

## Sliding Mode Control of the Double Fed Asynchronous Machine Applied by Current Sources

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**Abstract:** This study deals with the application of sliding mode control theory to wound rotor induction motor with its rotor fed by current sources in which the system operate in stator field oriented control. After determining the model of the machine, a set of simple surfaces have been applied to a cascade structure and the associated control laws have been synthesised. Furthermore, in order to reduce chattering phenomenon, smooth control functions with appropriate threshold have been chosen. Simulation study based on idealized motor is conducted to show the effectiveness of the proposed method.

**Key words:** Double fed induction motor, vector control, sliding mode control, current sources, machine

### INTRODUCTION

In the area of the control of the electric machines, the research works are oriented more and more towards the application of the modern control techniques. These techniques involve in a vertiginous way with the evolution of the computers and power electronics. This allows to lead to the industrial processes of high performances. These techniques are the fuzzy control, the adaptive control, the sliding mode control etc. The recent interest accorded to the latter is due primarily to the availability of the high frequency commutation switches and of the increasingly powerful microprocessors.

It is regarded as one of the simplest approaches for the control of the nonlinear systems and the systems having an imprecise model.

A considerable attention was concentrated on the control of the uncertain dynamic non-linear systems which are subject to the disturbances and the variations of the external parameters.

The sliding mode control concept consists of moving the state trajectory of the system towards and to maintain it around the sliding surface with the appropriate logic commutation. This latter gives birth to a specific behaviour of the state trajectory in a neighbourhood of the sliding surfaces known as sliding regimes.

In this study, we propose a control scheme to achieve the goal of speed regulation with stator field oriented DFIM drive and sliding mode control (Tang and Xu, 1995; Walczyna, 1991).

**System description and machine modelling:** Using the frequently adopted assumptions, thus by assuming sinusoidally distributed air gap, flux density distribution and linear magnetic conditions, in the referential axis linked to rotating field, the following electrical equations are deduced (Machmoum *et al.*, 1992; Wang and Ding, 1993):

$$\begin{cases} \overline{V_s} = V_s \cdot e^{j\omega t} = R_s \overline{I_s} + j\omega_s \overline{\Psi_s} + \frac{d}{dt} \overline{\Psi_s} \\ \overline{V_r} = V_r \cdot e^{j\omega t} = R_r \overline{I_r} + j\omega_r \overline{\Psi_r} + \frac{d}{dt} \overline{\Psi_r} \\ \overline{\Psi_s} = \Psi_s \cdot e^{j\omega t} = L_s \overline{I_s} + M \overline{I_r} \\ \overline{\Psi_r} = \Psi_r \cdot e^{j\omega t} = M \overline{I_s} + L_r \overline{I_r} \end{cases} \quad (1)$$

The torque is:

$$\Gamma_e = \frac{2}{3} p M L_m \left( \overline{I_s} \cdot e^{j\theta_s} \overline{I_r}^* \cdot e^{-j(\theta_r + \theta)} \right) \quad (2)$$

Knowing that  $\theta_x = \theta_r + \theta$ , by introducing the latter into the (Eq. 1) the torque can be expressed by:

$$\Gamma_e = \frac{2}{3} p M L_m \overline{I_s I_r}^* \quad (3)$$

by taking account of :

$$\begin{aligned} \overline{I_s} &= I_s \cdot e^{j\omega_s t} \\ \overline{I_r} &= I_r \cdot e^{j\omega_r t} \end{aligned}$$

the torque becomes equal to:

$$\Gamma_e = \frac{2}{3} p.M.I_s.I_r.\sin \varphi_i \quad (4)$$

In this model expressed in coordinates of the synchronous reference frame, the parameters are a fonction only of the module and the position of the vector associated with the considered parameter.

The variables which appear in this model are the currents  $(\bar{I}_s, \bar{I}_r)$ , the voltages  $(\bar{V}_s, \bar{V}_r)$ , the flux,  $(\bar{\Psi}_s, \bar{\Psi}_r)$  and pulsations,  $\omega_s, \omega_r, \omega$ .

