Asian Journal of Information Technology 5 (9): 933-937, 2006
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Numerical Solution of Mechanical Vibrating Systems Using Cellular Neural Networks

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Abstract: In this study, Cellular Neural Network (CNN's) is applied to compute the transient response of mechanical vibrating systems along with popular numerical integration algorithms. The result of RK-Butcher algorithm is compared with Euler, RK-Gill algorithms and with analytic solutions of the mechanical vibrating system. Stability analysis for the RK-Butcher algorithm has been discussed. Error graphs for the mechanical vibrating systems are presented in a graphical form to show the efficiency of this RK-Butcher algorithm.

Key words: Mechanical vibrating, integration

INTRODUCTION

This study is concerned with mechanical vibrating systems and the solution involves either the solution of Ordinary Differential Eq. (ODE) (systems of mass points) or the solutions of Partial Differential equation (PDE's).

While the first can directly be mapped onto the CNN architecture and the latter a spatial discretization with numerical integration must be carried out in advance, to reduce the PDE's to a system of ODE's. The spatial numerical integration rule is crucial for preserving certain qualitative properties of the physical system to be simulated. Such problems with appropriate boundary and initial conditions occur in a number of engineering areas, including transient stress and deformation analysis, fluid flow or thermal conduction. For example, the basic elliptic equation of the linear elasticity is the Lame Eq.[1]

$$\operatorname{div}\left(\operatorname{grad}\underline{u}\right) + k\operatorname{grad}\left(\operatorname{div}\underline{u}\right) = \rho / G\left(\partial^2 \underline{u} / \partial t^2\right) \qquad (1)$$

where $\underline{u}$ (x, t) denotes the displacement field which is a function of space ($\underline{x}$) and time ($\underline{t}$) (k, ρ and G are material parameters; $\underline{u}$ and $\underline{x} \in R^3$). The finite element method discretizes a structure into a set of structural components with a specific pattern, called finite element[2]. The elements are interconnected with the adjacent elements by nodal points. The number of geometrical points in the discretized system is N and f denotes the number of degrees of freedom (the number of possible displacement directions) per node.The finite element method via spatial discretization converts the governing Eq. 1 into a set of $n = N^f$ ordinary differential Eq:

$$M\ddot{\underline{U}} + D\dot{\underline{U}} + K\underline{U} = \underline{F} \qquad (2)$$

with initial conditions $\underline{U}(0) = \underline{U}_0$ and $\dot{\underline{U}}(0) = \dot{\underline{U}}_0$

$\underline{U}$ (t) is the displacement vector of the discrete nodes. The displacement of the i th node is defined by three consecutive elements of $\underline{U}$ representing the displacement in x, y and z direction. In (2) M, K and D are the mass, stiffness and the damping matrices respectively and F is the external load or force vector. M, K and D are symmetric matrices of size . It follows from the boundary conditions that certain elements of $\underline{U}$ are fixed during the motion. If the mass matrix M is not singular, (2) can be given in the following form

$$\ddot{\underline{U}} = M^{-1}\left(-K\underline{U} - D\dot{\underline{U}} + \underline{F}\right) \text{ or } \ddot{\underline{U}} = \left(-A\underline{U} + C\dot{\underline{U}} + B\underline{F}\right) \qquad (2a)$$

THE CNN PARADIGM AND THE MECHANICAL VIBRATING SYSTEM

Cellular neural networks are locally connected, analog, dynamic, nonlinear processing arrays[3,4]. The processing elements are arranged on a 2D gird (one layer), which can be replicated to have a multilayer CNN. The dynamics of mechanical vibrating

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systems when discretized in space can be described by second order ODEs.

