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Transducer (Accelerometer) Modeling and Simulation

S. Saad, Z. Ghemari, L .Herous and O.E. Hadjadj
Laboratoire des Systèmes Electromécanique, Université Badji-Mokhtar Annaba, Algeria

Abstract: In this research, the accelerometer is modelled and simulated, in order to choose the accelerometer corresponding to the vibration generated by the machine. A mathematical model of the accelerometer and a program of simulation working in MATLAB environment are proposed. This simulation enables to calculate the relative movement modulus, the measurement errors, damping ratio and also the choice of the accelerometer frequency band. The results obtained are discussed and can make easier the choice of the accelerometer that suits the vibration generated by the machine.

Key words: Accelerometer, simulated MATLAB, damping, machine

INTRODUCTION

The importance of the vibration in the industrial field has encouraged many studies and research work to be carried on the sensor recording (Froging Mills, 1999; Placko *et al.*, 2000; Giachno, 1986; Favennec, 1987; Asch *et al.*, 1998) these vibratory movements. These sensors provide information on machine state (Bigret and Feron, 1995; Arques, 1996; Boulenger, 1998) which enables to establish a program of preventive and predictive maintenance.

Any machine under operation produces a degree of vibration generated by its rotary or linear movements. These vibrations are detected by using a sensor called accelerometer or vibration sensor (Schenck, 1994; Boulenger and Pachaud, 1998). Its function is to transform the level of vibration into a time electric signal. The rest of the measuring equipment (amplifier, analyzer FFT) converts the time electric signal to a frequency electric signal from the operations of signal processing using the software. These operations are signal generation, signal decomposition and calculation of the signal spectrum. The purpose of the signal conversion from the time to the frequency domain is to identify the frequencies of vibration (Rasolofondraire, 1995; Peterson, 1998; Thomess, 1999).

This study is focused only on the accelerometer and its parameters which are the relative movement modulus, the measurement error, damping factor and the frequency band. In order to realise this, it is necessary to determine the accelerometer mathematical model and then simulate it to find the parameters indicated above.

Accelerometer modelling: An accelerometer is a sensor which is fixed directly on the vibrating structure to measure its vibrations. As it vibrates with the structure, it does not measure the absolute movement y(t) of the structure, but the relative movement z(t) which have to be interpreted to extract information about the absolute movement.

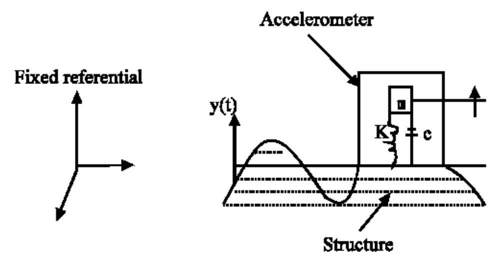

Fig. 1: The Accelerometer modeling

The accelerometer is regarded as a system composed from a mass, a spring and a damper, defined by m, K and C, respectively Fig. 1.
m: is the mass, K: the elasticity modulus, C: the friction factor.

The relative movement is defined as:

$$z(t) = x(t)-y(t) \tag{1}$$

Where, z(t) represents the relative movement of the mass (m) according to the structure base.
By applying Newton law, the equation of the movement will be:

$$m\ddot{x}(t) = -k(x(t)-y(t))-c(\dot{x}(t)-\dot{y}(t)) \tag{2}$$

$$m\ddot{x}(t)+c(\dot{x}(t)-\dot{y}(t))+k(x(t)-y(t)) = 0 \tag{3}$$

Corresponding Author: S. Saad, Laboratoire des Systèmes Electromécanique, Université Badji-Mokhtar Annaba, Alegria

From (1) we obtain the following equations:

$$\dot{x}(t)-\dot{y}(t) = \dot{z}(t) \quad (4)$$

$$\ddot{x}(t) = \ddot{y}(t)+\ddot{z}(t) \quad (5)$$

Substituting (1), (4) and (5) in (3), we obtain a second-order differential equation with its variable represents the relative movement.

$$m\ddot{z}(t)+c\dot{z}(t)+kz(t) = -m\ddot{y}(t) \quad (6)$$

Supposing a harmonic movement of the structure and then finding the amplitude Y of the following equation:

$$y(t) = Y e^{tax} \quad (7)$$

$$z(t) = Z e^{tax} \quad (8)$$

Using Laplace transformation, the equation presented below is obtained:

$$(-m\omega 2+i\omega c+k)Z = m\omega 2Y \quad (9)$$

Simplifying by m, will lead to:

$Z = m\omega^2Y/(-m\grave{u}^2+i\omega c+Z = \omega^2Y/(-\omega^2+i\omega c/m+k/m)$
$Z = \omega^2Y/(-\omega^2+i.\omega.2.\omega_n c/2\omega_n\, m+\omega_n^2)$

$$Z=\frac{\omega^2 Y}{(\omega_n^2-\omega^2)+i(2\xi\omega\omega_n^2)} \quad (10)$$

Where:

$\omega_n = (k/m)^{1/2}$ represents the accelerometer natural frequency and $\xi = c/(2m\,\xi_n)$ represents its dumping factor After the determination of sensor (accelerometer) mathematical model it is necessary to test by simulation this model using a developed program working in MATLAB environment (Brian and Breiner, 1999) by introducing real data.

