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Impact of Current and Voltage Harmonics on Measurement of Active Power in the Electrical Systems

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Abstract: When the voltage and the current are sinusoidal there is a single definition for the active power. When both the voltage and the current or just one of them are not sinusoidal, the measurement and the calculation of this power are complicated. The results of the experiment carried out in this study show that the total active power is a sum of the active powers produced by the voltage and current harmonics using the same serial number.

Key words: Harmonics, active power, electrical system

INTRODUCTION

In modern power systems, more energy is transmitted by no sinusoidal voltages and currents as a result of widely used nonlinear loads. Such no sinusoidal voltages and currents create problems concerning the measurement, determination and calculations of their harmonic contents.

The tariffs of the electric companies for billing the energy user depend on methods of measurement and calculation of the active power under no sinusoidal conditions. This is why it is necessary to determine correctly the active power in no sinusoidal circuits. The main goal of this research is to develop a model in order to perform experimental measurements of this power component and to compare the measured and the calculated ones.

EXPERIMENTAL MEASURING SETUP OF ACTIVE POWER

Joule-Lenz's law associates the active energy W with the heat H released as a result of a current flowing through a conductor with a resistance. It determines the energy and the average active power P, transferred during the periodτ, by:

$$H = W = \int_0^{\tau} u(t).i(t)dt = P.\tau \quad (1)$$

Thus:

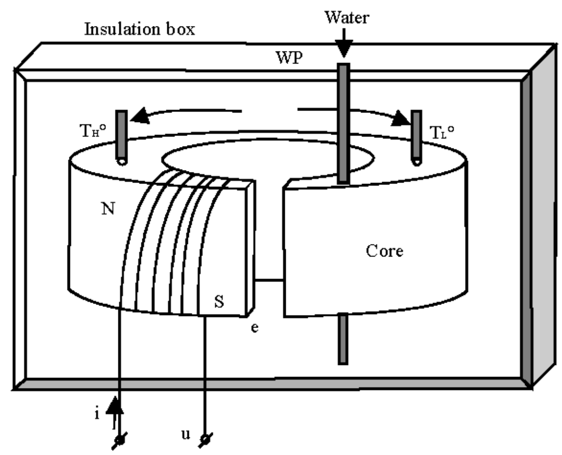

Fig. 1: Experimental model

$$P = \frac{W}{\tau} = \frac{H}{\tau} \quad (2)$$

In order to measure the active power experimentally a modified electrical model of Searle's apparatus is developed . Searle's apparatus allows direct measurement of the energy used in order to heat up a certain body. Since the heat is a form of energy transfer to the body, the developed model is capable of measuring this energy.

The model (Fig. 1) consists of a cylindrical ferromagnetic solid core with an air gap e intended to provide air isolation between the two core ends.

On one end of the core a copper coil N is wound. The voltage u applied across the winding causes the current *I* to flow in it. The current heats up the model core. The

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temperature rises at the core edge, where the winding N is wound and it becomes the hot end point of the core.

A temperature sensor measures the high temperature $T°_H$at this end. A water pipe WP is inserted through the other core end.

The cold water passing through the pipe cools down the second core edge. It becomes the *cold end point* of the core. A second temperature sensor measures the low temperature $T°_L$.

The temperature measurements are carried out at the moment when the temperatures at the two edges of the core stop changing. In order to reduce heat losses, the model is encased in a special insulation box.

The heat produced in the solid core due to the current flow is expressed by IEEE (2002):

$$H = C.\frac{F}{d}.(T_H^0 - T_L^0).\tau \quad (3)$$

Where:

C is the thermal conductivity Coefficient in [W m^{-1}.°c];

F the core cross-section in [m^2];

D: Distance between the temperature sensors in [m];

τ : Time measurement of the gradient temperature ΔT.

$$\Delta T = T_H^0 - T_L^0 \quad (4)$$

Thus using (2), (3) and (4):

$$P = C.\frac{F}{d}.\Delta T \quad (5)$$

EXPERIMENTAL MEASUREMENT OF ACTIVE POWER UNDER SINUSOIDAL CURRENTS

Six series of measurements are carried out with sinusoidal currents, whose RMS-values I are 0.5, 1, 1.5, 2, 3 and 5A, respectively. During each measurement series the RMS-value of the current is kept constant, while its frequency is changed from 50 Hz up to 1500 Hz. The voltage RMS-value, U, the phase angle φ between the voltage and the current and the temperature gradient, are measured. The active power is calculated by the formula:

$$P = UI\cos\varphi \quad (6)$$

The experimental and calculated results of the electrical and temperature parameters obtained for sinusoidal currents are given as a function of the frequency in Fig. 2-5, respectively. The voltage U

