Asian Journal of Information Technology 5(6): 573-577, 2006

A Scheme of Combined Quad/Triangle Subdivision Surface

Ahsan Ullah and Koichi Harada
Graduate School of Engineering, Hiroshima University, Japan

Abstract: Subdivision has become a staple of the geometric modeling community allowing coarse, polygonal shapes to represent highly refined, smooth shapes with guaranteed continuity properties. Unlike regular surface spines, such as NURBS, subdivision surface can handle shapes of arbitrary topology in a unified framework which is important in designing aesthetically pleasing shapes. This study presents a scheme of combined quad/triangle subdivision surface that has many important features resulting it's superiority over some other existing methods like individual triangle or individual quadrilateral subdivision schemes. In general, the scheme produces nicer surface for combined quad/triangle meshes.

Key words: Surface subdivision, linear subdivision, catmull-clark method, loop method and object modeling

INTRODUCTION

Usually when an artist using a 3D modeling package wants to create a perfectly smooth surface, they must use a NURBS (non-rational uniform b-spline surface) patch. This is a grid of control points joined by smooth curves in three dimensions that approximates a smooth surface. While this works, the main problem with these patches is that if two are to be placed next to each other, the number of vertices along their width and their height has to be equal on both patches; otherwise discontinuities may become visible in the surface of the final mesh. This gives the artist another thing to worry about during creation time. It also limits the artist to creating something that is composed entirely of quadrilaterals, as opposed to using a strictly triangular based mesh.

Subdivision surfaces can be designed so that they will act on any arbitrary mesh. This way, an artist does not have to worry about what size the patches on their mesh are. It also allows the artist to no longer worry about how the mesh is constructed: i.e. whether it is composed entirely of quadrilaterals, triangles, or any n-sided polygon. There are schemes that will break arbitrary polygons into triangles, quadrilaterals, or into any other arbitrary polygon.

Subdivision surfaces were introduced in 1978 by both Catmull-Clark[1] and Doo-Sabin[2] methods. They both generalized tensor product B-Splines of bi-degree three and two respectively to arbitrary topologies by extending the refinement rules to irregular parts of the control mesh. Later in 1987 Loop generalized triangular Box splines of total degree four to arbitrary triangular meshes[3].

The visual quality of a subdivision surface depends in a crucial way on the initial or base mesh of control vertices. For general shapes designers often want to model certain region with triangle patches and others with quad patches.

Both Catmull-Clark and Loop surfaces require that all patches be quadrilateral or triangular, respectively. However Catmull-Clark surfaces behave very poorly on triangle-only base meshes. The resulting surface exhibits annoying undulating artifacts. Similarly Loop schemes do not perform well on quad-only meshes. It is often desirable to have surfaces that have a combined quad/triangle patch structure.

Designers often want the added flexibility of having both quads and triangles in there model. It is also well known that triangle meshes generate poor limit surface when using a quad scheme, while quad-only meshes behave poorly with triangular schemes. This study uses the triangular and quadrilateral subdivision scheme. The scheme is a generalization of the well known Catmull-Clark and Loop subdivision algorithms.

Subdivision scheme for mixed triangle/quad meshes are second curvature continuity everywhere except for isolated, extraordinary points where this is first curvature continuity. The rules used are the same as Stam/Loops[4] scheme except that an unzippering pass prior to subdivision is performed. This single modification improves the smoothness along the ordinary triangle/quad boundary from first continuity to second continuity and creates a scheme capable of subdivision arbit meshes. With a proof base on Lavin/Lavins[5] joint spectral radius calculation to show our scheme is indeed second continuity along the triangle/quad boundary. However the rules that produce surfaces with bounded curvature at the regular quad/ triangle boundary are provided and the optimal masks that minimize the curvature divergence elsewhere are also

Corresponding Author: Ahsan Ullah, Graduate School of Engineering, Hiroshima University, Japan

given. The visual quality of our surface with several examples is demonstrated.

SURFACE SUBDIVISIONS

A surface is generally represented as a list of vertices {x, y, z} and a list of faces where every face is a list of indices into the vertex list. This kind of indexed data structure is common in graphics as this facilitates the rendering of polygons and explicitly separates the topology from the geometric positions of the vertices. Subdivision surfaces are polygon mesh surfaces generated from a base mesh through an iterative process that smoothes the mesh while increasing its density. Complex smooth surfaces can be derived in a reasonably predictable way from relatively simple meshes.