Simulation of accelerometer relative movement modulus and measurement error

Relative movement modulus simulation: Modulus graphical representation of the standardized equation of the relative movement for different values of æ, is obtained by using a developed program working in Matlab environment.

Having:

$$Z = \frac{Y}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \quad (11)$$

Z: the relative movement of the accelerometer
Y: The movement amplitude
ξ : Le dumping factor
ω_n: The natural frequency of the accelerometer
ω: The relative frequency
ω/ω_n : Frequency ratio

Values of the relative movement modulus parameters are presented in the Table 1.

The curves presented below are obtained by varying the accelerometer dumping factor from 0.1 to 0.9 with a natural frequency of 100 Hz and a relative frequency varying from 0 to 40 Hz to determine the relative movement Z(t) is showing in Fig. 2.

It is necessary to find the measurement error of the relative movement to choose the dumping factor of the accelerometer.
The measurement error is given as follows:

By multiplying the expression (10) by ω^2 we obtain:

$$\omega^2 Z = \frac{\omega^2 Y}{\sqrt{(1-(\omega/\omega_n)^2)^2+(2\xi\omega/\omega_n)^2}} \quad (12)$$

According to Eq. 12, it can be noticed, that the structure acceleration amplitude is expressed as $\ddot{Y} = \omega^2$ Õ (absolute acceleration), which is the required expression.

If an accelerometer is chosen with $\omega/\omega_n \ll 1$ to decrease the measurement error to its lowest value, then the Eq. 12 is simplified to:

$$\ddot{z} = \omega_n z \approx \ddot{Y} \quad (13)$$

The relative movement measurement, defined by $\omega_n^2 Z$, gives a representative value of the amplitude of the structure acceleration at ω frequency.

Table 1: The movement amlitude and values of the relative movement modulus Parameters

Input parameters	Values
The movement amplitude (m)	0.015
Dumping factor	For ξ = 0.1, 0.2, 0.3, 0.5, 0.9
the natural frequency (Hz)	100
the relative frequency (Hz)	0, 5, 10, 15, 20, 25, 30, 35,40
Frequency ratio	0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4

Table 2: The values of the parameters used for the accelerometer measurement error determination

Input parameters	Values
Dumping factor	For ξ = 0.4, 0.6, 0.65, 0.7, 0.8
The nature Frequency (Hz)	100
The relative Frequency (Hz)	0.5, 10, 15, 20, 25, 30

Table 3: The values of the simulation parameters of the relative movement modulus

Input parameters	Values
The movement amplitude (m)	0.015
Dumping factor	0.65
The nature frequency (Khz)	100
The relative frequency (Khz)	0,5,10,15,20

Table 4: The result obtaine by simulation are summarized in simulation results

Y	ξ	ωn	ω	ω/ω_n	Z	E
0.015	0.65	100	0	0	0	0
0.015	0.65	100	5	0.05	0.3751	0.001
0.015	0.65	100	10	0.1	1.5023	0.003
0.015	0.65	100	15	0.15	3.386	0.0065
0.015	0.65	100	20	0.2	6.0326	0.0109

The measurement error carried out is defined as:

$$E = (\ddot{z}/\ddot{Y}) - 1 = \frac{1}{\left((1 - (\omega/\omega_n)^2)^2 + (2\xi\omega/\omega_n)^2\right)} - 1 \quad (14)$$

Measurement error simulation: A graphical representation of accelerometer measurement error according to frequency ratio for different values of æ is obtained by using a developed program working in Matlab environment.

Where:

$$E = (\ddot{z}/\ddot{Y}) - 1 = \frac{1}{\left((1 - (\omega/\omega_n)^2)^2 + (2\xi\omega/\omega_n)^2\right)} - 1$$

$\omega/\omega_n << 1$ for the accelerometer choice
E : The accelerometer measurement error
are presented in Table 2.

Simulation of the relative movement modulus and the measurement error of an accelerometer: The mathematical model obtained by the finite element method has made possible the simulation by a developed program working in Matlab environment, which calculates the frequency ratio ω/ω_n, the relative movement modulus and the measurement errors of an accelerometer with different values of relative frequency of the movement.

