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Switching Controller Design for Nonlinear Systems via Fuzzy Models

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Abstract: This study presents a Lyapunov based switching controller design method for non linear systems using Takagi-Sugeno fuzzy models. The basic idea of the proposed approach is to represent the fuzzy model as a set of uncertain linear systems. The switching controller consists of local controllers obtained by solving the corresponding set of Algebraic Ricatti Equations (AREs). A stabilizable switching strategy is proposed to guarantee the global stability. A simulation example is given to illustrate the effectiveness of this study.

Key words: Switching control, fuzzy systems, stability covering condition, uncertain system

INTRODUCTION

- In recent years, Fuzzy Logic Control (FLC) has attracted considerable attention from scientists and engineers. FLC design methods are generally based on a fuzzy model composed by a set of *if-then* rules. Fuzzy modeling is an efficient method to represent complex non-linear systems by fuzzy sets and fuzzy reasoning. By using a Takagi-Sugeno fuzzy model, a non linear system can be expressed as a weighted sum of simple subsystems. This model gives a fixed structure to some non linear systems and thus facilitates their analysis. There are two ways to obtain the fuzzy model:
- by applying identification methods with input-output data from the plant,
- or directly from the mathematical model of the non linear plant[1-3]. Recently, there have been appeared a number of systematic stability analysis and controller design results in fuzzy control literature. Tanaka *et al.* discussed the stability and the design of fuzzy control systems in[4]. They gave some checking conditions for stability, which can be used to design fuzzy control laws. Unfortunately, the stability conditions require the existence of a common positive definite matrix for all the local linear models. However, this is a difficult problem to be solved in many cases, especially when the number of rules is large. Representation of fuzzy models by a set of linear uncertain systems has been suggested by Cao *et al.*[5], based on linear uncertain system theory several control design approaches has been proposed. The drawback of the precedent approaches is that the LMIs or the algebraic Riccati equations used to check the stability can be infeasible. Based on the representation of Cao *et al.*[5-7] we propose, in this study, a switching control design approach. The proposed approach is based on the resolution of a set of independent algebraic Ricatti equation. The fulfillment degree of each rule is incorporated in the algebraic Riccati equation to overcome the problem of infeasibility. A minimization program is used to compute the minimal rule degree for which the local algebraic Ricatti equation has a solution. If the stability covering condition is fulfilled than the global stability can be assured by using a suitable switching strategy. One of the advantages of this approach is that we can minimize the number of the local controllers.

Takagi-sugeno fuzzy model: Many physical systems are very complex in practice so that rigorous mathematical models can be very difficult to obtain, if not impossible. However, many of these systems can be expressed in some form of mathematical models. Takagi-Sugeno fuzzy models have been largely used to model complex non

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linear systems[1]. The continuous-time Takagi-Sugeno fuzzy dynamic model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by a set of fuzzy if-then rules. The ith rule of the fuzzy model takes the form:

Rule I:

$$\text{If } z_1(t) \text{ is } F_1^i, \cdots, \text{and } z_g(t) \text{ is } F_g^i \text{ Then} \quad \dot{x}(t) = A_i x(t) + B_i u(t) \qquad (1)$$

where x (t) ∈ R^n denotes the state vector, u (t) ∈ R^m the control vector, y (t) ∈ R^p the output vector, F^i_j is the j^{th} fuzzy set of the i^{th} rule, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are the state matrix and the input matrix for the i^{th} local model, r is the number of if-then rules and z_1 (t), z_2 (t),..., z_g (t) are some measurable system variables. The final output of the fuzzy model can be expressed as:

$$\dot{x}(t) = \sum_{i=1}^{r} \alpha_i(z(t))\{A_i x(t) + B_i u(t)\} \qquad (2)$$

where

$$\alpha_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r} \omega_i(z(t))} \qquad (3)$$

and

$$\omega_i(z(t)) = \prod_{j=1}^{g} F_j^i(z(t)) \qquad (4)$$

F^i_j is the grade of membership of z (t) in F^i_j.
The scalars α_i (z (t)) are characterized by:

$$0 \le \alpha_i(z(t)) \le 1, \qquad \sum_{i=1}^{r} \alpha_i(z(t)) = 1 \qquad (5)$$