Quad subdivision: For quad subdivision, we start with quadrilateral subdivision because this method is the most similar to the curve method. For the case of curve subdivision, two steps namely linear subdivision and averaging are used.

In order to perform the linear subdivision on a polygonal face, we use Catmull-Clark splitting on each of the face in the mesh. As for the first step we insert new vertices at the midpoints of each edge of the face and one new vertex at the centroid of the face. Then, with the view to form m quads from the m-sided polygon, we connect the required vertices as shown in Fig. 1.

After the linear subdivision is performed, one round of averaging on the mesh is to be performed. This operation on quadrilateral meshes is analogous to the averaging operation for curves. For each vertex, we place that vertex at the average of the centroids of all quads containing that vertex. Fig. 2(a) shows the centroid calculation for each quad and Fig. 2(b) shows the composite rule formed by averaging the centroids together.

We can implement this averaging as single pass over the list of faces. Before the pass, we initialize each entry of a Table of new vertex positions to have value {0, 0, 0}. Next, for each quad q, we compute the centroid of q and add this centroid's position to the four entries in this Table indexed by the vertices of q.

After processing all of the faces in the mesh, we divide each entry in the Table by the valence of the vertex associated with the entry. (This valence information can also be computed during the centroid calculations.) Note that dividing by the valence forces the coefficients of the associated averaging rule (shown in Fig. 2 (b)) to sum to one and makes the resulting subdivision scheme affinely invariant.

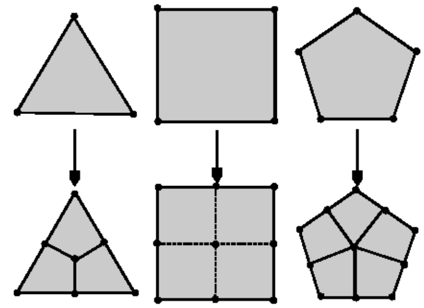

Fig. 1: Linear subdivision of polygonal faces for quad subdivision schemes. After one round, all faces are quads

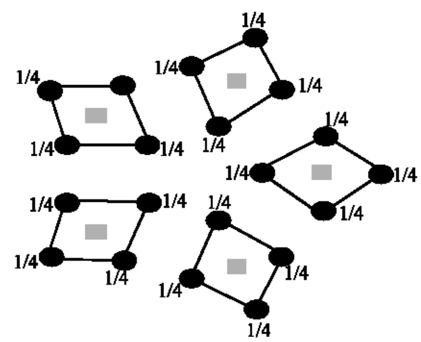

Fig. 2(a): Computation of centroids with squares denothing the position of the centroid

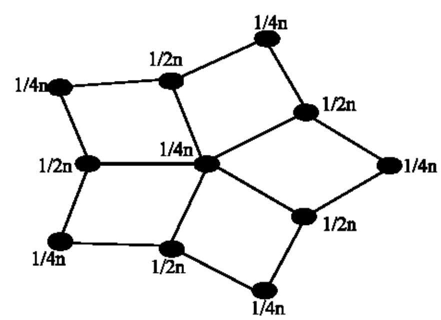

Fig. 2(b): Averaging the centroids together generates the composite averaging rule at an arbitrary valence vertex

Figure 3 (left) illustrates an example surface produced by subdividing a cube using this subdivision scheme. The surfaces produced by this method are C^2 everywhere except at extraordinary vertices where the surface is only C^1. Though the surface is smooth everywhere, the shading of the surface varies rapidly near the valence three vertices of the cube. These discontinuities are due to the fact that the surface normals do not vary smoothly in these regions. (Technically, the surface is strictly C^1)

Triangle subdivision: Many surfaces encountered in real world are not composed of quads. Instead, the huge

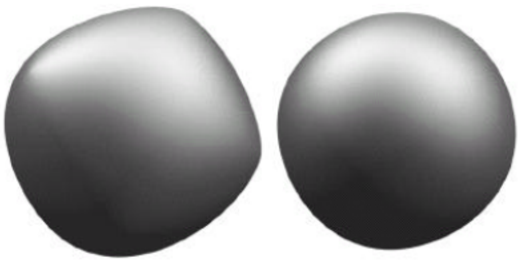

Fig. 3: Subdivision of a cube. Uncorrected averaging(left). Catmull-clark subdivision (right)

