

Multi-Objective Short-Term Hydrothermal Scheduling Based on Heuristic Search Technique

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Abstract: A new heuristic search technique based on binary successive approximation method is applied to the problem of determining the optimal schedule of power generation in a hydro thermal power system. The main objective for hydrothermal operation is not only to minimize the total system operating cost, represented by the fuel cost required for system's thermal generation subject to the operating constraints of hydro and thermal plants, over the optimization interval but also to consider the environmental and system security objectives. Normally, the decision making input system data were assumed to be well behaved and deterministic. But in practical situations the input system data cannot be predicted and estimated with hundred percent certainties. It is bound to vary depending upon the uncertainties, load changes, load forecasting errors, ageing of equipment etc. It is worthwhile to assume the system data as variable and uncertain for more realistic approach. In this study an attempt has been made to solve fixed head short-term hydro-thermal generation scheduling problem in the multi objective framework by taking into account the statistical variation of various system parameters such as variance of cost and emission coefficients of generators, variance of power, generation mismatch etc.

Key words: Hydrothermal, multi-objective, heuristic search technique

INTRODUCTION

Economic operation and control of interconnected power systems involves the solution of difficult optimization problems that require good computational tools. A new heuristic search technique based on binary successive approximation method is one such tool that has shown its ability in solving complex problems. Because of insignificant marginal cost of hydroelectric power, the problem of minimizing the operational cost of a hydro-thermal system essentially reduces to that of minimizing the fuel cost for thermal plants under the constraints of the water available for hydro generation in a given period of time. Mostly, hydrothermal optimal scheduling is achieved (Aggarwal and Nagrath, 1972; Rao *et al.*, 1974; Edgardo and Jao, 2004) with the assumption, that the water inflows to the reservoirs and the load demands are known with complete uncertainty. However it is not true. The availability of limited amount

of hydroelectric energy, as stored water in the system reservoirs, makes the optimal operation complex, because it creates a link between an operating decision in a given stage and the future consequences of this decision. Further, it is impossible to have perfect forecasts of the future inflow sequence and the load variation during a given period. Therefore, for long-term storage regulation, it becomes necessary to account for the random nature of the load and the river inflow and a stochastic representation of these must be used.

Most of the methods that have been used to solve the hydrothermal co-ordination problem make a number of simplifying assumptions in order to make the optimization problem more tractable. The study by Hara and Suzuki (1967) is one of the earliest which considers the stochastic nature of the system load fluctuations and river inflows. Arvantidis and Rosing (1970) did not deal with the optimal integrated operation of hydrothermal system as such and they considered water inflow as random

zvariables. Booth (1972) presented the method of simulation based on probability distributions with dynamic programming. Aggarwal (1973) used the gradient method to solve the optimal scheduling of hydrothermal system considering deterministic water inflow. Dhillon *et al.* (1980) developed a model of hydrothermal system suitable for long term regulation using stochastic representation of the inflows to the individual reservoirs, the load and unit availability. Kothari and Nagrath (1980) in another attempt obtained the optimal stochastic scheduling of hydrothermal system using the Discrete Maximum Principle. The stochastic dual programming algorithm has been successfully applied to the stochastic scheduling of multi-reservoir systems, without transmission limitations by Pereira (1989). In another study, the methodology has been extended to handle the transmission network, represented by a linearised flow model by Pereira *et al.* (1992). A long-term, large scale hydrothermal production scheduling method has been proposed by Yu *et al.* (1998). They used both composite hydro and composite thermal representations, based on the monthly or weekly energy requirement. The study selected a probabilistic representation of the water inflow. A major source of uncertainty in optimal dispatch is that associated with cost coefficients (Dhillon *et al.*, 1985). However with the increasing concern recently given to the environmental considerations (Dhillon *et al.*, 1985, 2000) a revised generation scheduling for hydrothermal power system is required that meets the constraints of available water at hydro plants and load demands for power while accounting for both cost and emission.

Therefore, in this study the authors have formulated stochastic multi objective hydrothermal generation scheduling problem as a multi objective problem with explicit recognition of uncertainties in the system production cost coefficients, emission coefficient and system load, which are treated as random variables. The objectives are clubbed in a single objective with the help of the weighting method. The weighting pattern is simulated by selecting suitable step size to generate the non-inferior solution surface. Fuzzy methodology has been exploited for solving a decision making problem involving multiplicity of objectives and selection criterion for best compromised solution. The objectives are quantified by eliciting the corresponding membership function. The shape of fuzzy membership function is decided by the Decision Maker (DM) and depends upon the type of the problem. The effectiveness of the proposed method is demonstrated on a sample system.