The dynamics of the array can be described by

$$\frac{dv_{xij}(t)}{dt} = -v_{xij}(t) + \sum_{kl\in N_r(ij)} A_{ij;kl}\left(v_{ykl,} v_{yij}\right) + \sum_{kl\in N_r(ij)} B_{ij;kl}(v_{ukl,} v_{uij}) + I_{ij}\ \frac{dv_{yij}}{dt} = -v_{yij} + f\left(v_{xij}\right); \quad (3)$$

In study of a linear mechanical vibrating system with one degree of freedom the following two-coupled layer CNN can be considered:

$$\frac{dv_{xij}(t)}{dt} = -v_{xij}(t) + \sum_{kl\in N_r(ij)} A_{ij;kl} v_{ykl} + \sum_{kl\in N_r(ij)} B_{ij;kl} v_{ukl} \quad (4)$$

$$\frac{dv_{yij}}{dt} = f\left(v_{xij}\right);$$

It is supposed that the linear part of $f(v_{xij})$ is used, i.e., $f(v_{xij}) = v_{xij}$.

The CNN transient response of the discretized Eq. in time can be calculated by different integration algorithms. In this study, we consider the mechanical vibrating system discussed by Szolgay[5]) but presenting a different approach using the numerical integration algorithms discussed by Murugesh and Murugesan[6,7] such as Euler algorithm, RK-Gill algorithm discussed by oliveira[8] and RK-Butcher algorithm discussed by Murugesan[9,10] with more accuracy.

STABILITY ANALYSIS

Consider the test Eq. $\dot{y} = \lambda y$ where λ is a complex constant and it is used to determine the stability region of this method.

$$k_1 = f(y_n) = \lambda y_n$$

$$k_2 = f\left(y_n + \frac{hk_1}{4}\right) = \lambda y_n\left(1 + \frac{h\lambda}{4}\right)$$

$$k_3 = f\left(y_n + \frac{hk_1}{8} + \frac{hk_2}{8}\right) = \lambda y_n\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)$$

$$k_4 = f\left(y_n - \frac{hk_2}{2} + hk_3\right) = \lambda y_n\left(1 - \frac{h\lambda}{2}\left(1 + \frac{h\lambda}{4}\right) + h\lambda\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)\right)$$

$$k_5 = f\left(y_n + \frac{3hk_1}{16} + \frac{9hk_4}{16}\right) = \lambda y_n\left(1 + \frac{3h\lambda}{16} + \frac{9h\lambda}{16}\left(\begin{array}{l}1 - \frac{h\lambda}{2}\left(1 + \frac{h\lambda}{4}\right)\\ + h\lambda\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)\end{array}\right)\right)$$

$$k_6 = f\left(y_n - \frac{3hk_1}{7} + \frac{2hk_2}{7} + \frac{12hk_3}{7} - \frac{12hk_4}{7} + \frac{8hk_5}{7}\right)$$

$$= \lambda y_n\left(\begin{array}{l}1 - \frac{3h\lambda}{7} + \frac{2h\lambda}{7}\left(1 + \frac{h\lambda}{4}\right) + \frac{12h\lambda}{7}\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)\\ -\frac{12h\lambda}{7}\left(1 - \frac{h\lambda}{2}\left(1 + \frac{h\lambda}{4}\right) + h\lambda\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)\right)\\ + \frac{8h\lambda}{7}\left(1 + \frac{3h\lambda}{16} + \frac{9h\lambda}{16}\left(1 - \frac{h\lambda}{2}\left(1 + \frac{h\lambda}{4}\right) + h\lambda\left(1 + \frac{h\lambda}{8} + \frac{h\lambda}{8}\left(1 + \frac{h\lambda}{4}\right)\right)\right)\right)\end{array}\right)$$

substituting $z = h\lambda$ we get

$$k_1 = f(y_n) = \lambda y_n$$
$$k_2 = \lambda y_n\left(1+\frac{z}{4}\right)$$
$$k_3 = \lambda y_n\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)$$
$$k_4 = \lambda y_n\left(1-\frac{z}{2}\left(1+\frac{z}{4}\right)+z\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)\right)$$
$$k_5 = \lambda y_n\left(1+\frac{3z}{16}+\frac{9z}{16}\left(1-\frac{z}{2}\left(1+\frac{z}{4}\right)+z\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)\right)\right)$$
$$k_6 = \lambda y_n\left(\begin{array}{l}1-\frac{3z}{7}+\frac{2z}{7}\left(1+\frac{z}{4}\right)+\frac{12z}{7}\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)-\frac{12z}{7}\left(1-\frac{z}{2}\left(1+\frac{z}{4}\right)+z\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)\right)\\+\frac{8z}{7}\left(1+\frac{3z}{16}+\frac{9z}{16}\left(1-\frac{z}{2}\left(1+\frac{z}{4}\right)+z\left(1+\frac{z}{8}+\frac{z}{8}\left(1+\frac{z}{4}\right)\right)\right)\right)\end{array}\right)$$