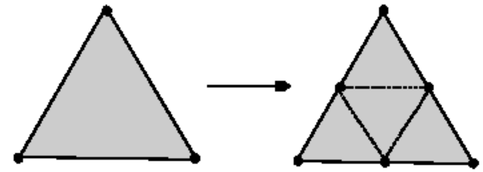

Fig. 4: Linear subdivision of triangles for triangular subdivision schemes

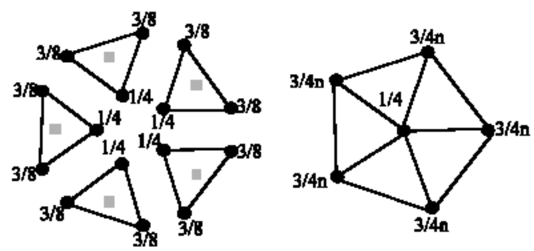

Fig. 5: Centroid calculation for triangles (left). Repositioning the vertex at the average of these centroids generates the triangle averaging rule (right)

number of surfaces is composed of triangles as the base modeling primitive. The quadrilateral subdivision is not at all fit for these surfaces. So we should have an alternative and that is the triangular subdivision scheme. Loop subdivision is considered as a very popular subdivision scheme for triangular meshes. Loop's method can also be represented in terms of linear subdivision and averaging scheme similar to that for quad meshes. Unlike the quadrilateral subdivision method, the triangular subdivision scheme only processes surfaces composed entirely of triangles. It is noteworthy that this requirement is very simple to fulfill as all faces in the mesh can be triangulated.

To perform linear subdivision on triangles, we insert new vertices on the edge of each polygon using the hash Table technique. Each triangle is then split into four triangles as shown in Fig. 4. Notice that all new vertices will have valence six in the mesh. Since triangular subdivision produces surfaces with valence six vertices almost everywhere, valence six vertices are ordinary while other valence vertices are extraordinary vertices.

Averaging for triangular surfaces is similar to quadrilateral surfaces. For each vertex in the mesh, we place the vertex at the average of the centroids of all polygons containing that vertex. However, we use a weighted centroid for triangular surfaces shown in Fig. 5. The centroid takes 1/4 of the vertex being repositioned plus 3/8 of the two neighboring vertices. Notice that while the centroid calculation for quads is uniform (1/4 of all vertices), the centroid calculation for triangles is not uniform and depends upon which vertex the centroid is being accumulated into.

COMBINED QUAD/TRIANGLE SUBDIVISION

The developed method subdivides surfaces composed of nearly all quadrilateral polygons or completely of triangles. However, this separation of subdivision schemes between the two mostly commonly used modeling primitives is unnecessary. Some surfaces, such as cylinders/tori, are naturally parameterized by quads while other surfaces are more conveniently parameterized by triangles. To remedy this problem, Stam and Loop[4] presented a subdivision scheme that unified these two methods (quads and triangles) into one subdivision scheme that produces Catmull-Clark subdivision for all quadrilateral surfaces, Loop subdivision for all triangular surfaces and generates smooth surfaces when both quads and triangles are present in the surface. Here a variant of Stam and Loop's method except recast the scheme in terms of a generalized averaging pass has been applied. The mixed quad/triangle subdivision scheme produces Catmull-Clark subdivision on all quadrilateral surfaces, a variant of Loop subdivision on all triangular surfaces and smooth surfaces when the model contains both quads and triangles.

Once again the scheme has been formulated as linear subdivision and averaging. During linear subdivision, we split all quadrilaterals as done for Catmull-Clark subdivision (Fig. 1) and all triangles as in Loop subdivision (Fig. 4).

Averaging precedes as before with centroids for quads computed as the average of the four vertices and for triangles as in Fig. 5 (left). However, each centroid is weighted by the angular contribution of that polygon in the ordinary case. For instance, the ordinary case for quad

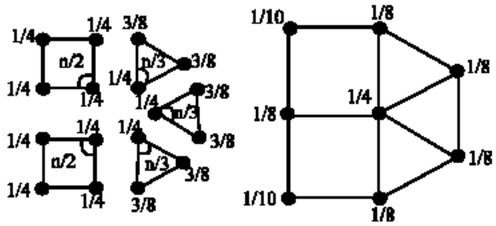

Fig. 6: Quad/Triangle subdivision is performed using the centroids from quad and triangle subdivision weighted by their angular contribution. The resulting subdivision rule along a ordinary quad/triangle boundary