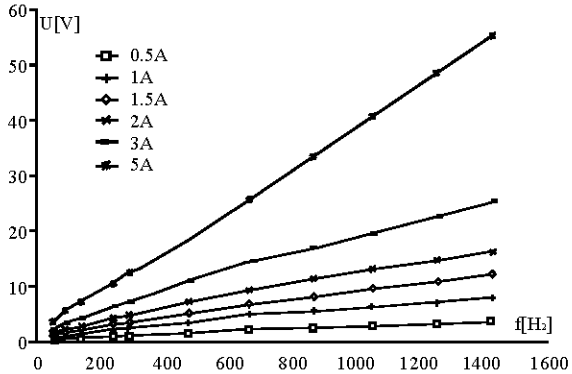

Fig. 2: Dependence U = f (*f*)

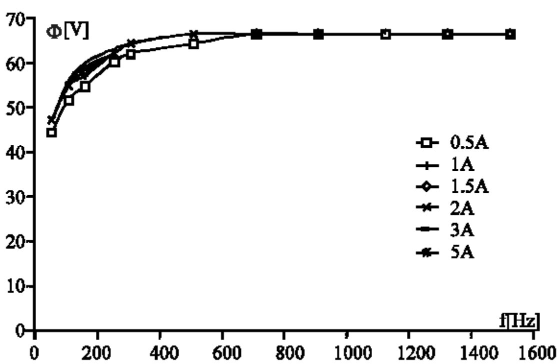

Fig. 3: Dependence φ = f (*f*)

applied across the winding grows with the increasing frequency f (Fig. 2). Since the current is kept constant follow that the model's resistance grows also with the increasing frequency. This can be explained by the existence of Skin effect in the solid core (Neiman, 1954; Ferrero, 1998; Gandelli *et al.*, 2000). In the presence of Skin effect the ratio between the core reactance and resistance is constant (Morando, 2001) i.e., the phase angle φ is constant too. The change of the phase angle φ during the experiment is shown in Fig. 3. In the range of the frequency between 50Hz and 500-700Hz, the angle φ increases and afterwards it is constant at the higher frequencies, because of the Skin effect in the solid body. It is equal to 65°.

The graphs of the active power P and the temperature gradient ΔT versus the frequency *f* are shown in Fig. 4 and 5. The two graphs are quite similar as the increase of the active power leads to the increase of the temperature gradient according to Eq. 5. Figure 6 shows the graph of the temperature gradient ΔT versus the active power P based on data from Fig. 4 and 5. All graph points in Fig. 6 lie on or are very close to a straight line. Therefore, it can be assumed that the

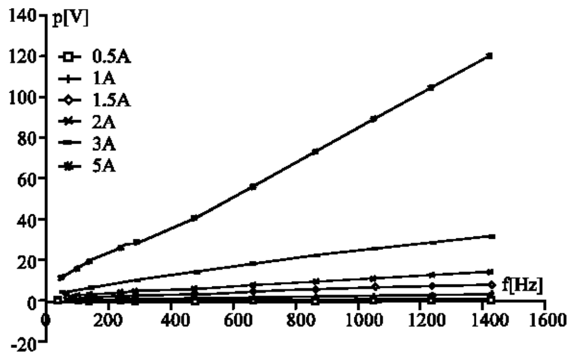

Fig. 4: Dependence P = f (*f*)

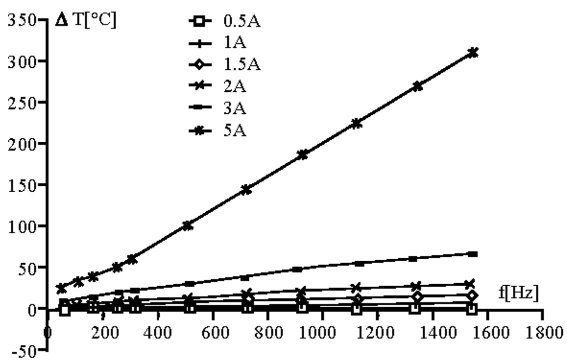

Fig. 5: Dependence ΔT = f (*f*)

dependence P = f (ΔT) is linear, i.e. the relation between the active power and the temperature gradient is constant:

$$K = C.\frac{F}{d} \qquad (7)$$

For the used study model it equals to *K* = 0.476 W/°C. Such a result leads to the conclusion that the temperature gradient versus the active power is independent on the frequency changes. The value of the particular active power produced in the solid core corresponds to the concrete temperature gradient value according to the formula:

$$P = K.\Delta T \qquad (8)$$

This conclusion will be used for the analysis of active power under no sinusoidal conditions.