The T-S fuzzy model (2) has strong non-linear interactions among its fuzzy rules which complicate the analysis and the control. In order to overcome these difficulties, the TS fuzzy model can be represented as a set of uncertain linear systems[5]. The global state space $\Omega \subseteq R^n$ is partitioned into r subspaces, each subspace is defined as:

$$\Omega_l = \{\Omega | \alpha_l(z(t)) > 0\} \qquad (6)$$

Each subspace Ω_l is the union of two subsets:

$$\Omega_l = \overline{\Omega_l} \cup \partial\Omega_l \qquad (7)$$

where

$$\overline{\Omega_l} = \{\Omega | \alpha_l(z(t)) = 1\} \qquad (8)$$

and

$$\partial\Omega_l = \{\Omega | \, 0 < \alpha_l(z(t)) < 1\} \qquad (9)$$

These subspaces are characterized by:

$$\bigcup_{i=1}^{r} \Omega_i = \Omega \qquad (10)$$

If the rules i and j can be inferred in the same time then:

$$\Omega_i \cap \Omega_j \ne \phi \qquad (11)$$

If the rules i and j can't be inferred in the same time then:

$$\Omega_i \cap \Omega_j = \phi \qquad (12)$$

In each subspace the TS fuzzy model (2) can be represented as:

$$\dot{x}(t) = \left[A_l + \sum_{R_i \in \Re_l} \alpha_i(z(t)) A_{il}\right] x(t) + \left[B_l + \sum_{R_i \in \Re_l} \alpha_i(z(t)) B_{il}\right] \qquad (13)$$

Where

$$A_{il} = A_i - A_l, \quad B_{il} = B_i - B_l \qquad (14)$$

and $\Re_l$ is a rule subset containing rules that can be inferred in the same time as the rule R_l. Since

$$\sum_{R_i \in \Re_l} \alpha_i(z(t)) = 1 - \alpha_l(z(t)) \qquad (15)$$

The TS fuzzy model can be written as:

$$\dot{x}(t) = \left[A_l + (1-\alpha_l(z(t)))\Delta A_l(z(t))\right]x(t) + \left[B_l + (1-\alpha_l(z(t))\Delta B_l(z(t))\right]u(t) \quad (16)$$

Where

$$\Delta A_l\left(z(t)\right) = \sum_{R_i \in \Re_l} \alpha_i'(z(t))(A_i - A_l)$$
$$\Delta B_l\left(z(t)\right) = \sum_{R_i \in \Re_l} \alpha_i'(z(t))(B_i - B_l) \quad (17)$$

and

$$\alpha_i'\left(z(t)\right) = \frac{\alpha_i\left(z(t)\right)}{1-\alpha_l\left(z(t)\right)} \quad (18)$$

If α_l (z (t)) = 1 then the fuzzy system can be represented by the corresponding linear local model. In each subspace, the fuzzy model consists of a dominant nominal system (A_l, B_l) and a set of interacting systems representing the effect of other active rules.

In this study we suppose that the state vector is measurable.. The fuzzy system can be represented by:

$$\dot{x}(t) = \tilde{A}_l\left(\alpha\left(z(t)\right)\right)x(t) + \tilde{B}_l\left(\alpha\left(z(t)\right)\right)u(t) \quad (19)$$

with

$$\tilde{A}_l\left(\alpha\left(z(t)\right)\right) = A_l + \left(1-\alpha_l\left(z(t)\right)\right)\Delta A_l\left(\alpha\left(z(t)\right)\right)$$
$$\tilde{B}_l\left(\alpha\left(z(t)\right)\right) = B_l + \left(1-\alpha_l\left(z(t)\right)\right)\Delta B_l\left(\alpha\left(z(t)\right)\right) \quad (20)$$

We assume that the matrices ΔA_l (α' (z (t)) and ΔB_l (α' (z (t)), l = 1,..., r are bounded and their bounds are known *a priori*.

$$\Delta A_l(\alpha'(z(t))) \cdot \left[\Delta A_l(\alpha'(z(t))\right]^T \le \Delta\overline{A}_l \cdot \Delta\overline{A}_l^T \quad (21)$$

$$\Delta B_l(\alpha'(z(t))) \cdot \left[\Delta B_l(\alpha'(z(t))\right]^T \le \Delta\overline{B}_l \cdot \Delta\overline{B}_l^T \quad (22)$$

Those bounds can be computed by:

$$\Delta\overline{A}_l = \max_{R_i \in \Re_l} A_i - A_l \quad (23)$$

$$\Delta\overline{B}_l = \max_{R_i \in \Re_l} B_i - B_l \quad (24)$$

CONTROLLER DESIGN

We assume that the fuzzy system (1) is locally controllable, that is, the pairs (A_l, B_l) I = 1,2,...,r, are controllable. The basic idea is to design local feedback controllers that maximize the stability region of each closed loop local model. The switching controller, represented in Fig. 1 consists of r_c linear state feedback controllers linear state feedback controllers that will be switched from one to another to control the system. The number of controllers may be different from the number of rules. The switching controller can be described by:

$$u(t) = \sum_{l=1}^{r_c} \zeta_l\left(z(t)\right)K_l x(t) \quad (25)$$

with:

$$K_l = -\rho_l B_l^T P_l \quad (26)$$

and

$$\sum_{l=1}^{r_c} \zeta_l\left(z(t)\right) = 1, \qquad \zeta_l\left(z(t)\right) \in \{0,1\} \quad (27)$$

K_l is the local state feedback gain in subspace Ω_l^c to be designed. It can be seen that (25)-(27) is a linear combination of r_c linear state feedback controllers. At each moment, only one of the linear state feedback controllers is chosen to generate the control signal.