PROBLEM FORMULATION

Consider an electric power system network having N thermal generating units and M-N hydro plants, where M is the total number of generating plants. The basic

problem is to find the active power generation of each plant in the system as a function of time over a time period from 0 to T.

Stochastic thermal model: The objectives to be minimized in electrical thermal power system are economy and environmental impacts because of No_x, SO₂ and CO₂ emissions.

The cost objective can be defined as

$$J_1 = \int_0^T \sum_{i=1}^N (a_{i1}P_i^2 + b_{i1}P_i + c_{i1}) dt \text{ Rs h}^{-1} \quad (1a)$$

No_x emission objective can be defined as

$$J_2 = \int_0^T \sum_{i=1}^N (a_{i2}P_i^2 + b_{i2}P_i + c_{i2}) dt \text{ kg h}^{-1} \quad (1b)$$

CO₂ emission objective can be defined as

$$J_3 = \int_0^T \sum_{i=1}^N (a_{i3}P_i^2 + b_{i3}P_i + c_{i3}) dt \text{ kg h}^{-1} \quad (1c)$$

SO₂ emission objective can be defined as

$$J_4 = \int_0^T \sum_{i=1}^N (a_{i4}P_i^2 + b_{i4}P_i + c_{i4}) dt \text{ kg h}^{-1} \quad (1d)$$

where

- a₁₁, b₁₁ and c₁₁ are cost coefficients of ith unit
- a₂₁, b₂₁ and c₂₁ are NO_x emission coefficients of ith unit
- a₃₁, b₃₁ and c₃₁ are SO_x emission coefficients of ith unit
- a₄₁, b₄₁ and c₄₁ are CO_x emission coefficients of ith unit
- P_i is real power generation of ith unit.
- N is the number of thermal generators

The stochastic model of multi-objective problem is formulated by considering cost coefficients, emission coefficients and load demand as random variables. Then the generator output automatically becomes random. Random variables are considered as normally distributed and statistically dependent to each other. By taking expectations, the stochastic model can be converted into its deterministic equivalent. The expected value of a function can be obtained by expanding the function, employing Taylor's series, about the mean. Deterministic equivalent of stochastic thermal model is stated as; Expected cost

$$\bar{J}_1 = \int_0^T \sum_{i=1}^N \left(\bar{a}_{i1} \bar{P}_i^2 + \bar{b}_{i1} \bar{P}_i + \bar{c}_{i1} + \bar{a}_{i1} \text{var}(P_i) + 2\bar{P}_i \text{cov} \left((a_{i1}, P_i) + \text{cov}(b_{i1}, P_i) \right) \right) dt \quad (2a)$$

Expected NO_x emission

$$\bar{J}_2 = \int_0^T \sum_{i=1}^N \left(\bar{a}_{i2} \bar{P}_i^2 + \bar{b}_{i2} \bar{P}_i + \bar{c}_{i2} + \bar{a}_{i2} \text{var}(P_i) + 2\bar{P}_i \text{cov} \left(\begin{matrix} \bar{a}_{i2}, P_i \\ \bar{a}_{i2}, P_i \end{matrix} \right) + \text{cov}(b_{i2}, P_i) \right) dt \quad (2b)$$

Expected CO₂ emission

$$\bar{J}_3 = \int_0^T \sum_{i=1}^N \left(\bar{a}_{i3} \bar{P}_i^2 + \bar{b}_{i3} \bar{P}_i + \bar{c}_{i3} + \bar{a}_{i3} \text{var}(P_i) + 2\bar{P}_i \text{cov} \left(\begin{matrix} \bar{a}_{i3}, P_i \\ \bar{a}_{i3}, P_i \end{matrix} \right) + \text{cov}(b_{i3}, P_i) \right) dt \quad (2c)$$

Expected SO₂ emission

$$\bar{J}_4 = \int_0^T \sum_{i=1}^N \left(\bar{a}_{i4} \bar{P}_i^2 + \bar{b}_{i4} \bar{P}_i + \bar{c}_{i4} + \bar{a}_{i4} \text{var}(P_i) + 2\bar{P}_i \text{cov} \left(\begin{matrix} \bar{a}_{i4}, P_i \\ \bar{a}_{i4}, P_i \end{matrix} \right) + \text{cov}(b_{i4}, P_i) \right) dt \quad (2d)$$

where, \bar{a}_{i1} , \bar{b}_{i1} and \bar{c}_{i1} are the expected cost coefficients of *i*th unit.