TEST WITH HARMONICS CURRENTS

Three experiments under no sinusoidal conditions are made. During these experiments the following procedure is used: The voltage and current harmonic spectra and the

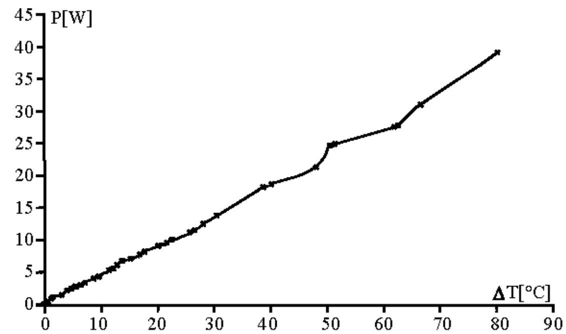

Fig. 6: Dependence P = f (ΔT)

temperature gradient ΔT are measured. Then, the temperature gradient ΔT is submitted into Eq. 8 in order to estimate the heat (active) power P_{mes} released into the solid core due to the no sinusoidal current flow through the winding.

As the active power is independent of the frequency, it is assumed that each "harmonic" *h* heats the core separately.

Then, taking into consideration the frequency and the current value, the phase angle is determined by the graph in Fig. 3. The active power of each harmonic is:

$$P_h = U_h I_h \cos\varphi_h \qquad (9)$$

Where: U_h and I_h are the measured RMS values of the voltage and the current, φ_h is the phase angle between them.

Finally, the total active power is calculated as:

$$P_{cal} = \sum_h P_h \qquad (10)$$

This sum P_{cal} is compared with the active power P_{mes} measured directly by the thermal method. The error ε is estimated by the following formula:

$$\varepsilon = \frac{P_{mes} - P_{cal}}{P_{mes}}.100\% \qquad (11)$$

Where: P_{mes} is the active power measured directly by the thermal method; P_{cal} is the calculated active power. An additional investigation is carried out. The temperature gradient for each P_h-value is read from the graph in Fig. 6 and summed up. The obtained sum is substituted into Eq. 8:

$$P_{th} = K\sum_h \Delta T_h \qquad (12)$$

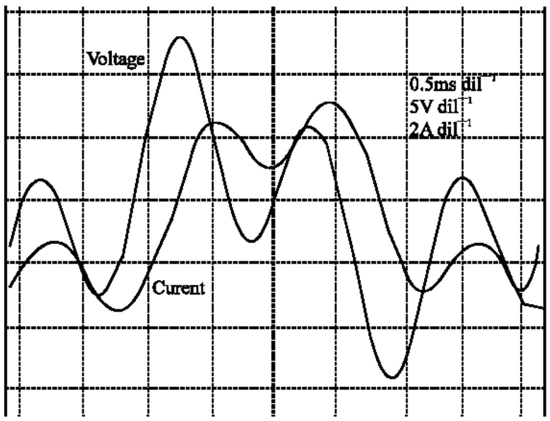

Fig. 7: Combination of voltage and current sinusoidal waveforms

The obtained active power P_{th} is compared with the active power P_{mes} measured directly by the thermal method. Then, it may prove that each current harmonic or respectively each harmonic power component of the active power contributes to the heating of the core. In the next three experiments this statement will be verified.

First experiment: During the first experiment two voltage waveforms of 300Hz and 900Hz are mixed. The obtained no sinusoidal voltage, shown in Fig. 7, is applied across the winding *N* of the study model (Fig. 1). As a result a no sinusoidal current flows through the winding N. Its shape is also shown in Fig. 7.

The current heats up the model core and the measured temperature gradient is ΔT = 14,67°C. Thus, substituting this value into Eq. 8 the active power, obtained experimentally by the thermal method, is found and it equals P_{mes} = 6.98W.

The voltage and current RMS harmonic spectra and the phase angles values (φ_h) are shown in Table 1. By using these data the harmonic active power for each harmonic (first and third) is calculated by Eq (9). Their sum ($P_{cal} = P_1 + P_3$ = 6,82W) is very close to the active power measured directly by the thermal method (P_{mes} = 6.98W). The error calculated is 2.29%. Therefore, it may be supposed that each current harmonic (first and third) heats the core of the study model separately. To check this statement for each calculated value of the active power P_h (the basic and the third harmonic) the corresponding temperature gradient ΔT_h (Table 1) is read from the graph of the dependence of the active power on the released heat in Fig. 6.