Theorem 1: If there exist symmetric positive definite matrix $Q_l \in R^{n \times n}$ and positive scalars $\rho_l > 0$, $\mu^A_l > 0$, $\mu^B_l > 0$ and $0 \le \underline{\alpha}_l < 1$ such that the following algebraic Ricatti Eq.

$$A_l^T P_l + P_l A_l - P_l S_l P_l + T_l = 0 \quad (28)$$

has a solution Pl = P^T_1 where:

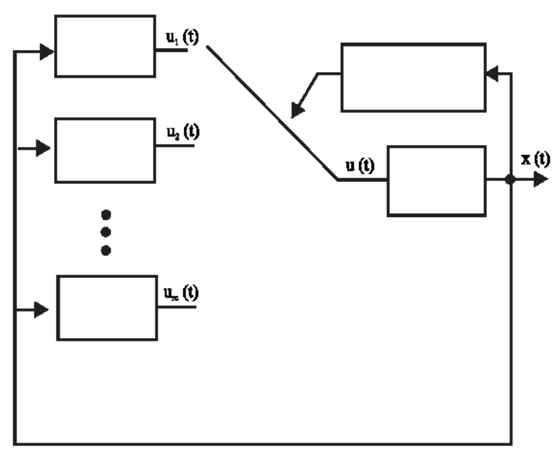

Fig. 1: Structure of the switching controller

$$S_1 = \rho_1\left[2-(1-\underline{\alpha}_1)\mu_1^B\right]B_1B_1^T + (1-\underline{\alpha}_1)\left(\mu_1^A I_n + \frac{\rho_1}{\mu_1^B}\Delta\bar{B}\Delta\bar{B}^T\right) \tag{29}$$

$$T_1 = Q_1 + \frac{1-\underline{\alpha}_1}{\mu_1^A}\Delta\bar{A}\Delta\bar{A}^T \tag{30}$$

then the state feedback control law :

$$u(t) = -\rho_1 B_1^T P_1 x(t) \tag{31}$$

quadratically stabilizes the fuzzy system (19) for $\alpha_1(z(t))$ such that:

$$\alpha_1(z(t)) \geq \underline{\alpha}_1 \tag{32}$$

Proof: Consider the following Lyapunov function candidate:

$$V_1(t) = x^T(t)P_1x(t) \tag{33}$$

where P_1 is a symmetric positive definite matrix. The time derivative of V (t) along the trajectory of the fuzzy system is given by:

$$\begin{aligned}\dot{V}(t) &= \dot{x}^T(t)P_1x(t) + x^T(t)P_1\dot{x}(t)\\ &= x^T(t)\tilde{A}_1^T(\alpha(z(t)))P_1x(t) + x^T(t)P_1\tilde{A}_1(\alpha(z(t)))x(t)\\ &+ u^T(t)\tilde{B}_1^T(\alpha(z(t)))P_1x(t) + x^T(t)P_1\tilde{B}_1(\alpha(z(t)))u(t)\end{aligned}$$

For simplicity of notation $\alpha(z(t))$ and t will be omitted from matrix and function expressions.

$$\begin{aligned}\dot{V}(t) &= x^T\tilde{A}_1^TP_1x + x^TP_1\tilde{A}_1x + u^T\tilde{B}_1^TP_1x + x^TP_1\tilde{B}_1u\\ &= x^T\left[A_1 + (1-\alpha_1)\Delta A_1\right]^T P_1x\\ &+ x^TP_1\left[A_1 + (1-\alpha_1)\Delta A_1\right]x\\ &+ u^T\left[B_1 + (1-\alpha_1)\Delta B_1\right]^T P_1x\\ &+ x^TP_1\left[B_1 + (1-\alpha_1)\Delta B_1\right]u\\ &= x^T\left[A_1^TP_1 + P_1A_1 + (1-\alpha_1)\left(\Delta A_1^TP_1 + P_1\Delta A_1\right)\right]x\\ &+ x^T\left[K_1^TB_1^TP_1 + P_1B_1K_1\right.\\ &\left.+ (1-\alpha_1)\left(K_1^T\Delta B_1^TP_1 + P_1\Delta B_1K_1\right)\right]x\end{aligned}$$