Then the 5th order predictor formula is

$$y_{n+1} = y_n + \frac{h}{90}\left(7k_1 + 32k_3 + 12k_4 + 32k_5 + 7k_6\right),$$

substituting the values of k_1, k_2, k_3, k_4, k_5 and k_6 then we obtain

$$y_{n+1} = y_n + \frac{h\lambda y_n}{90}\left(90+\frac{90}{2}z+\frac{30}{2}z^2+\frac{30}{8}z^3+\frac{30}{40}z^4+\frac{30}{240}z^5\right)$$

divide both sides by y_n then the stability polynomial $Q(z) = y_{n+1}/y_n$ is given as

$$Q(z) = 1+z+\frac{z^2}{2!}+\frac{z^3}{3!}+\frac{z^4}{4!}+\frac{z^5}{5!}+\frac{z^6}{6!}$$

A comparative study of the stability regions of the RK-Gill algorithm and the RK-Butcher algorithm is carried out. Stability regions for these two algorithms with $y' = y$, $y(0) = 1$ for the exact solution $y(x) = \exp(x)$ are plotted in Fig. 1 for comparison. Here the stability region range for real part of λ for RK-Gill algorithm is $-3.463 < \text{Re}(z) < 0.0$ where as for RK-Butcher algorithm it is $-2.780 < \text{Re}(z) < 0.0$. From this, it is evident that RK-Butcher algorithm is more stable.

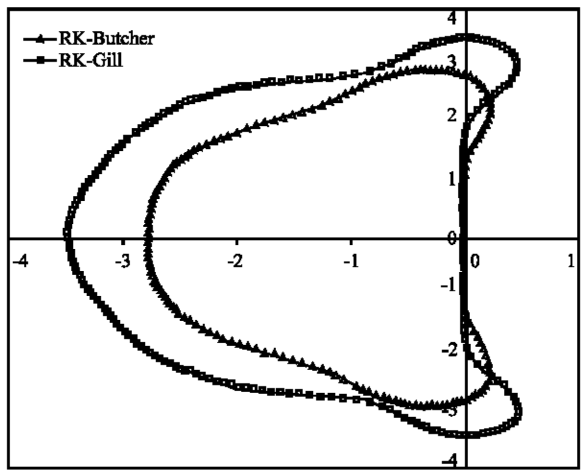

Fig. 1: Stability region for RK-Gill and RK-Butcher algorithm

MECHANICAL VIBRATING SYSTEM

Consider the mechanical vibrating system of five bodies connected by springs. The motion of the system shown in Fig. 2 is represented by the $x_i(t)$ functions which are the solutions of the following set of linear ODE's:

$$\begin{aligned}\ddot{x}_1 &= \frac{-2cx_1}{m}+\frac{cx_2}{m}+\frac{F_1}{m}\\ \ddot{x}_2 &= \frac{cx_1}{m}-\frac{2cx_2}{m}+\frac{cx_3}{m}+\frac{F_2}{m}\\ \ddot{x}_3 &= \frac{cx_2}{m}-\frac{2cx_3}{m}+\frac{cx_4}{m}+\frac{F_3}{m}\\ \ddot{x}_4 &= \frac{cx_3}{m}-\frac{2cx_4}{m}+\frac{cx_5}{m}+\frac{F_4}{m}\\ \ddot{x}_5 &= \frac{cx_4}{m}-\frac{2cx_5}{m}+\frac{F_5}{m}\end{aligned} \tag{5}$$