The obtained results for the temperature gradient of each harmonic ΔT_h are summed up ($\Delta T_1 + \Delta T_3$ = 15 °C). This sum is submitted in Eq (8).

The calculated active power value of P_{th}= 7.14 W is very close to the active power P_{mes}= 6.98W.

Table 1: Combination of two sinusoidal waveforms-measured and calculated results

Measured values			*Calculated values*		
f[Hz]	300	900	ΔT [°C]	P [W]	ε%
U_h [V]	4.9	5.6	-	-	-
I_h [A]	2	1	-	-	-
φ_h[°]	63	65	-	-	-
P_h[W]	4.45	2.37	-	6.82	2.29
ΔT_h [°C]	10	5	15	7.14	2.29
$\Delta T = T_1° - T_2°$ = 46°C – 31.33°C			14.67	6.98	-

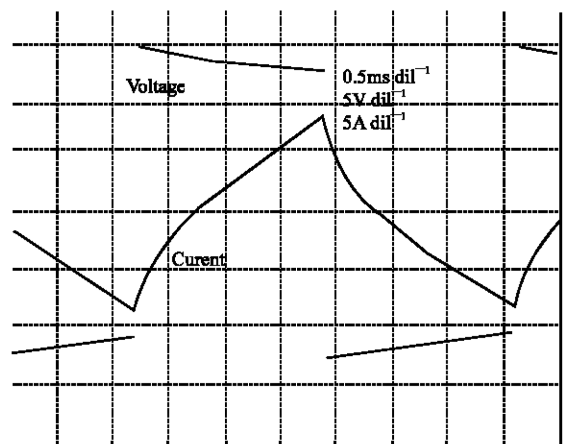

Fig. 8: A rectangular voltage waveforms and the obtauned current waveforms

The error calculated is 2.29%. Therefore, the following conclusion is drawn: each active power harmonic contributes independently to the heating of the core.

Second experiment: During the second experiment a rectangular voltage waveform of basic frequency 300Hz (Fig. 8) is applied to the winding N of the experimental model.

The current, which flows through the winding, has also a no sinusoidal waveform (Fig. 8). There, as in the previous experiment, the same approaches to find the active power is applied. The measured and the calculated results are shown in Table 2.

The 2 values of the active power obtained by using the electrical (P_{cal} = 30.4 W) and thermal (P_{th} = 31.41W) calculations are very close to the measured directly by the thermal method value (P_{mes} = 30.04 W), Table 2. Therefore, the same conclusion holds for this case - every active power harmonic contributes independently to the heating of the core.

Third experiment: During the third experiment a no sinusoidal current of 50 Hz basic frequency (Fig. 9) flows through the winding *N* of the study model. This current is obtained as a combination of the input currents of several

Table 2: A rectangular voltage waveform and the obtained current-measured and calculated results

Measured values					*Calculated values*		
f[Hz]	300	900	1500	2100	ΔT [°C]	P [W]	ε %
U_h [V]	12.47	4.21	2.37	1.6	-	-	-
I_h [A]	5	0.9	0.35	0.2	-	-	-
φ_h[°]	63	65	65	65	-	-	-
P_h[W]	28.31	1.6	0.35	0.14	-	30.4	1.2
Δt_h [°C]	62	3	0.71	0.27	65.98	31.41	4.56
$\Delta T = T_1° - T_2° = 108.7°C - 45.6°C$					63.1	30.04	-

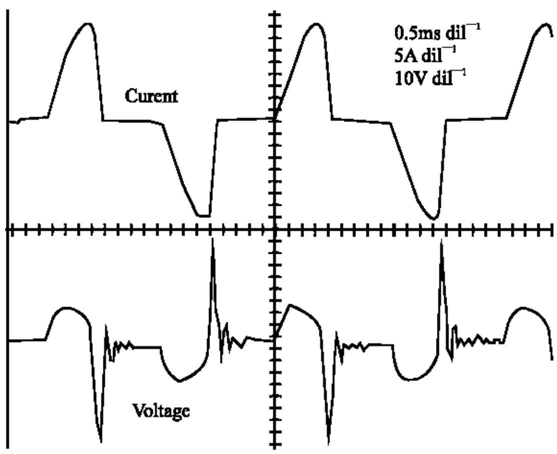

Fig. 9: Eds-Expermental voltage and current waveforms

Electronic Devices (EDs). The voltage across the winding of the study model is also no sinusoidal and its waveform is shown in Fig. 9.