$$\begin{aligned}\dot{V}(t) &= x^T\left[A_1^TP_1 + P_1A_1 + K_1^TB_1^TP_1 + P_1B_1K_1\right]x\\ &+ (1-\alpha_1)x^T\left[\Delta A_1^TP_1 + P_1\Delta A_1 + K_1^T\Delta B_1^TP_1 + P_1\Delta B_1K_1\right]x\end{aligned}$$

$$\begin{aligned}\dot{V}(t) &= x^T\left[A_1^TP_1 + P_1A_1 - 2\rho_1P_1B_1B_1^TP_1\right]x\\ &+ (1-\alpha_1)x^T\left[\Delta A_1^TP_1 + P_1\Delta A_1 + K_1^T\Delta B_1^TP_1 + P_1\Delta B_1K_1\right]x\\ &- \rho_1(1-\alpha_1)x^TP_1\left[B_1\Delta B_1^T + \Delta B_1B_1^T\right]P_1x\end{aligned}$$

Since for any positive scalar μ_1 and real matrices Y and Z we have[8]:

$$ZY^T + YZ^T \leq \mu YY^T + \frac{1}{\mu}ZZ^T \tag{34}$$

It follows that:

$$\Delta A_1^TP_1 + P_1\Delta A_1 \leq \mu_1^AP_1P_1 + \frac{1}{\mu_1^A}\Delta A_1^T\Delta A_1$$

$$B_1\Delta B_1^T + \Delta B_1B_1^T \leq \mu_1^BB_1B_1^T + \frac{1}{\mu_1^2}\Delta B_1\Delta B_1^T$$

$$\Delta A_1^TP_1 + P_1\Delta A_1 \leq \mu_1^AP_1P_1 + \frac{1}{\mu_1^A}\Delta\bar{A}_1^T\Delta\bar{A}_1$$

$$B_1\Delta B_1^T + \Delta B_1B_1^T \leq \mu_1^BB_1B_1^T + \frac{1}{\mu_1^2}\Delta\bar{B}_1^T\Delta\bar{B}_1$$

$$\begin{aligned}\dot{V}(t) &\leq x^T\left[AP_1 + P_1A_1 - 2\rho_1P_1B_1B_1^TP_1\right]x\\ &+ (1-\alpha_1)x^T\left[\mu_1^AP_1P_1 + \frac{1}{\mu_1^A}\Delta\bar{A}_1^T\Delta\bar{A}_1\right]x\\ &+ \rho_1(1-\alpha_1)x^TP_1\left[\mu_1^BB_1B_1^T + \frac{1}{\mu_1^B}\Delta\bar{B}_1^T\Delta\bar{B}_1\right]P_1x\end{aligned}$$

Since:

$$\alpha_1 \geq \underline{\alpha}_1 \quad \Rightarrow \quad 1-\alpha_1 \leq 1-\underline{\alpha}_1$$

$$\begin{aligned}\dot{V}(t) &\leq x^T\left[AP_1 + P_1A_1 - 2\rho_1P_1B_1B_1^TP_1\right]x\\ &+ (1-\underline{\alpha}_1)x^T\left[\mu_1^AP_1P_1 + \frac{1}{\mu_1^A}\Delta\bar{A}_1^T\Delta\bar{A}_1\right.\\ &\left.+ \rho_1P\left(\mu_1^BB_1B_1^T + \frac{1}{\mu_1^B}\Delta\bar{B}_1^T\Delta\bar{B}_1\right)P_1\right]x\end{aligned}$$

$$\begin{aligned}\dot{V}(t) &\leq x^T\left[AP_1 + P_1A_1 + \frac{1-\underline{\alpha}_1}{\mu_1^A}\Delta\bar{A}_1^T\Delta\bar{A}_1\right]x\\ &- x^TP_1\left[\rho_1\left(2-(1-\underline{\alpha}_1)\right)\mu_1^BB_1B_1^T\right.\\ &\left.-(1-\underline{\alpha}_1)\left(\mu_1^AI_n + \frac{\rho_1}{\mu_1^B}\Delta\bar{B}_1^T\Delta\bar{B}_1\right)\right]P_1x\end{aligned}$$

$$\dot{V}(t) \le x^T \left(AP_l + P_l A_l - P_l S_l P_l + T_l - Q_l \right) x$$

Since:

$$AP_l + P_l A_l - P_l S_l P_l + T_l = 0$$

It yields

$$\dot{V}(t) \le -\lambda_{\min}(Q_l) \| x(t) \|^2 < 0 \tag{35}$$

In each subspace, the command is given by:

$$u(t) = -\rho_l B_l^T P_l x(t) \tag{36}$$

In order to maximize the region of stability of each subregion Ω^s_l, the minimal value hat guarantee the stability is obtained by solving the following minimization program:

$$\begin{aligned} &\underset{P_l, Q_l, R_l, \mu_l^1, \mu_l^2}{\text{Minimize}} \alpha_l \\ &\text{Suject to } \; P_l = P_l^T > 0, Q_l > 0, \rho_l > 0, \mu_l^A > 0, \mu_l^B > 0 \\ &A_l^T P_l + P_l A_l - P_l S_l P_l + T_l = 0 \end{aligned} \tag{37}$$

Note that this minimization program has always a solution $\underline{\alpha}_l < 1$ since we assume that the local systems are controllable.

Definition 1: We say that the stability covering condition is verified if:

$$\bigcup_{i=1}^{r} \Omega_l^s = \Omega \tag{38}$$

Lemma 1: The stability covering condition is verified if there exists, at each moment t, at least one integer k ϵ {1,2,...,r} such that:

$$\alpha_k(z(t)) \ge \underline{\alpha}_k \tag{39}$$

Proof:

$$\forall t \; \exists k \left| \alpha_k(z(t)) \ge \underline{\alpha}_k \right. \Leftrightarrow \forall t \; \exists \Omega_K^S \left| x(t) \in \Omega_k^s \right.$$

$$\forall t \; \exists \Omega_K^S \left| x(t) \in \Omega_k^s \right. \Rightarrow \bigcup_{i=1}^{r} \Omega_i^s = \Omega$$

The resolution of the r independent minimization programs (37) leads to three possible cases as shown in Fig. 2:

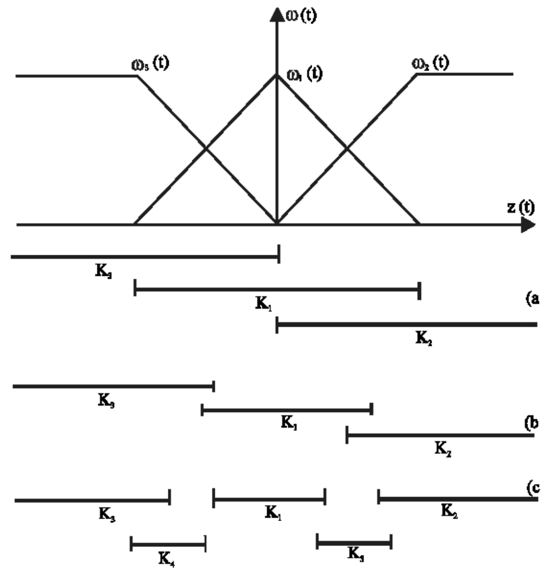

Fig. 2: Possible casas

Case 1: Several or all $\underline{\alpha}_l = 0$, l = 1,2,...,r, Fig. 2a, a local controller can be used to stabilize the fuzzy system in its own local sub-region and in adjacent sub-regions. The number of controllers can be reduced. The number of controllers is inferior to the number of rules. In Fig. 2a, the state feedback gain K_1 is sufficient to control the fuzzy system.

Case 2: If the number of controllers can't be reduced and the condition (38) is fulfilled then the number of controllers is equal to the number of rules, Fig. 2b.

Case 3: If the condition (38) is not fulfilled, the global system may be instable. To solve this problem, we can add new rules to the model since we know exactly in which region, in the state space, we need new ones. Or we can add new controllers, K_4 and K_5 in Fig. 2c, without changing the model by using new nominal local systems, which is equivalent to the addition of new rules to the model.

Let $\Omega^c_l \subseteq \Omega^s_l$ the state subspace associated with the state feedback K_l and τ_i, I = 1,2,...,N the i[ith] time instant at which the state meets the boundary of a sub-region Ω^c_l, j = 1,2,...,r. We assume that the state x (t) does not jump at the transition time τ_i, that is[7]

$$x(\tau_i^-) = x(\tau_i) = x(\tau_i^+), \qquad i = 1,2,\ldots,N \tag{40}$$

Lemma 2: The fuzzy system (19) is globally stable if the transition time instants are finite (N < ∞) and the stability covering condition is verified.