Current  $\bar{I}_s$  and flux  $\bar{\Psi}_r$  are determinates according to the state variables  $\bar{I}_r$  and  $\bar{\Psi}_s$ :

$$\begin{cases} \bar{I}_s = -\frac{M}{L_s} \bar{I}_r + \frac{\bar{\Psi}_s}{L_s} \\ \bar{\Psi}_r = \sigma L_r \bar{I}_r + \frac{M}{L_s} \bar{\Psi}_s \end{cases} \quad (5)$$

By introducing the expression (5) into the voltage equations of the general mathematical model, one obtains:

$$\begin{cases} \bar{V}_s = -\frac{M}{L_s} R_s \bar{I}_r + \left[ \frac{R_s}{L_s} + j\omega_s \right] \bar{\Psi}_s + \frac{d}{dt} \bar{\Psi}_s \\ \bar{V}_r = (R_r + j\sigma L_r \omega_r) \bar{I}_r + \sigma L_r \frac{d}{dt} \bar{I}_r + \\ + \frac{M}{L_s} \left[ j\omega_r \bar{\Psi}_s + \frac{d}{dt} \bar{\Psi}_s \right] \end{cases} \quad (6)$$

The choice of  $\bar{I}_r$  and  $\bar{\Psi}_s$  as a state variable allows to lead to a simple presentation of the machine.

The decomposition of the state equations for the rotor currents is:

$$\begin{cases} \frac{d\bar{I}_r}{dt} = -\frac{1}{\sigma.T_s'} \bar{I}_r + \omega_r \bar{I}_q + \frac{1-\sigma}{\sigma.M.T_s} \Psi_{sd} + \\ + \frac{1-\sigma}{\sigma.M} \omega \Psi_{sq} + \frac{1}{\sigma.L_r} \bar{V}_r - \frac{1-\sigma}{\sigma.M} \bar{V}_s \\ \frac{d\bar{I}_q}{dt} = -\frac{1}{\sigma.T_s'} \bar{I}_q - \omega_r \bar{I}_d + \frac{1-\sigma}{\sigma.M.T_s} \Psi_{sq} - \\ - \frac{1-\sigma}{\sigma.M} \omega \Psi_{sd} + \frac{1}{\sigma.L_r} \bar{V}_q - \frac{1-\sigma}{\sigma.M} \bar{V}_s \end{cases} \quad (7)$$

With:

$$\frac{1}{T_s'} = \frac{1}{T_r} + \frac{1-\sigma}{T_s}$$

and for the flux by:

$$\begin{cases} \frac{d\Psi_{sd}}{dt} = \frac{M}{T_s} \bar{I}_r - \frac{1}{T_s} \Psi_{sd} + \omega_s \Psi_{sq} + V_{sd} \\ \frac{d\Psi_{sq}}{dt} = \frac{M}{T_s} \bar{I}_q - \frac{1}{T_s} \Psi_{sq} - \omega_s \Psi_{sd} + V_{sq} \end{cases} \quad (8)$$

The torque equation is given by:

$$\Gamma_e = \frac{2}{3} p \frac{M}{L_s} (\Psi_{sd} \bar{I}_q - \Psi_{sq} \bar{I}_d) \quad (9)$$

The position of the chosen reference frame are obtained starting from the following law:

$$\omega_s = \left[ \frac{M}{T_s} \bar{I}_q - \frac{1}{T_s} \Psi_{sq} + V_{sd} - \frac{d}{dt} \Psi_{sq} \right] \cdot \frac{1}{\Psi_{sd}} \quad (10)$$

The equations of the stator flux are determinate from the system (8):

$$\begin{cases} \Psi_{sd} = [M \bar{I}_r + \omega_s T_s \Psi_{sq} + V_{sd} T_s] \cdot \left[ \frac{1}{T_s s + 1} \right] \\ \Psi_{sq} = [M \bar{I}_q - \omega_s T_s \Psi_{sd} + V_{sq} T_s] \cdot \left[ \frac{1}{T_s s + 1} \right] \end{cases} \quad (11)$$

In this study, the stator is supplied by a network of fixed amplitude and frequency voltage and the rotor by a three-phase current source.