The measured and the calculated results are given in Table 3.

The two values of the active power obtained by using the electrical (P_{cal} = 12.26W) and thermal (P_{th} = 12.57W) calculations are very close to the measured directly by the thermal method value (P_{mes}= 12.28 W) (Table 3).

Therefore, the same conclusion is made: Each active power harmonic contributes independently to the heating of the core.

VALIDATION OF THE CONCEPTUAL MODELLING

Currently there are various methods for power calculations under no sinusoidal conditions. In 1927 Budeanu was the first one to develop a theory for calculating power under no sinusoidal conditions by using Frequency Domain Methods (Budeanu *et al*., 1938). A few years later, in 1932, Fryze developed a theory using Time Domain Method (Fryze, 1932).

Budeanu and Fryze obtain the active power expression in the same way by integrating the

Table 3: EDs experimental voltage and current-measured and calculated results

Measured values						*Calculated values*		
f[Hz]	50	150	250	350	450	ΔT [°C]	P [W]	ε %
U_h [V]	1.95	6.93	3.45	2.54	2.33	-	-	-
I_h[A]	3.02	1.74	0.6	0.4	0.28	-	-	-
φ_h[°]	47	58	59	62	63	-	-	-
P_h[W]	4.02	6.39	1.07	0.48	0.3	-	12.26	0.16
ΔT_h [°C]	9	14	2	0.9	0.5	26.4	12.57	2.36
$\Delta T = T_1° - T_2° = 59.8°C - 34°C$						25.8	12.28	-

instantaneous power. Budeanu expresses it in the Frequency domain:

$$P_{cal} = \sum_h U_h I_h \cos\varphi_h \quad (13)$$

While Fryze-in the Time domain:

$$P = \frac{1}{\tau}\int_0^{\tau} u(t).i(t)dt \quad (14)$$

Nowadays a number of authors have suggested methods for power calculation, which are further developments of Budeanu's and Fryze's theories Although none of the authors that suggest a method for calculation of power under no sinusoidal conditions contest the nature of the active power, there are several suggested methods for active power calculations under no sinusoidal conditions.

Shepherd and Zakhihani (1972) define differently the active power component:

$$P_R = \sqrt{\sum_h U_h^2 \sum_h I_h^2 \cos^2\varphi_h} \quad (15)$$

This power is influenced by the presence of the active nonlinear loads in the power systems.

Slonim and Van Wyk (1988) accept Budeanu-Fryze's definition of the active power, but they claim that the active power value is influenced by the presence of the linear and nonlinear active loads in the power system. Thereafter, they add to the active power defined by Budeanu and Fryze, the distortion power component related to the presence of the linear and nonlinear active loads in the power system:

$$P_T = \sqrt{\left(\sum_h U_h I_h \cos\varphi_h\right)^2 + \sum_{K \neq h} (R_h - R_K)^2 I_h^2 I_K^2} \quad (16)$$

Where: R_h and R_k are the resistance with harmonic numbers *h* and *k*; I_h and I_k are the current harmonics with numbers *h* and *k*.

Emanuel (1994) accepts also the active power of Budeanu and Fryze, but he states that the basic harmonic of the active power

$$P_1 = U_1 I_1 \cos\varphi_1 \quad (17)$$

must be measured separately from the remaining active power, due to the presence of the high harmonic in the system:

$$P_H = \sum_{h \neq 1} U_h I_h \cos\varphi_h \quad (18)$$

Comparing the formulae for calculation of the active power suggested by the authors mentioned above, it is obviously that the formulae of Budeanu and Fryze coincide with the experimentally proved statement that the total active power is a sum of the active powers produced by the voltage and current harmonics with the same serial number.

CONCLUSION

The experiments with sinusoidal currents changing widely in amplitude and frequency lead to the conclusion that the generated active power is independent of the current frequency.

It is linearly dependent on the temperature gradients produced by the currents, because the ratio *K* between the active power and the heat released (temperature gradient) is constant.

This conclusion is applied to the analysis of the active power under no sinusoidal conditions.

As the active power is independent of the frequency, it is assumed that when both the voltage and the current or just one of them is no sinusoidal each "harmonic" heats the core separately.

The experiments with no sinusoidal voltages and currents help us to prove experimentally that the total active power under no sinusoidal conditions is a sum of the active powers produced by the voltage and current harmonics with the same serial number.

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