} + ad^{-1}a^{-1}) + g(-g^{-1}dd^{-1} + ad^{-2} - aa^{-1}g^{-1} + gg^{-2})} \quad (16)$$

Then

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^2(m, n) = \frac{1}{2\pi j} \oint_{|z|=1} \frac{q_0(z_1^{-1})}{b_0(z_1^{-1})} \frac{dz_1}{z_1} \quad (17)$$

We can use Matlab for both evaluating the RHS (16) and then finding

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^2(m, n)$$

by the residue method. Well it should be noted here that to use the residue m ethod to evaluate the integral in (17), we have to find out all the poles. A Matlab program has been written for both evaluating the RHS of Eq. (16) and then finding the 1-D integral (17) to finally arrive at the value of

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^2(m,n)$$

As discussed in Section III, the 1-D integral of (17) should give always a positive value if

$$z_1 = e^{jw_1}$$
$$z_2 = e^{jw_2}$$

are substituted in the Parseval's integral given in (13) since

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^2(m,n)$$

is always positive. It should also give a finite value if $B(z_1, z_2) \neq 0$ on $|z_1| = 1$ and $|z_2| = 1$.

In case the given transfer function is unstable and is not infinite on the unit bicircle, the 1-D integral value in (17) will naturally have to be negative since it cannot give infinite value when we use residue method to evaluate the 1-D integral (17).

Thus we conclude that if the 1-D integral gives a finite positive value the given $H(z_1, z_2)$ is stable and it gives a negative value it must be considered that the 2-D transfer function is unstable.

Thus we have the following theorem on 2-D recursive fitter stability.

Theorem 3: The 2-D transfer function $H(z_1, z_2)$ given by

$$H(z_1, z_2) = \frac{A(z_1, z_2)}{B(z_1, z_2)}$$

is BIBO stable if

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^2(m,n)$$

is finite and positive, where $h(m,n)$ is the inverse z-transform of $H(z_1, z_2)$.

We now give a few of examples to illustrate the above theorem.

Example 5: Let

$$H(z_1, z_2) = \frac{1}{(0.5z_1 + 0.2)z_2 + (0.5z_1 + 1)}$$

The Matlab program gives the value for

$$q_0(z_1^{-1})$$

and then

$$\frac{q_0(z_1^{-1})}{b_0(z_1^{-1})}$$

is given by

$$\frac{q_0(z_1^{-1})}{b_0(z_1^{-1})z_1} = \frac{-2.5}{(z_1 + 1.8633)(z_1 + 0.5367)}$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = \frac{1}{2\pi j}\oint_{|z_1|=1} \frac{-0.25\ dz_1}{(z_1+1.8633)(z_1+0.5367)}$$
$$= \frac{-2.5}{13266}$$

So the filter is unstable.

Example 6: Consider

$$H(z_1,z_2) = \frac{1}{(z_1-0.7)z_2 + (0.3-0.5z_1)}$$

This is the same transfer function tested in[5].

This gives for

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = 2.981424002$$

Since the variance is positive, the given transfer function is stable.

Example 7: Let [1, p-129, b_3]

$$H(z_1,z_2) = \frac{1}{z_1^2z_2^2 - 1.2z_1^2z_2 + 0.5z_1^2 - 1.5z_1z_2^2 + 1.8z_1z_2 - 0.75z_1 + 0.6z_2^2 - 0.72z_2 + 0.2718}$$

This transfer function was found to be barely unstable. Let us see what we get for the variance by the method suggested in this section. We identify,

$$a = z_1^2 - 1.5z_1 + 0.6$$
$$d = -1.2z_1^2 + 1.8z_1 - 0.72$$
$$g = 0.5z_1^2 - 0.75z_1 + 0.2718$$

a^{-1}, d^{-1} and g^{-1} a re obtained by replacing z_1 by z_1^{-1} in a, d and g, respectively. We used a Matlab program to evaluate

$$\frac{q_0(z_1^{-1})}{b_0(z_1^{-1})}$$

of Eq. (16) and we obtained

$$\frac{q_0(z_1^{-1})}{b_0(z_1^{-1})} = \frac{0.464z_1^2 - 1.821z_1 + 2.7237 - 1.821z_1^{-1} + 0.464z_1^{-2}}{0.0737z_1^4 - 0.5805z_1^3 + 2.048z_1^2 - 4.0903z_1 + 5.106 - 4.0903z_1^{-1} + 2.048z_1^{-2} - 0.5805z_1^{-3} + 0.0737z_1^{-4}}$$

and by using the residue method (Again using a Matlab program) we obtained the value of

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n)$$

by evaluating the integral

$$\frac{1}{2\pi j}\oint_{|z_1|=1} \frac{q_0(z_1^{-1})}{b_0(z_1^{-1})}\frac{dz_1}{z_1}$$

as

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = -4.1152$$

Since the variance is negative, we conclude that the filter is unstable.

Example 8: Consider [1, p-124]

$$H(z_1,z_2) = \frac{1}{z_1^2z_2^2 - 1.2z_1^2z_2 + 0.5z_1^2 - 1.5z_1z_2^2 + 1.8z_1z_2 - 0.75z_1 + 0.6z_2^2 - 0.72z_2 + 0.29}$$

This transfer function gave by residue method

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = 0.5125$$

so the filer is stable.

Example 9: Consider [1, p-129,b_2]

$$H(z_1,z_2) = \frac{1}{z_1^2z_2^2 - 0.75z_1^2z_2 + 0.9z_1^2 - 1.5z_1z_2^2 - 1.2z_1z_2 1.3z_1 + 1.2z_2^2 + 0.9z_2 + 0.5}$$

The variance for this example has been found to be

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h^2(m,n) = -0.07236$$

so the filer is unstable.

CONCLUSIONS

In this paper alternative perhaps slightly simpler methods are given for testing the BIBO stability of 1-D and 2-D recursive digital filters. The methods basically follow the approaches suggested by Hwang[4,5] to evaluate the Parseval's integral which will give the variance. Probably, the method suggested for 1-D filters is slightly more efficient than Jury-Marden table method. In the case of 2-D filters though several methods do exist, both mapping and numerical, they are either less accurate or cumbersome even for second order filters. The method given in this paper for second order filters is perhaps made simple because we have derived using Hwang's work[5] a readily usable expression for the 1-D integrand.

Since second order digital filters form the building blocks for the design of higher order filters in the cascade connection which reduces coefficient sensitivity and quantization error the results of this papers gain importance. The technique used here for the 2-D filters is very tedious for higher order filters like the existing methods.

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