This machine is controlled by action on the rotor. The control laws are as follows:

$$\begin{cases} \Psi_{sd} = [M \bar{I}_r + \omega_s T_s \Psi_{sq} + V_{sd} T_s] \cdot \left[ \frac{1}{T_s s + 1} \right] \\ \Psi_{sq} = [M \bar{I}_q - \omega_s T_s \Psi_{sd} + V_{sq} T_s] \cdot \left[ \frac{1}{T_s s + 1} \right] \\ \Gamma_e = \frac{2}{3} p \cdot \frac{M}{L_s} (\Psi_{sd} \bar{I}_q - \Psi_{sq} \bar{I}_d) \\ \Omega = (\Gamma_e - \Gamma_r) \cdot \frac{1}{J_s} \\ \omega_r = \omega_s - \omega \\ \theta_r = \int (\omega_s - \omega) dt \end{cases} \quad (12)$$

Projections of  $\bar{V}_s$  on the axes d and q are given by:

$$\begin{cases} V_{sd} = \frac{3\sqrt{2}}{2} V_s \cos \varphi_{vs} \\ V_{sq} = \frac{3\sqrt{2}}{2} V_s \sin \varphi_{vs} \end{cases} \quad (13)$$

In this research, one considers the orientation of stator flux:

$$\Psi_{sd} = \Psi_{eq} \quad \Psi_{sq} = 0 \quad (14)$$

### SLIDING MODE CONTROL

**General concept:** The variable structure system and their associated sliding regimes are characterised by a discontinuous nature of the control action with which a desired dynamic of the system is obtained by choosing appropriate sliding surfaces. The control actions provide the switching between subsystems which give a desired behavior of the closed loop system (Goh *et al.*, 2004a, b; Utkin, 1993; Chekireb *et al.*, 2004; Abdessemed *et al.*, 2002). Figure 1 illustrate a sliding mode phenomenon, which consists of an infinite switching of the control action within the neighbourhood of the sliding surface.

Assuming that the system is controllable and observable, the sliding mode control objectives consist of the following steps:

- Design of the switching surface  $S(x)$  so that the state trajectories of the plant restricted to the equilibrium surface have a desired behavior such as tracking, regulation and stability.
- Determine a switching control strategy,  $U(x)$  so that to drive the state trajectory to the equilibrium surface and maintain it on the surface. This strategy has the form:

$$U = \begin{cases} U_{max} & \text{if } S(x) > 0 \\ U_{min} & \text{if } S(x) < 0 \end{cases} \quad (15)$$

where  $S(x)$  is the switching manifold;

- Reduce the chattering phenomenon due to discontinuous nature of the control.
- A well known surface chosen to obtain a sliding mode regime which guarantees the convergence of the state  $x$  to its reference  $x_{ref}$  s given as follows:

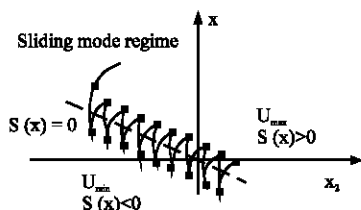


Fig. 1: State trajectory in sliding mode regime

$$S(x) = \left( \frac{d}{dt} + \lambda \right)^{r-1} (x_{ref} - x) \quad (16)$$

where  $r$  is the degree of the sliding surface.

Two parts have to be distinguished in the control design procedure. The first one concerns the attractivity of the state trajectory to the sliding surface and the second represents the dynamic response of the representative point in sliding mode. This latter is very important in terms of application of non-linear control techniques. Because it eliminates the uncertain effect of the model and external perturbation. Among the strategies of the sliding mode control available in the literature, one can chose for the controller the following expression:

$$U_c = U_{eq} + U_n \quad (17)$$

Where  $U_{eq}$  is the control function defined by Utkin and noted equivalent control, for which the trajectory response remains on the sliding surface (Wang and Ding, 1993; Tang and Xu, 1995) In this case, the invariance condition is expressed as:

$$\begin{cases} S(x) = 0 \\ \dot{S}(x) = 0 \end{cases} \quad (18)$$

The equivalent control can be interpreted as the average value of control switching representing the successive commutation in the range  $(U_{min}, U_{max})$  (Machmoum *et al.*, 1992).

Let us consider the system described by (Eq.7), when the sliding mode regime arise, the dynamic of the system in sliding mode is subjected to the following equation  $S(x) = 0$  thus for the ideal sliding

$\dot{S}(x) = 0$  mode we have also

$$\begin{aligned} \dot{S}(x) &= \frac{\partial S}{\partial x} \frac{dx}{dt} = \frac{\partial S}{\partial x} [f(x) + g(x).U_{eq}] + \\ &+ \frac{\partial S}{\partial x} [g(x).U_n] \end{aligned} \quad (19)$$

and  $U_n=0$  (on  $S(x)=0$ ), we obtain:  $\dot{S}(x) = 0$  for

$$U_{eq} = - \left[ \frac{\partial S}{\partial x} g(x) \right]^{-1} \cdot \left[ \frac{\partial S}{\partial x} f(x) \right] \quad (20)$$

by replacing  $U_{eq}$  we obtain:

$$\dot{S}(x) = \frac{\partial S}{\partial x} g(x) \cdot U_n \quad (21)$$

The term  $U_n$  is added to the global function of the controller in order to guarantee the attractiveness of the chosen sliding surface. This latter is achieved by the condition:

$$S(x) \cdot \dot{S}(x) < 0 = S(x) \cdot \frac{\partial S}{\partial x} g(x) \cdot U_n < 0 \quad (22)$$