\bar{a}_{i2} , \bar{b}_{i2} and \bar{c}_{i2} are the expected NO_x emission coefficients of *i*th unit.

\bar{a}_{i3} , \bar{b}_{i3} and \bar{c}_{i3} are the expected SO₂ emission coefficients of *i*th unit.

\bar{a}_{i4} , \bar{b}_{i4} and \bar{c}_{i4} are the expected CO₂ emission coefficients of *i*th unit.

Stochastic hydro model: In short range hydrothermal scheduling problem, an insignificant fuel cost is incurred in the operation of hydro-units (Booth, 1972; Aggarwal, 1973). The input-output characteristic of a hydro-generator is expressed by the variation of water discharge, *q_j* as a function of power output, *P_j* and net head, *h* (Booth, 1972). According to Glimm- Kirchmayer model, the discharge is

$$q_j = K\phi(h_j)\tau(P_j), \quad j = N + 1, 2, \dots, M \quad (3a)$$

where ϕ τ are functions of head and hydro-generations, respectively.

For a large capacity reservoir, it is practical to assume that the effective head is constant over the optimization interval. Therefore, $\phi(h_j)$ becomes constant and (3a) is written as

$$q_j = K' \tau(P_j), \quad j = N + 1, 2, \dots, M \quad (3b)$$

where *K* is a constant. Each hydro plant is constrained by the amount of water available for the optimization interval.

$$\int_0^T q_j dt = R_j, \quad j = N + 1, 2, \dots, M \quad (3c)$$

where *R_j* is predefined volume of water available for *j*th hydro-plant.

The performance of *q_j* is represented by

$$q_j = x_j P_j^2 + y_j P_j + z_j, \quad j = N + 1, 2, \dots, M \quad (3d)$$

where *x_j*, *y_j* and *z_j* are the discharge coefficients of *j*th hydro-plant.

Since the thermal generations and load demand are random, the hydro-generations also become random. A stochastic model of function *q_j* is obtained deeming the hydro-generation and discharge coefficients to be random variables. The random variables are assumed to be normally distributed and dependent, so the expected value of discharge becomes

$$\bar{q}_j = \bar{x}_j \bar{P}_j^2 + \bar{y}_j \bar{P}_j + \bar{z}_j + \bar{x}_j \text{var}(P_j) + 2\bar{P}_j \text{cov}(x_j, P_j) + \text{cov}(y_j, P_j), \quad j = N + 1, 2, \dots, M \quad (3e)$$

Equality and inequality constraints:

- The expected load demand equality constraint is expressed as

$$\sum_{i=1}^M \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (4a)$$

- The expected limits are imposed as

$$\bar{P}_i^L \leq \bar{P}_i \leq \bar{P}_i^U, \quad i = 1, 2, \dots, M \quad (4b)$$

where, \bar{P}_D is the expected power demand,

\bar{P}_L is the expected transmission loss,

\bar{P}_i^L and \bar{P}_i^U are the expected lower and upper limits of real power, respectively.

Expected transmission losses: A common approach to model transmission losses in the system is to use Kron's loss formula through B- coefficients.

$$P_L = \sum_{i=1}^M \sum_{j=1}^M P_i B_{ij} P_j + \sum_{i=1}^M P_i B_{io} + B_{oo} P_o \quad (5a)$$

With normally distributed random variables, the expected transmission losses, \bar{P}_L are

$$\begin{aligned} \bar{P}_L = & \sum_{i=1}^M \sum_{j=1}^M \bar{P}_i \bar{B}_{ij} \bar{P}_j + \sum_{i=1}^M \bar{B}_{ii} \text{var} P_i + \sum_{i=1}^{M-1} \sum_{j=i+1}^M 2.0 \bar{B}_{ij} \text{cov}(P_i, P_j) \\ & + \sum_{i=1}^M 2.0 \bar{P}_i \text{cov}(P_i, B_{ii}) + \sum_{i=1}^M \sum_{j=1}^M 2.0 \bar{P}_j \text{cov}(P_i, B_{ij}) \quad (5b) \\ & + \sum_{i=1}^M \bar{B}_{i0} \bar{P}_i + \bar{B}_{00} \bar{P}_0 \end{aligned}$$