Where the mass of an element is denoted by m and the parameter of springs is denoted by c. Let us suppose that F=0 and the third element has a unit displacement and all other elements have a zero

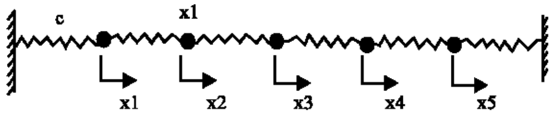

Fig. 2: The homogeneous chain of masses

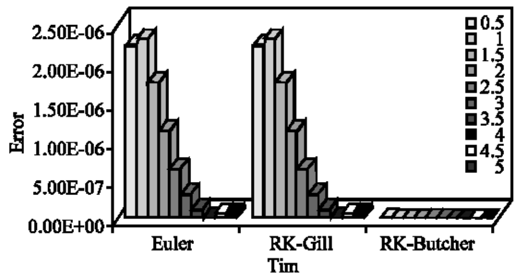

Fig. 3: Error graph of x_1

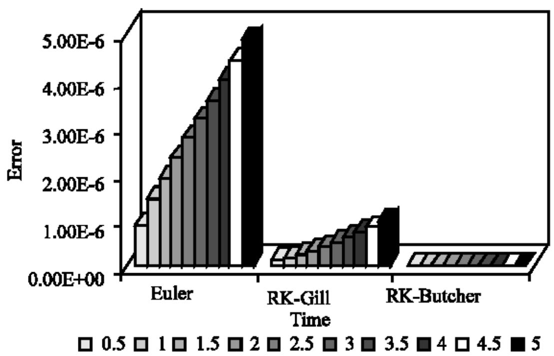

Fig. 4: Error graph of x_2

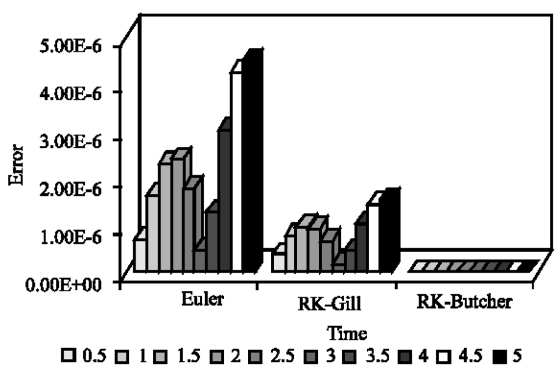

Fig. 5: Error graph of x_3

displacement as an initial condition. The initial speed of the masses is zero. The eigen frequencies of the system obtained from the characteristic Eq. with m=1 and c=1 are as follows: α_1 = 0.51763, α_2 = 1.00, α_3 = 1.41421, α_4 = 1.73205 and α_5 = 1.93185. The resulting motion is composed of five harmonic components of various amplitudes depending on the initial conditions and the exciting forces F (here F=0). In that study the following analytic solution describes the motion of the masses:

$$x_1 = x_5 = \frac{1}{6}\begin{bmatrix}\cos(0.51763t) - 2\cos(1.41421t) \\ +\cos(1.93185t)\end{bmatrix}$$
$$x_2 = x_4 = \frac{1}{(2\sqrt{3})}\left[\cos(0.51763t) - \cos(1.93185t)\right] \quad (6)$$
$$x_3 = \frac{1}{3}\begin{bmatrix}\cos(0.51763t) + \cos(1.4142t) \\ +\cos(1.93185t)\end{bmatrix}$$

The set of Eq. represented in Eq. 5 is solved by CNN using the RK-Butcher algorithm and the results are compared with the solutions obtained by using Euler algorithm, RK-Gill algorithm along with the exact solutions calculated using Eq. 6. An Error graph is presented for the variable x_1, x_2 and x_3 in Fig. 3-5 at various time intervals.

CONCLUSION

It is to be noted that from Fig. 3-5 we can observe that the RK-Butcher algorithm yields very less error (almost no error) when compared to the Euler and RK-Gill algorithms and hence this RK-Butcher algorithm is more suitable for transient response computation of mechanical vibrating system.

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