A simple form of the control action using sliding mode theory is a relay function (Fig. 2). However, this latter produces a drawback in the performances of a control system, which is known a chattering phenomenon.

$$S(x) = K \cdot \text{sgn}(S(x)) \quad (23)$$

Replacing  $U_n$ , we obtain:

$$\dot{S}(x) \cdot S(x) = \frac{\partial S}{\partial x} g(x) \cdot K \cdot |S(x)| \quad (24)$$

The term  $(\partial S/\partial x) \cdot g(x)$  is negative for the class of the system considered, whereas the gain  $K$  is chosen positive to satisfy attractivity and stability conditions. In this context, we can verify the stability of the sliding surface with using theorem of Lyapunov. Let us chose the following positive function ( $V(x) > 0$ ) such us:

$$V(x) = \frac{1}{2} S^2(x) \quad (25)$$

Its derivative is given by:

$$\dot{V}(x) = S(x) \cdot \dot{S}(x) \quad (26)$$

One must decreasing of the Lyapunov function to zero. for this purpose it is sufficient to assure that its derivative is negative.

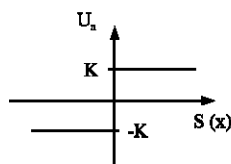


Fig. 2: Relay function

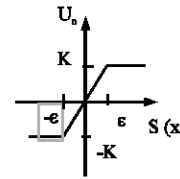


Fig. 3: Smooth sign

In order to reduce the chattering phenomenon due to the discontinuous nature of the controller, a smooth function is defined in some neighbourhood of the sliding surface with a threshold (Fig. 3). If a representative point of the state trajectory moves within this interval, a smooth function replaces the discontinuous part of the control action. Thus, the controller becomes:

$$U_n = \begin{cases} \frac{K}{\epsilon} \cdot S(x) & \text{if } |S(x)| < \epsilon \neq 0 \\ K \cdot \text{sgn}(S(x)) & \text{if } |S(x)| > \epsilon \end{cases} \quad (27)$$

where  $K$  takes the admissible value.

**Application to the DFIM:** The surface of speed regulation has a following form:

$$S(\omega) = \omega^* - \omega \quad (28)$$

The derivative of the surface is:

$$\dot{S}(\omega) = \dot{\omega}^* - \dot{\omega} \quad (29)$$

By taking account of  $\omega$  expression given by the system (12), knowing that  $\Omega = \frac{\omega}{p}$ , the (Eq. 28) becomes:

$$\dot{S}(\omega) = \dot{\omega}^* - \left[ \frac{2 \cdot P^2 \cdot M}{3 \cdot j \cdot Ls} \Psi_{sd} \cdot Irq - \frac{P}{j} \cdot \Gamma r \right] \quad (30)$$

By replacing the  $Irq$  by the control current  $Irq^*$  such as  $= Irq_{eq} + Irq_n$ , the (Eq. 29) can be written as follows:

$$\dot{S}(\omega) = \dot{\omega}^* - \left[ \frac{2 \cdot P^2 \cdot M}{3 \cdot j \cdot Ls} \Psi_{sd} \cdot Irq_{eq} + \frac{2 \cdot P^2 \cdot M}{3 \cdot j \cdot Ls} \Psi_{sd} \cdot Irq_n - \frac{P}{j} \Gamma r \right] \quad (31)$$