Expected deviation: The generator outputs are treated as random variables and the stochastic model is converted into its deterministic equivalent by taking its expected value. So, the solution will provide only the expected values of power generation. By virtue of the above consideration, there will be a mismatch in load demand. The variance of a random variable quantifies the degree of uncertainties associated with the mean value of the random variable. The active power loss, the system fuel cost and emission curves are quadratic functions of decision variable, P_i and there variances quantify the degree of uncertainties associated with the expected values. So, the expected mismatch can be estimated through minimization of the squared error of the unsatisfied power demand.

$$E \left[\left(P_D + P_L - \sum_{i=1}^M P_i \right)^2 \right] \quad (6a)$$

P_i is the actual power generation required to meet the load, which is considered to be random. Using (4a), (6a) can be rewritten as

$$E \left[\left(\sum_{i=1}^M P_i - \sum_{i=1}^M \bar{P}_i \right)^2 \right] \quad (6b)$$

$$\bar{J}_5 = \int_0^T \left(\sum_{i=1}^M \text{var}(P_i) - \sum_{i=1}^{M-1} \sum_{j=i+1}^M 2\text{cov}(P_i, P_j) \right) dt \quad (6c)$$

Multi-objective stochastic optimization problem formulation: In this study, variance and covariance are replaced by the coefficients of variation and correlation coefficient, respectively. In general variance and covariance are defined as:

$$\text{var}(x) = C^2(x) \bar{x}^2 \quad (7a)$$

$$\text{cov}(x, y) = R(x, y) C(x) C(y) \bar{x} \bar{y} \quad (7b)$$

where $C(x)$ and $C(y)$ are the coefficients of variation and \bar{x} and \bar{y} are the expected values of variables x and y , respectively. $R(x, y)$ is correlation coefficient and varies from -1 to 1.

The zero value of coefficient of variation implies no randomness, in other words, the complete certainty about the value of random variables. Using Eq. 7a and 7b, the stochastic hydrothermal optimization problem defined by Eq. 2-6 can be rewritten as;

$$\text{Minimize } [\bar{J}_1, \bar{J}_2, \bar{J}_3, \bar{J}_4, \bar{J}_5]^T \quad (8a)$$

$$\text{Subject to } \sum_{i=1}^N \bar{P}_i = \bar{P}_D + \bar{P}_L \quad (8b)$$

$$\int_0^T q_j dt = R_j, \quad j = N + 1, 2, \dots, M \quad (8c)$$

$$\bar{P}_i^L \leq \bar{P}_i \leq \bar{P}_i^U, \quad i = 1, 2, \dots, M \quad (8d)$$

$$\text{where } \bar{J}_j = \sum_{i=1}^N (\bar{A}_{ij} \bar{P}_i^2 + \bar{B}_{ij} \bar{P}_i + \bar{C}_{ij}) \quad j = 1, 2, 3, 4.$$

with

$$\bar{A}_{ij} = [1 + C^2(P_i) + 2R(a_{ij}, P_i)C(a_{ij})C(P_i)] \bar{a}_{ij},$$

$$\bar{B}_{ij} = [1 + R(b_{ij}, P_i)C(b_{ij})C(P_i)] \bar{b}_{ij}$$

$$\bar{C}_{ij} = \bar{c}_{ij}$$

$$\bar{J}_5 = \sum_{i=1}^M \sum_{j=1}^M P_i T_{ij} P_j \quad (8e)$$

with $T_{ii} = C^2(P_i)$ and $T_{ij} = R(P_i, P_j)C(P_i)C(P_j); i \neq j$

$$\bar{P}_L = \sum_{i=1}^M \sum_{j=1}^M \bar{P}_i U_{ij} \bar{P}_j + \sum_{i=1}^M \bar{B}_{i0} \bar{P}_i + \bar{B}_{00} \quad (8f)$$

with

$$U_{ii} = 1.0 + C(P_i)^2 + 2.0 R(P_i, B_{ii})C(P_i)C(B_{ii})$$

$$U_{ij} = 1.0 + R(P_i, P_j)C(P_i)C(P_j) + 2.0 R(P_i, B_{ij})C(P_i)$$

$$C(B_{ij})P_i B_{ij} P_j; i \neq j$$

SOLUTION APPROACH

To generate the non-inferior solutions of the multi-objective problem, the weighting method is used. In this method, the multi-objective optimization problem is converted into a scalar optimization problem as:

$$\text{inimize } \sum_{k=1}^5 w_k \bar{J}_k \quad (9a)$$

$$\text{subject to } (8b-8d) \text{ and } \sum_{k=1}^5 w_k = 1 \quad (9b)$$

To solve the scalar optimization problem, the Lagrangian function is defined as:

$$L = \sum_{k=1}^5 w_k J_k + \int_0^T \left[\sum_{j=N+1}^M v_j q_j + \lambda \left(\bar{P}_D + \bar{P}_L - \sum_{i=1}^M \bar{P}_i \right) \right] dt - \sum_{j=N+1}^M v_j R_j \quad (9c)$$

where λ is the Lagrangian multiplier and v_j are the water conversion factors.

The necessary conditions to minimize the unconstrained Lagrangian function are obtained by taking the first order partial derivatives of the Lagrangian function with respect to the decision variables as follows:

$$\frac{\partial L}{\partial P_i} = \sum_{n=1}^5 w_n \frac{\partial \bar{J}_k}{\partial P_i} + \lambda \left(\frac{\partial \bar{P}_L}{\partial P_i} - 1 \right) = 0; \quad i = 1, 2, \dots, N \quad (10a)$$

$$\frac{\partial L}{\partial P_j} = v_j \frac{\partial \bar{q}_j}{\partial P_j} + w_s \frac{\partial \bar{J}_s}{\partial P_j} + \lambda \left(\frac{\partial \bar{P}_L}{\partial P_j} - 1 \right) = 0, \quad j = N+1, 2, \dots, M \quad (10b)$$

$$\frac{\partial L}{\partial v_j} = \int_0^T q_j dt - R_j = 0; \quad j = N+1, 2, \dots, M \quad (10c)$$

$$\frac{\partial L}{\partial \lambda} = \bar{P}_D + \bar{P}_L - \sum_{i=1}^N \bar{P}_i = 0 \quad (10d)$$

DECISION MAKING

Considering the imprecise nature of the DM's judgment, it is natural to assume that the DM may have fuzzy or imprecise goals for each objective function. The fuzzy sets are defined by equations called membership functions. These functions represent the degree of membership in certain fuzzy sets using values from 0 to 1. The membership value 0 indicates incompatibility with the sets, while 1 means full compatibility. By taking account of the minimum and maximum values of each objective function together with the rate of increase of membership satisfaction, the DM must determine the membership function $\mu(J_i)$ in a subjective manner. It is assumed that $\mu(J_i)$ is a strictly monotonic linear decreasing and continuous function and is defined as:

$$\mu(J_i) = \begin{cases} 1 & ; \bar{J}_i \leq \bar{J}_i^{\min} \\ \frac{\bar{J}_i^{\max} - \bar{J}_i}{\bar{J}_i^{\max} - \bar{J}_i^{\min}} & ; \bar{J}_i^{\min} \leq \bar{J}_i \leq \bar{J}_i^{\max} \\ 0 & ; \bar{J}_i \geq \bar{J}_i^{\max} \end{cases} \quad (11)$$

where \bar{J}_i^{\min} and \bar{J}_i^{\max} are the minimum and maximum values of i th objective function in which the solution is

expected. The value of membership function suggests how far (in the scale from 0 to 1) a non-inferior solution has satisfied the \bar{J}_i objective. The decision regarding the best solution is made by the selection of minimax of membership function as defined below (Tapia and Murtagh, 1991):

$$\mu_D^k = \text{Max} \left[\begin{matrix} \text{Min} \{ \mu(J_j)^k; j = 1, 2, \dots, L \}; \\ k = 1, 2, \dots, 2^L + 1 \end{matrix} \right] \quad (12)$$

The function μ_D^k in (12) can be treated as a membership function for non-dominated solutions. The solution which attains highest membership μ_D^k in the fuzzy set so obtained can be chosen as best solution or the one having highest cardinal priority ranking.

ALGORITHM FOR BINARY SEARCH OF INCREMENTAL COST

The operation diagram of the successive approximation method is shown in Fig. 1. It has been shown in the diagram that all the possibilities from 1 to 15 have been included when four binary bits are used to represent either the incremental cost or the weights. In the proposed method the number of binary bits to represent the incremental cost has been selected as twenty four to get accurate results. The successive approximation strategy to search the incremental cost, \bar{e} is elaborated here for combinations of four binary digit. (NB-1) $\times 2$, number of comparisons is required to arrive at the solution.