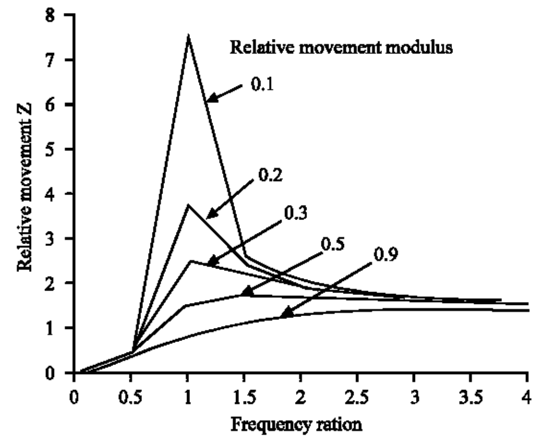

Fig. 2: Relative movement modulus as a function of ζ (simulation result)

With:

$$Z = \frac{Y}{\left[\left(1 - \left(\frac{\omega}{\omega_n^2}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

$$E = (\ddot{z}/\ddot{Y}) - 1 = \frac{1}{\left((1 - (\omega/\omega_n)^2)^2 + (2\xi\omega/\omega_n)^2\right)} - 1$$

and the measurement error of the accelerometer are presented in the Table 3.

The results obtained by simulation are summarized in the following Table 4.

The curve illustrated below is obtained by plotting the measurement error as a function of the relative movement.

RESULTS AND DISCUSSION

From the curves obtained and presented in Fig. 2, 3 and 4, it is noticed that the relative movement (z) module is having a decreasing relationship with dumping factor.

The relative movement modulus is at its maximum value when dumping factor is at its minimum value (0.1).The relative frequency equal to the natural frequency of the accelerometer (the frequency ration = 1).

The module of relative movement takes the minimal value in the case of the value of rate of depreciation equal to 0.9 (the maximum value) and the relative frequency equal to the natural frequency of the accelerometer (frequency ratio equal to 1).

The curves obtained by simulation show that the measurement error depends on the accelerometer internal damping. As shown in Fig. 3, a damping of 65% limits the error to 1%.

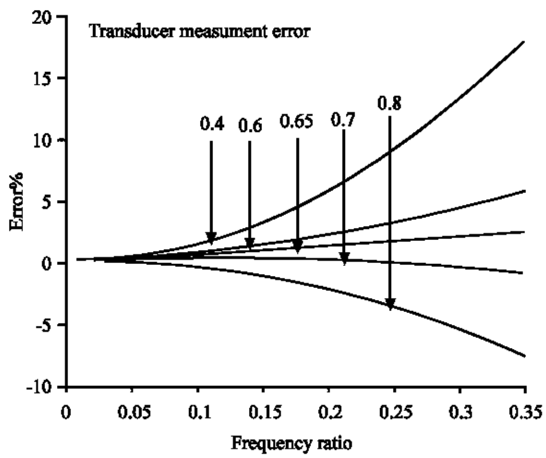

Fig. 3: The accelerometer measurement error (simulation esults)

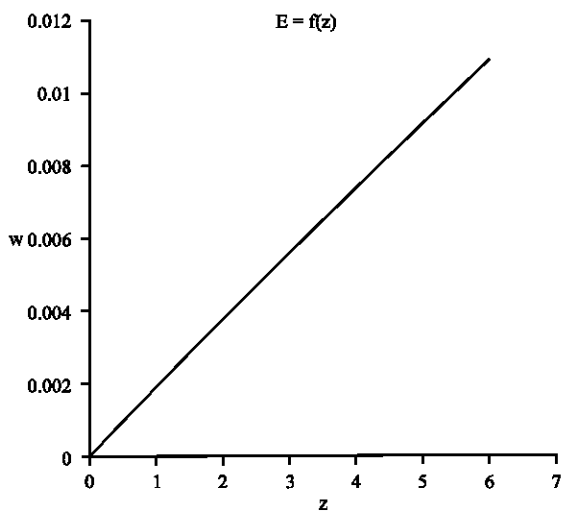

Fig. 4: Measurement error according to relative movement

According to the results obtained and presented in Table 3, it is noticed that, for a constant dumping factor of 0.65, the measurement error is proportional to the relative movement modulus with a variation of frequency ratio 0-0.2.

The accelerometer natural frequency must be selected so that $\omega_n > 3\omega_{max}$ ou ω_{max} and is determined from vibration curve. That is, a machine generating a maximal vibrating movement of 1 KHz, thus ω_n =3 KHz with a dumping factor of 0.65 to limit the measurement error to 1%.

The curve in Fig. 4 shows that, there is a linear relationship between the relative movement and the measurement error.

CONCLUSION

From the mathematical model, the equation of the accelerometer relative movement modulus and the measurement error according to the dumping factor and frequency ratio is determined. From this, it can be concluded that:

- The measurement error depends on the accelerometer internal dumping.
- A damping of 65% is required to limit the error to 1%.
- To obtain right values, the movement relative frequency must be lower than the accelerometer natural frequency ($\omega \ll \omega_n$).

In the future, it is also important to study the reliability of the accelerometer which can contribute to determine the vibration level with accuracy.

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