Proof: Consider the following piecewise quadratic Lyapunov function candidate:

$$V(t) = \sum_{l=1}^{r} \zeta_l(z(t)) x^T(t) P_l x(t) \qquad (41)$$

where:

$$\zeta_l(x(t)) = \begin{cases} 1 & x(t) \in \Omega_l^c \\ 0 & \text{otherwize} \end{cases} \qquad (42)$$

if τ_i is the time instant at which the state leaves the subregion Ω^c_j and enters in the subregion Ω^c_k then:

$$V(\tau_i^-) = x^T(\tau_i^-) P_j x(\tau_i^-) = x^T(\tau) P_j x(\tau) \qquad (43)$$

$$V(\tau_i^+) = x^T(\tau_i^+) P_k x(\tau_i^+) = x^T(\tau) P_k x(\tau) \qquad (44)$$

The local symmetric positive matrices P_l, $l = 1,2,...,r$, are determined so as to guarantee the local stability:

$$(35) \Rightarrow \frac{\dot{V}(t)}{V(t)} \le -\frac{x^T(t)(Q_l)x(t)}{x^T(t)P_l x(t)} \le -\sigma_l,\ \sigma_l = \frac{\lambda_{min}(Q_l)}{\lambda_{max}(P_l)}$$
$$x(t) \in \Omega_l^c,\quad \tau_i^+ < t < \tau_{i+1}^-,\quad i = 1,2,\dots,N$$

$$V(t) > 0,\quad x(t) \ne 0 \quad \Rightarrow \quad \frac{d(\ln(V(t)))}{dt} \le -\sigma_l$$

$$V(t) \le V(\tau_i^+) e^{-\sigma_l(t-\tau_i^+)},\quad \tau_i^+ < t < \tau_{i+1}^-,\quad i = 1,2,\dots,N$$

Since:

$$\lambda_{min}(P_l)\|x(t)\|^2 \le V(t) \le \lambda_{max}(P_l)\|x(t)\|^2,\quad \tau_i^+ < t < \tau_{i+1}^-,$$
$$i = 1,2,\dots,N$$

It follows that:

$$\|x(t)\| \le C_l \|x(\tau_i)\| e^{-\frac{\sigma_l}{2}(t-\tau_i^+)},\quad \tau_i^+ < t < \tau_i^-,\quad i = 1,2,\dots,N$$
$$C_l = \frac{\sqrt{\lambda_{max}(P_l)}}{\sqrt{\lambda_{min}(P_l)}}$$

Since the number of transition is finite, $N < \infty$ then:

$$\|x(t)\| \le C_{l_0} \|x(\tau_N)\| e^{-\frac{\sigma_{l_0}}{2}(t-\tau_N^+)},\quad t > \tau_N^+$$

At the N^{th} transition ($t = \tau^+_N$) the state enters into the sub region $\Omega^c_{l_0}$ containing the origin and converges to the origin at $t \to \infty$.

$$x(t) \in \Omega_{l_0}^c,\quad t > \tau_N^+ \qquad \|x(t)\|_{t\to\infty} \to 0$$

The fuzzy system is globally stable.

Switching strategy: In order to guarantee the global stability we need to impose restrictions on switching. A stabilizable switching for a particular controller is to let switching the switching occurs on a point in state-space whose Euclidean norm is less than Euclidean norm of the point-state space when the same controller was used last time[9]. The following lemma gives the switching strategy assuring global stability.

Lemma 3: If the stability covering condition is verified and the switching states are chosen such that:

$$\|x(\tau_{i+k})\| < \|x(\tau_i)\| \qquad (45)$$

where τ_i is the time instant in which the states enters into the sub region Ω^c_l and τ_{i+k} is the next time instant in which the states enters into the same sub region Ω^c_l.

Proof: Consider the piecewise quadratic Lyapunov function candidate given by (41)-(42).

At $t = \tau_i$, the state enters into the sub region Ω^c_l, we have:

$$\|x(t)\| \le C_l \|x(\tau_i)\| e^{-\frac{\sigma_l}{2}(t-\tau_i^+)},\quad \tau_i^+ < t < \tau_i^-,\quad i = 1,2,\dots,N$$
$$C_l = \frac{\sqrt{\lambda_{max}(P_l)}}{\sqrt{\lambda_{min}(P_l)}}$$

The state leaves the sub region Ω^c_l at $t = \tau_{i+k}$ with:

$$\|x(\tau_{i+1})\| \le C_l \|x(\tau_i)\| e^{-\frac{\sigma_l}{2}(\tau_{i+1}^- - \tau_i^+)} < C_l \|x(\tau_i)\|$$