The stepwise procedure is outlined below:

- 1 Read NB, number of binary digits to represent, λ .
- 2 Set binary digit counter, $i = 1$
- 3 $N = 2^{NB-i}$
- 4 Increment i ; $i = i+1$
- 5 If ($i \geq NB$) then go to 10
- 6 Determine N_1 and N_2 as
 $N_1 = N + 2^{NB-i}$
 $N_2 = N - 2^{NB-i}$
- 7 Determine λ_1 and λ_2 as

$$7.1 \quad \lambda_1 = \lambda^{\min} + \frac{N_1}{2^{NB} - 1} (\lambda^{\max} - \lambda^{\min})$$

7.1.1 Determine P_i^1 ; $i = 1, 2, \dots, N$ from (10a) using Gauss

Elimination method

7.1.2 Determine $\Delta P_D^1 = \left| P_D + P_L - \sum_{i=1}^N P_i^1 \right|$

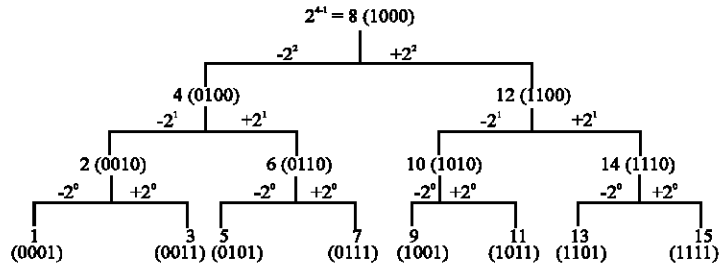


Fig. 1: Operation diagram of the successive approximation method

Table 1: Weight set corresponding to the best solution and the objectives

W ₁	W ₂	W ₃	W ₄	W ₅	Cost(Rs h ⁻¹)	NO _x (kg h ⁻¹)	SO ₂ (kg h ⁻¹)	CO ₂ (kg h ⁻¹)	Risk(MW) ²
0.1	0.2	0.1	0.1	0.5	16837.39	825.69	27023.97	32032.52	13485.20

$$7.2 \quad \lambda_2 = \lambda^{\min} + \frac{N_2}{2^{N_2} - 1} (\lambda^{\max} - \lambda^{\min})$$

7.2.1 Determine P_i²; i = 1, 2, ..., N from (10a) using Gauss

Elimination method

$$7.2.2 \text{ Determine } \Delta P_D^2 = \left| P_D + P_L - \sum_{i=1}^N P_i^2 \right|$$

8 If (ΔP_D¹ < ΔP_D²) then set N = N₁ and ΔP_D = ΔP_D¹

else set N = N₂ and ΔP_D = ΔP_D²

9 If (ΔP_D ≤ ε) then continue else go to 4

10 Stop.

Algorithm for short-term fixed head hydrothermal generation scheduling: To perform the short-term fixed head hydrothermal generation scheduling, the stepwise procedure is outlined below;

- 1 Input the data.
- 2 Compute the initial guess values of P_{ik}⁰; I = 1, 2, ..., M, λ_k⁰ and v_j⁰; j = 1, 2, ..., M-N.
- 3 Consider v_j = v_j⁰; j = 1, 2, ..., M-N.
- 4 Set iteration counter, r = 1
- 5 Set hourly count, r = 1.
- 6 Compute P_{ik}, λ_k using algorithm presented in section 5.
- 7 Check if k = T, then go to 8, else k = k + 1 and go to 6 and repeat.
- 8 Compute water withdrawals, v_j; j = 1, 2, ..., m-n.
- 9 Check |v_j - v_j⁰| ≥ ε then go to 10, else v_jⁿ = v_j⁰ + α (v_j - v_j⁰) / v_j^s; j = 1, 2, ..., M-N
v_j⁰ = v_jⁿ; j = 1, 2, ..., M-N
r = r + 1 and go to 5 and repeat.
- 10 Compute cost, loss etc. and stop.