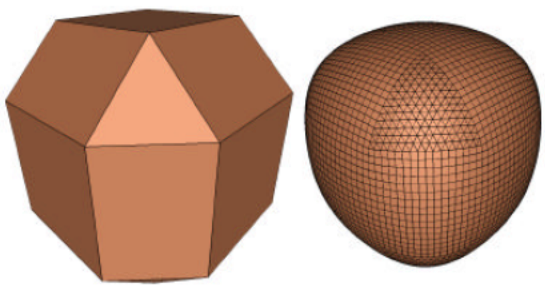

Fig. 7: Subdivision of a combined quad/triangle surface

subdivision is four quads containing a vertex so the weight is $2\pi/4=\pi/2$. Likewise, for triangular subdivision there are six triangles containing a vertex in the ordinary case so the weight for triangles is $2\pi/6=\pi/3$. Finally, we normalize by the sum of the weights of the polygons containing each vertex.

In the case of vertices contained by only quads or only triangles, this method produces the same results as the uncorrected quad and triangle averaging methods respectively. Notice that at the boundary where a triangle and a quad meet, linear subdivision will generate the polygonal structure shown in Figure 6 (right) all along the edge.

In Stam and Loop's study on quad/triangle subdivision, the authors define the polygonal configuration in Figure 6 to be an ordinary boundary between the two surfaces since that structure is replicated along the entire interface between quads and triangles (Fig. 7). The averaging rules (applied after linear subdivision) chosen by Stam and Loop are also shown on the right of Fig. 6.

They analyzed the smoothness of the surface at this edge and showed that the surface is C^1 across the edge. From this ordinary boundary, they generalized their subdivision scheme to vertices containing an arbitrary number of quads and triangles. The subdivision scheme that is presented here for quads and triangles differs in the rules used at extraordinary vertices; however, the rules used here reproduce the subdivision rules of Stam and Loop's method along the ordinary boundary and, therefore, share the same smoothness results along that edge.

To generate smooth surfaces everywhere, the correction term as shown below is presented.

$$w(n_t,n_q) = \begin{cases} 1.5 & n_q=0,\ n_t=3 \\ 12/(3n_q+2n_t) & \text{otherwise} \end{cases}$$

where n_q/n_t is the number of quads/triangles containing the vertex. This correction generates Catmull-Clark surfaces with all quadrilateral models and a variant of Loop surfaces with models composed completely of triangles. However, the combined correction term $12/(3n_q+2n_t)$ is not smooth at vertices contained by only three triangles. Hence, we use a piecewise function for $w(n_t,n_q)$ that uses the correction value for Loop subdivision at this valence to generate a smooth surface. Stam and Loop also provided a correction term Table in their study that was generated by an optimization method in an attempt to produce surfaces of bounded curvature at low valence vertices. Interestingly, the polynomial correction term used here is a surprisingly good approximation of that correction Table even though the rules differ slightly at extraordinary vertices. Instead of performing own optimization, the provided correction term is used for simplicity. Fig. 7 illustrates an example surface composed of quads and triangles subdivided several times using combined quad/triangle method.

CONCLUSION

The surface subdivision scheme that allows both quadrilaterals and triangles has been presented in this study. This is in contrast to other subdivision schemes which generate meshes which are either all triangles or all quadrilaterals. Artists often want to keep both triangles and quadrilaterals in their models. Also it is well known that triangles in a base mesh create artifacts in the refined meshes when a quad-based subdivision scheme is used. Both triangles and quadrilaterals are simultaneously incorporated in the subdivision scheme because the scheme is based on a decomposition of the rules into a linear step followed by smoothing.

REFERENCES

1. Catmull, E. and J. Clark, 1978. Recursively generated B-spline surfaces on arbitrary topological meshes. Computer-Aided Design, 10: 350-355.

2. DOO, D. and M. Sabin, 1978. Behaviour of recursive division surface near extraordinary points. Computer-Aided Design, pp: 356-360
3. Loop, C., 1987. Smooth Subdivision Surface Based on Triangles. Master's thesis, Department of Mathematics, University of Utah, USA.
4. Stam, J. and C. Loop, 2003. Quad/Triangle Subdivision, Computer Graphics Forum 22, 1: 1-7.
5. Levin, A., D. Levin, 2003. Analysis of quasi uniform subdivision. Applid and computational Harmonic Analysis 15: 18-32.