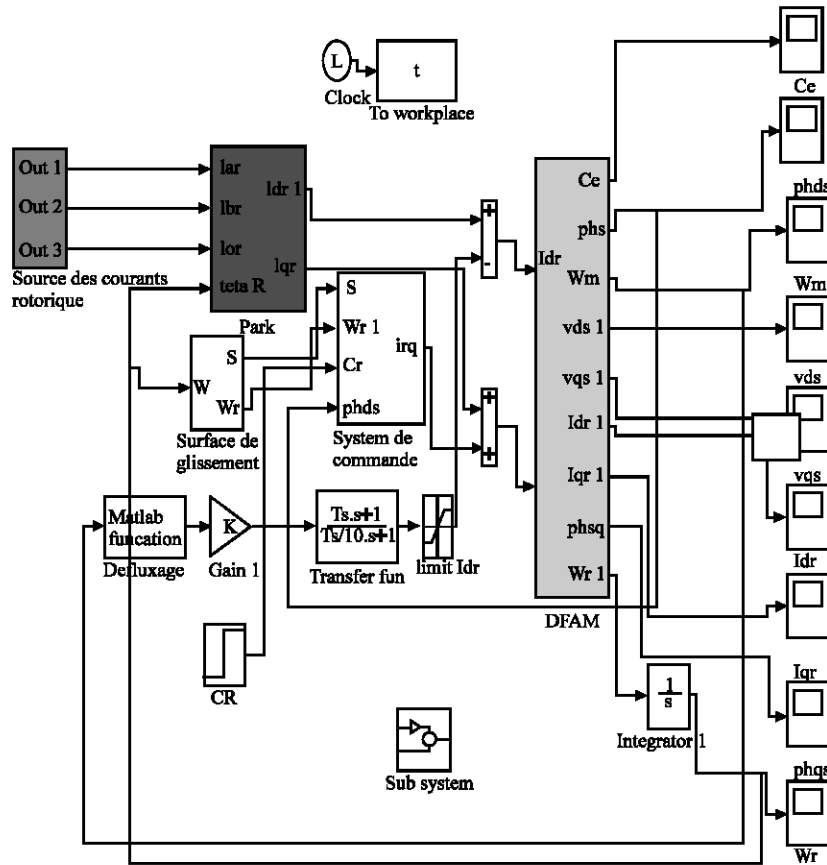


Fig.4. Block diagram of the sliding mode control of the DFAM

During the sliding mode and steady stat, we have  $S(\omega) = 0$  and consequently  $\dot{S}(\omega) = 0$  and  $I_{eq_n}$  and one obtains the equivalent control formula  $I_{req}$ :

$$I_{req} = \frac{1}{\Psi_{sd}} \frac{3.j.Ls}{2.P^2.M} \left( \dot{\omega}^* + \frac{P}{j} \Gamma r \right) \quad (32)$$

During the convergence mode, the condition must  $\dot{S}(\omega).S(\omega) < 0$  be checked. By replacing  $I_{req}$  formula in (31):

$$\dot{S}(\omega) = -\frac{2.P^2.M}{3.j.Ls} \Psi_{sd}.I_{req_n} \quad (33)$$

The Fig. 4 shows the block diagram of the sliding mode control of the DFAM.

From the choice smoothed control one can write:

$$I_{q_n} = \begin{cases} \frac{K}{\varepsilon} |S(\omega)| \dots |S(\omega)| < \varepsilon \\ K.\text{sign}(S(\omega)) \dots |S(\omega)| > \varepsilon \end{cases} \quad (34)$$

For attenuating all the overshoot of the current  $I_{rq}$ , we bound the reference current  $I_{rq}^*$ , the bounded current  $I_{rq_{lim}}$  has the following expression:

$$I_{rq_{lim}} = I_{rq_{max}}.\text{sign}(I_{rq}^*) \quad (35)$$

From these equations we can simulate the sliding mode control; note we are in the case of setting by the sliding mode with a non linear surface, only one surface is sufficient for setting the speed with direct bounding of the rotor current in quadrature

## RESULTS AND DISCUSSION

A sliding mode control of the stator flux oriented control has been simulated using the parameters: (220/380) V; (3.8/2.2) A ; 0.8 kW; 1500 tr/mn;  $p = 2$ ;  $L_s = 0.0605$  H;  $L_r = 0.0736$  H;  $R_s = 11.98 \Omega$ ;

$R_r = 9.08 \Omega$ ;  $M = 0.209$  H. Thus, the speed regulation is obtained using such a controller in spite of the presence stern disturbances such as reference speed