TEST SYSTEM AND RESULTS

The system under study comprise of two thermal plants and two hydro plants (Dhillon *et al.*, 2002). A short

Table 2: Power demand, error in demand, incremental cost and water discharges

IT	Demand (MW)	Error (MW)	λ (Rs/MWh)	q ₁ (m ³ h ⁻¹)	q ₂ (m ³ h ⁻¹)
1	455.0	-0.000036	6.137995	2.433098	1.763288
2	425.0	0.000044	5.649754	2.312947	1.593472
3	415.0	-0.000002	5.487859	2.273238	1.537282
4	407.0	0.000088	5.358646	2.241594	1.492477
5	400.0	-0.000089	5.245810	2.213996	1.453382
6	420.0	0.000063	5.568753	2.293071	1.565351
7	487.0	0.000098	6.663045	2.562960	1.946480
8	604.0	-0.000092	8.621158	3.052851	2.634487
9	665.0	-0.000004	9.666658	3.317748	3.004623
10	675.0	0.000042	9.839703	3.361799	3.066055
11	695.0	0.000021	10.187210	3.450433	3.189555
12	705.0	-0.000105	10.361670	3.495016	3.251626
13	580.0	-0.000080	8.214495	2.950419	2.491016
14	605.0	-0.000025	8.638158	3.057141	2.640491
15	616.0	0.000027	8.825472	3.104445	2.706678
16	653.0	-0.000015	9.459622	3.265119	2.931187
17	721.0	0.000000	10.641800	3.566717	3.351382
18	740.0	0.000048	10.976050	3.652456	3.470554
19	700.0	0.000055	10.274380	3.472702	3.220563
20	678.0	0.000063	9.891707	3.375049	3.084525
21	630.0	0.000074	9.064675	3.164957	2.791286
22	585.0	0.000024	8.298999	2.971676	2.520805
23	540.0	-0.000085	7.542499	2.781931	2.254588
24	503.0	0.000053	6.927239	2.628553	2.038876

range hydrothermal load scheduling problem of 24 h duration has been undertaken. The optimization period has been divided into 24 intervals of 1 h each. The economy, environmental impacts because of NO_x, SO₂ and CO₂ emissions and variance of power are the five objectives considered which have weights w₁, w₂, w₃, w₄ and w₅, respectively. The cost and emission coefficients are treated as random variables. The non-inferior solutions for 130 different weight combinations are generated for the following values of coefficients of variation and correlation coefficients:

$$C(a_{ij}) = C(b_{ij}) = C(P_i) = 0.01; i = 1, 2, \dots, 6$$

$$R(a_{ij}, P_i) = R(b_{ij}, P_i) = R(P_i, P_j) = 0.2; i = 1, 2, \dots, 6;$$

$$i = 1, 2, \dots, 6; i \neq j$$

Table 3: Generation schedule

IT	Demand (MW)	P ₁ (MW)	P ₂ (MW)	P ₃ (MW)	P ₄ (MW)
1	455.0	67.12674	68.67648	234.0927	94.72745
2	425.0	61.26991	62.55711	223.7146	85.90372
3	415.0	59.32160	60.52567	220.2585	82.96497
4	407.0	57.76433	58.90346	217.4947	80.61480
5	400.0	56.40280	57.48626	215.0774	78.55911
6	420.0	60.29550	61.54086	221.9863	84.43418
7	487.0	73.39362	75.24535	245.1789	104.15160
8	604.0	96.48135	99.63536	285.8570	138.71590
9	665.0	108.62850	112.58870	307.1554	156.80370
10	675.0	110.62710	114.72800	310.6528	159.77330
11	695.0	114.63050	119.02010	317.6529	165.71620
12	705.0	116.63530	121.17300	321.1555	168.68960
13	580.0	91.72292	94.58397	277.4943	131.61200
14	605.0	96.67987	99.84637	286.2056	139.01200
15	616.0	98.86494	102.17050	290.0418	142.27050
16	653.0	106.23290	110.02740	302.9606	153.24180
17	721.0	119.84730	124.62700	326.7630	173.44950
18	740.0	123.66840	128.74380	333.4276	179.10600
19	700.0	115.63260	120.09600	319.4039	167.20270
20	678.0	111.22710	115.37070	311.7024	160.66440
21	630.0	101.64950	105.13620	294.9273	146.41980
22	585.0	92.71327	95.63424	279.2357	133.09140
23	540.0	83.81808	86.22071	263.5777	119.78800
24	503.0	76.53470	78.54602	250.7284	108.86830

The non-inferior solution is chosen as the best solution that attains maximum membership by applying fuzzy decision making. The results are shown in Table 1-3 corresponding to the best solution. Table 1 gives the weight set corresponding to the best solution and the objectives. Table 2 shows the error in demand, incremental cost and water discharges in each interval of one hour. Table 3 gives the generation schedule of 24 h duration.