If the state enters into the sub region Ω^c_l a second time at $t = \tau_{i+k}$ and leaves this region at $t = \tau_{i+k+1}$, we have:

$$\|x(\tau_{i+k+1})\| < C_l \|x(\tau_{i+k})\|$$

If the condition (43) is verified than:

$$\|x(\tau_{i+k})\| < \|x(\tau_i)\| \Rightarrow \|x(\tau_{i+k+1})\| < C_l \|x(\tau_{i+k})\| < C_l \|x(\tau_i)\|$$

If we assume that the transition time instants are finite ($N < \infty$) then the fuzzy system (19) is globally stable, otherwise if the condition (45) is verified we have:

$$\frac{\|x(t)\|}{C_1} < \cdots < \|x(\tau_{i+k})\| < \|x(\tau_i)\| < \cdots < \|x(\tau_{i-j})\| < \cdots < \|x(0)\|$$
$$\Rightarrow \|x(t)\| \underset{t\to\infty}{\to} 0$$

and the fuzzy system (19) is globally stable.

Simulation example: To show the effectiveness of the proposed method, we simulate the control of the chaotic Lorenz system. The control objective is to drive its chaotic trajectory to the origin. The Lorenz equations are as follows[10]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma x_1(t) + \sigma x_2(t) \\ r x_1(t) - x_2(t) - x_1(t)x_3(t) \\ x_1(t)x_2(t) - b x_3(t) \end{bmatrix} \quad (46)$$

The nominal values of (σ, r, b) are (10,28,8/3) for chaos to emerge. An exact fuzzy modeling is employed to construct fuzzy models for chaotic systems. It utilizes the concept of sector nonlinearity[1]. Assume that $x_1(t) \in [-d, d]$ then we can have the following fuzzy model which exactly represents the non-linear Eq. under $x_1(t) \in [-d, d]$.

- R^1 : if $x_1(t)$ is about $-d$ Then $\dot{x}(t) = A_1 x(t)$
- R^2 : if $x_1(t)$ is about d Then $\dot{x}(t) = A_2 x(t)$

Where

$$A_1 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & d \\ 0 & -d & -b \end{bmatrix}, \; A_2 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -d \\ 0 & d & -b \end{bmatrix} \quad (47)$$

and

$$d = 30 \quad (48)$$

The membership functions, shown in Fig. 3, are chosen as:

$$\omega_1(x(t)) = \begin{cases} \frac{-x_1(t)}{2d} + \frac{1}{2} & \text{if } -d \le x_1(t) \le d \\ 1.0 & \text{if } x_1(t) < -d \\ 0 & \text{if } x_1(t) > d \end{cases} \quad (49)$$

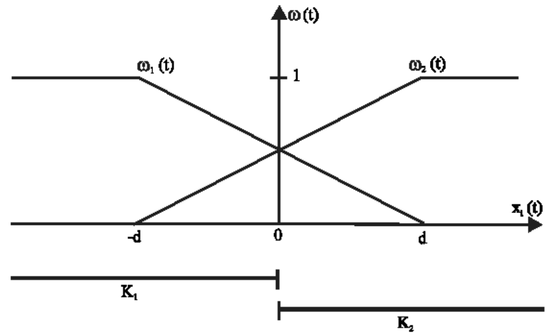

Fig. 3: Membership functions

$$\omega_2(x(t)) = \begin{cases} \frac{x_1(t)}{2d} - \frac{1}{2} & \text{if } -d \le x_1(t) \le d \\ 1.0 & \text{if } x_1(t) > d \\ 0 & \text{if } x_1(t) < -d \end{cases} \quad (50)$$

The input matrices B_1 and B_2 are chosen as:

$$B_1 = B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (51)$$

The fuzzy model can be decomposed into two subsystems:

- Subsystem 1 :

$$\dot{x}(t) = [A_1 + (1-\alpha_1)\Delta A_1]x(t) + [B_1 + (1-\alpha_1)\Delta B_1]u(t)$$

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 30 \\ 0 & -30 & -2.6667 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta A_1 = \alpha_2'(t)(A_2 - A_1) = \alpha_2'(t)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 60 \\ 0 & -60 & 0 \end{bmatrix}$$

$$\Delta B_1 = 0$$

- Subsystem 2 :

$$\dot{x}(t) = \left[A_2 + (1-\alpha_2)\Delta A_2\right]x(t) + \left[B_2 + (1-\alpha_2)\Delta B_2\right]u(t)$$

$$A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & -2.6667 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Delta A_2 = \alpha_1'(t)(A_1 - A_2) = \alpha_1'(t)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -60 \\ 0 & 60 & 0 \end{bmatrix},$$

$$\Delta B_2 = 0$$

The bounds $\Delta\bar{A}_1$ and $\Delta\bar{A}^2$ are:

$$\Delta\overline{A}_1 = A_2 - A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 60 \\ 0 & -60 & 0 \end{bmatrix}$$

$$\Delta\overline{A}_2 = A_1 - A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -60 \\ 0 & 60 & 0 \end{bmatrix}$$

and

$$\Delta\overline{B}_1 = \Delta\overline{B}_2 = 0$$

The values obtained after the resolution of the minimization programs (37):