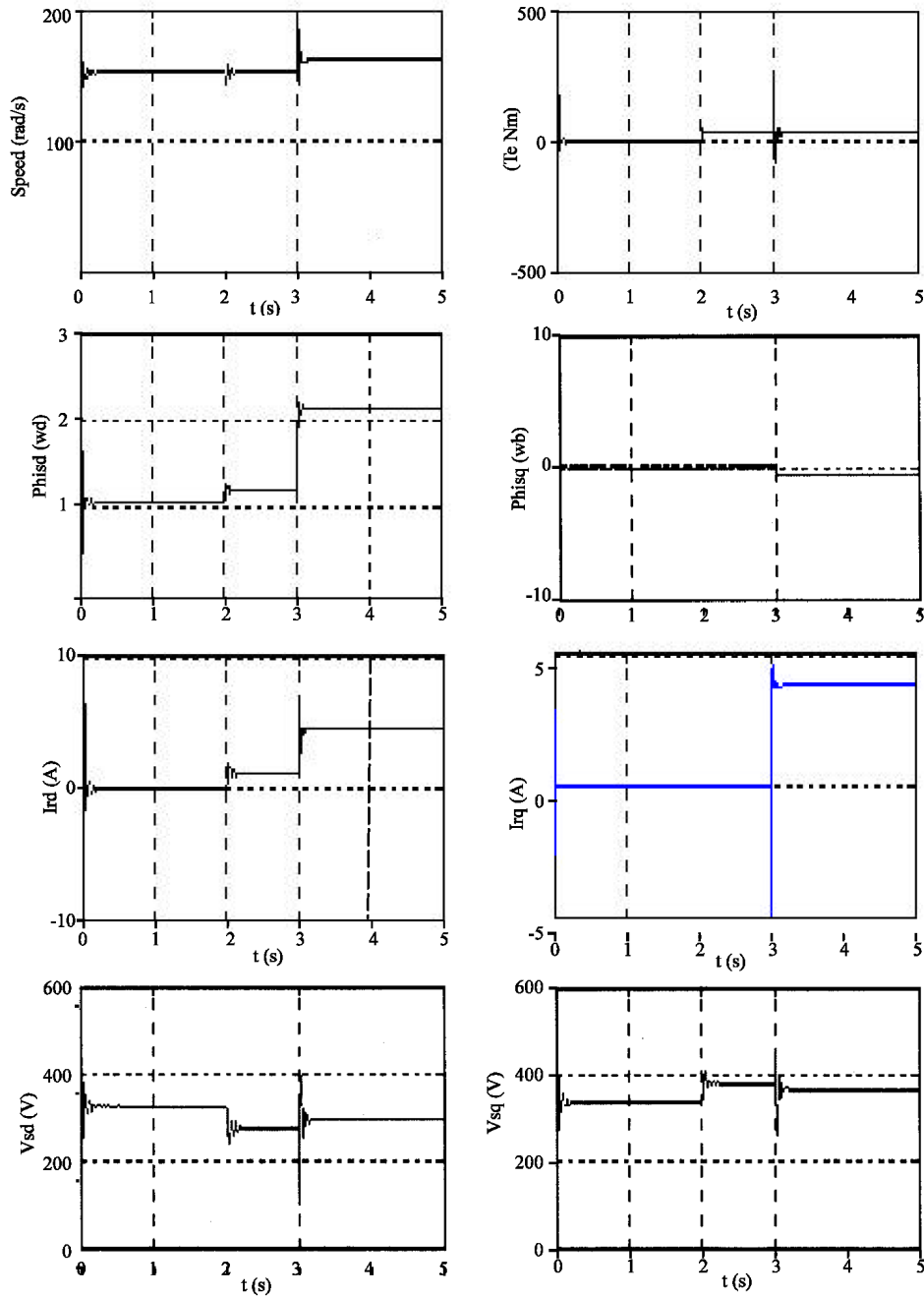


Fig. 5: The dynamic responses of the speed and the electromagnetic torque

variation and step changing of the load torque. Figure 5 shows the dynamic responses of the speed and the electromagnetic torque when a load torque perturbation is imposed in the system at  $t = 0.2$ s and at  $t = 3$ s an application of speed reference change with step changing load torque. It is clearly shown from the results that the input reference is perfectly tracked by the speed and the introduced perturbation is immediately rejected by the control system.

The control by the sliding mode of the D.F.A.M. give high dynamic and static performances. It offers a good pursuit and a rejection of disturbances.

### CONCLUSION

In this study, a sliding mode control of the stator field oriented doubly fed induction machine is proposed. Satisfying results are obtained. In order to reduce the

chattering phenomenon, a smooth function has been applied. The robustness quality of the proposed controllers appears clearly in the test results by changing machine operations and especially the load torque variation.

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