CONCLUSION

This study presents a new approach for the economic operation of hydrothermal power systems. The conventional short-term fixed head hydro-thermal generation dispatch method allocates a generation schedule to the generating units based on deterministic cost function and load demand, ignoring inaccuracies and uncertainties. Such generation schedules result in the lowest expected total cost, but this cost is also associated with a relatively large variance that can be interpreted as a risk measure. Moreover, in power system operation planning, there are multiple objectives which need to be attained, which conflict with each other and are subjected to a mutual interface. Thus any objective can only be improved at the expense of other objectives. In the multi-objective framework, the analysis of hydrothermal short-range fixed-head is undertaken with explicit recognition of uncertainties in production cost, NO_x, SO₂ and CO₂

emissions and load demand. The analysis allows the facilities to consider: The inaccuracies and uncertainties in the hydro-thermal schedule, it allows an explicit trade-off between total operating cost, NO_x, SO₂ and CO₂ emission pollutants and risk levels with the given weightage or importance and it provides the DM with the most efficient solution from the non-inferior set, with the help of fuzzy set theory. The practical utility of the stochastic formulation is illustrated through numerical example. The algorithm requires small computing resources. It is fast and efficient and has the potential for application to online economic load dispatch in hydro thermal power systems.

REFERENCES

- Aggarwal, S.K. and I.J. Nagrath, 1972. Optimum scheduling of hydrothermal systems, Proc. IEEE., 119: 169-173.
- Aggarwal, S.K., 1973. Optimal stochastic scheduling of hydrothermal systems, Proc., IEEE., 120: 674-4.
- Arvantidis, N.V. and J. Rosing, 1970. Optimal operation of multi reservoir systems using a composite representation. IEEE. Trans. Power Sys., 89: 327.
- Booth, R.R., 1972. Optimal generation planning considering uncertainties. IEEE. Trans. Power Sys., 91: 70.
- Dhillon, J.S., S.C. Parti and D.P. Kothari, 1985. Multiobjective decision making in Stochastic economic dispatch. Elec. Machines Power Sys., 23: 289-301.
- Dhillon, J.S., S.C. Parti and D.P. Kothari, 2002. Fuzzy decision making in stochastic multiobjective short-term hydrothermal scheduling. IEEE. Proc., Generation Transmission Distribution, 149: 191-200.
- Dillon, T.S., R.W. Martin and D. Sjelvgren, 1980. Stochastic optimization modeling of large hydrothermal Systems for long-term regulation. Int. J. Elec. Power Energ. Sys., 2: 220.
- Edgardo D. Castronuova and Joa˜o A. Pecos Lopes, 2004. Optimal operation and hydro storage sizing of a wind-hydro power plant, Elec. Power Energ. Sys., 26: 771-778.
- Gornstein, B.G., N.M. Campodonico, J.P. Costa and M.V.F. Pereira, 1992. Stochastic optimization of a hydrothermal system including network constraints, IEEE. Trans. Power Sys., 7: 791-7.
- Hara, H. and H. Suzuki, 1967. A method for planning annual economic operation of combined hydrothermal power system, Elec. Eng. J., 87: 70.

- Kothari, D.P. and I.J. Nagrath, 1980. Optimal stochastic scheduling of hydrothermal systems using Discrete Maximum Principle. *J. Inst. Eng.*, 61: 22-6.
- Pereira, M.V.F., 1989. Optimal stochastic operations scheduling of hydroelectric systems. *Int. J. Elec. Power Energ. Sys.*, 11: 16.
- Rao, K.P.S., S.S. Prabhu and S.K. Aggarwal, 1974. Optimal scheduling of hydorthermal systems by method of local variations, IEEE. PES. Winter Meeting, New York, p\Paper No. C-74-025-3.
- Tapia, C.G. and B.A. Murtagh, 1991. Interactive fuzzy programming with preference criteria in multiobjective decision making. *Comput. Oper. Res.*, 18: 307-316.
- Yu, Z., F.T. Sparrow and D. Nderito, 1998. Long-term hydorthermal scheduling using composite thermal and composite hydro representation. *Proc. Gener. Trans. Distrib.*, 143: 210-6.