- Subsystem 1:

Pour $Q_1 = 0.5I_3, \quad \mu_1^1 = 2, \quad \rho_1 = 5$

$$\underline{\alpha}_1 = 0$$

$$P_1 = \begin{bmatrix} 2.3710 & 2.8392 & 0.5717 \\ 2.8392 & 14.8472 & 0.0069 \\ 0.5717 & 0.0069 & 14.6598 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -11.8548 & -14.1958 & -2.8587 \\ -14.1958 & -74.2360 & 0.0344 \\ -2.8587 & 0.0344 & -73.2992 \end{bmatrix}$$

- Subsystem 2:

$$Q_2 = 0.5I_3, \quad \mu_2^1 = 2, \quad \rho_2 = 10$$

$$\underline{\alpha}_2 = 0$$

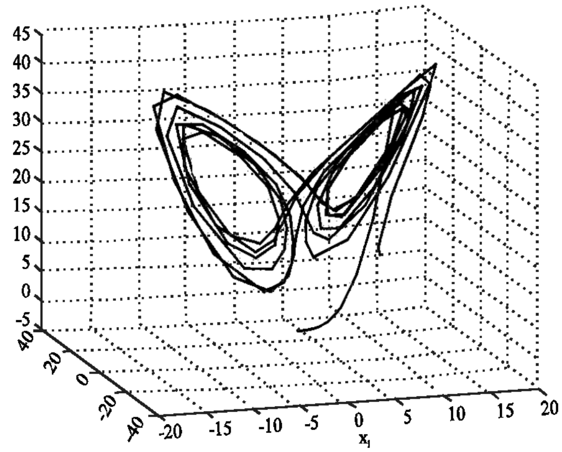

Fig. 4: The phase trajectory of the controlled lorenz system

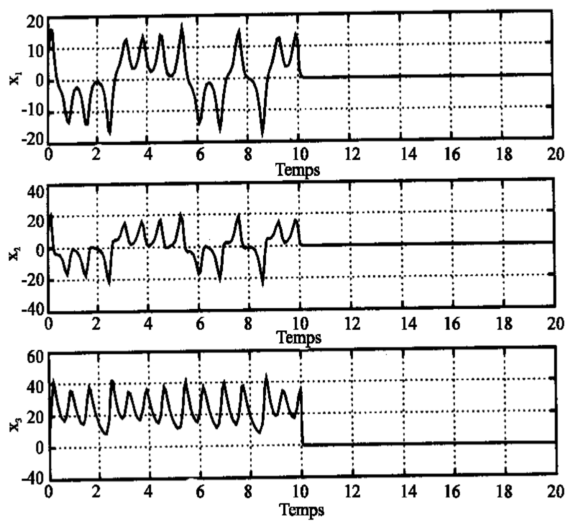

Fig. 5: Controlled lorenz system states

$$P_2 = \begin{bmatrix} 1.0808 & 1.3530 & -0.1938 \\ 1.3530 & 9.9298 & 0.0013 \\ -0.1938 & 0.0013 & 9.8523 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -10.8078 & -13.5300 & 1.9376 \\ -13.5300 & -99.2982 & -0.0126 \\ 1.9376 & -0.0126 & -98.5225 \end{bmatrix}$$

The initial values of states are $x(0) = [10\ 10\ 10]^T$. The simulation time is 20s. The control input is activated at t = 10s. Before the activation of the control the phase trajectory of the Lorenz system was chaotic. However, after the activation of the control the phase trajectory is quickly directed to the origin as shown in Fig. 4 and 5. In this example the boundary of the two sub-spaces are determined by $\underline{\alpha}_1$ $\underline{\alpha}_2$, Fig. 3, which means that the chaotic system can be controlled using only one state feedback u (t) = K_1x (t) or u (t) = K_2x (t).

CONCLUSION

In this study a Lyapunov based method has been proposed to design a fuzzy model based switching controller for non linear systems. The fuzzy model is represented as a set of uncertain linear systems. A local controller is designed such that the stability region of the corresponding local subsystem is maximized. If the stability covering condition is fulfilled and a suitable switching strategy is used than the switching controller has the ability to stabilize the non linear system. The control of the chaotic Lorenz system has been used demonstrate the effectiveness of this approach.

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