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Fault Tree Reliability Modelling of Complex Systems

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Abstract: By this research project we hope to contribute to the automation of the reliability of complex systems with failure delay by introducing the Fault Tree (FT) technique. So, we present gates delays transformation method in a tree logic and intermediate gates delay elimination. We defined gate delay domains notion and developed a transformation algorithm of gates delays in F.T. We have considered the cases of systems with non reparable and reparable components with random and determinist failure delays. To calculate the probability of occurrence of the unwanted events in an FT, we used common laws of life expectancy to determine the distribution functions and probability density of life expectancies of system components.

Keys words: Reliability systems, fault-tree, failure delay systems, evaluation systems

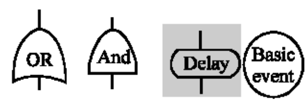
INTRODUCTION

The increasing complexity of the industrial equipments of systems with important dimensions and the importance of the incurred risks of default. This has given the FT technique great importance and an extremely useful tool in the analysis of complex systems safety of exploitation. We investigated in particular systems characterized by failure delays (Limnio, 1992) strong interactions between ageing components. The FT method (Barlow and Lambert, 1975) is well adapted and consists in representing the logic of the interactions between diverse elements at various levels of a system, with the help of intermediate logical operators. This is based on two main steps:

- Tree construction
- Qualitative and quantitative evaluation.

After the introduction of the pseudo Boole algebra (Schneeweiss, 1981) to process the delays of failure and the recursive approach (Page and Perry, 1986) who has simplified the qualitative evaluation (cut sets) in an FT. The FT technique has became a major tool in the analysis of complex systems reliability. The non simultaneity principle (Schanthikumar, 1986) is integrated into the modelling and the representation of failure process. We investigated the failure delay in the representation of fault tree logic and presented a gates delays transformation method. We developed a corresponding algorithm. Certain events (components) have been modelled by determinist and random life expectancy processes (Dunbar, 1984).

We have done the calculation of distribution functions and probability density of components and system life expectancy (Theologou and Limnios, 1990). This would help in the evaluation of occurrences probabilities of events in a considered FT example (Pages and Gondran, 1980).



MATERIALS AND METHODS

Delay gate processing in FT:

The input DG is an AND gate: Before the transformation of delay (Fig. 1) we have:

$$S_1(t) = \begin{cases} 0 & \text{if } 0 \le t \le \tau \\ Y(t-\tau) = X_1(t-\tau)X_2(t-\tau) & \text{if } t \le \tau \end{cases}$$

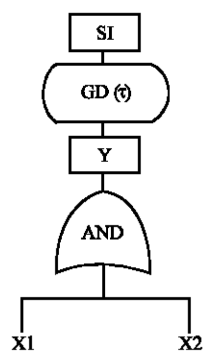


Fig. 1: Before the transformation of delay

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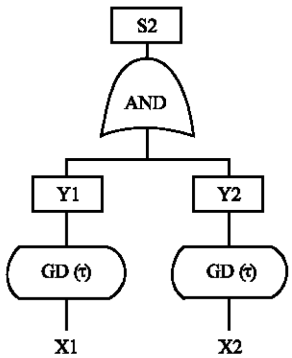

Fig. 2: After the transformation of delay

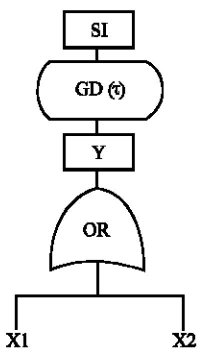

Fig. 3: Before delay transformation

After the transformation of delay (Fig .2) we have:

$$S_2\ (t) = \begin{cases} 0 & \text{if } 0 \le t \le \tau \\ Y_1(t)\ .\ Y_2(t) = X_1(t-\tau)\ .\ X_2(t-\tau) & \text{if } t \le \tau \end{cases}$$

Then we can write:

$$S_1(t) = S_2(t), \forall t \ge 0$$

The input DG is an OR gate: Before delay transformation (Fig. 3) we have:

$$S_1(t) = \begin{cases} 0 & \text{if } 0 \le t \le \tau \\ Y(t-\tau) = X_1(t-\tau)\ +\ X_2(t-\tau) & \text{if } t \le \tau \end{cases}$$

After the delay transformation (Fig. 4) we have:

$$S_2\ (t) = \begin{cases} 0 & \text{if } 0 \le t \le \tau \\ Y_1(t)\ +\ Y_2(t) = X_1(t-\tau) + X_2(t-\tau) & \text{if } t \le \tau \end{cases}$$

We can write:

$$S_1(t) = S_2(t)\ \ ;\text{ for all } t \ge 0$$

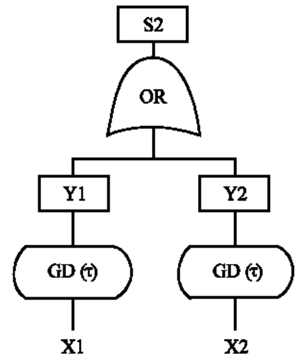

Fig. 4: After the delay transformation

Transformation algorithm: This algorithm developed for power plant reliability calculation program (Abderrezak, 2004) allows to establish domains and associated time delays in FT, once the FT is introduced in the program

$$\text{If } DG_i \in S \text{ Then } D(DG_i) = \{i_1,\ldots,i_k\}$$

In every ii ∈ °D (DGi), we associate the delay τi corresponding to the gate delay GDi.

If i belongs to the domains of several gates delays, then the delay (τ) associated is equal to the sum of the gates delays.

RELIABILITY QUANTITATIVE EVALUATION

We only consider the evaluation of the occurrences probabilities occurrences of components events from the distributions functions and those of probabilities densities according to Pagès and Gondran (1980) where we have:

The component Reliability i

$R_i(t) = P_i$ {the i component failing between (0, t)}

The repartition function F

$$F(t) = 1 - e^{-\lambda t}$$

The probability of density of T

$$f(t) \frac{dF(t)}{dt} = \lambda\ e^{\lambda t}$$

If the component is of time delay (τ) with constant λ, its occurrence probability is:

$$\text{Prob} = 1 - e^{-\lambda(t-\delta)}$$

For complex systems with default time delay we considered the cases of non reparable components and reparable with fixed and random default delays.

Systems with non reparable components: According to the associated delay nature of FT basic components, we have:

Fixed delay (τ): The F of T of component is formulated by:

$$F_T(t) = P\{T \leq t\} = P\{X+\tau \leq t\} \text{ hence :}$$

$$F_T(t) = \begin{cases} F_x(t-\tau) & \text{for } t \geq \tau \\ 0 & \text{for } 0<t<\tau \end{cases}$$

the f(t) :

$$f_T(t) = \lambda\, e^{-\lambda(t-\tau)}$$

Random delay: According to the associated delay nature of FT basic components, we have:

$$F_T(t) = P\{X+\tau \leq t\}$$

If X and τ follow, respectively, an exponential and Weibull laws.

The system f(t) and F(t) are deduced from their respective functions convolution (Theologou and Limnios, 1990) where:

$$F_T(t) = (F_X * F\tau)(t) = F \int_0^t F_\tau(t-u)\,dF_X(u)$$

the f(t) is :

$$^T(t) = f_\tau(t) * f_X(t) = \int_0^t \lambda\, e^{-\lambda\ (t-u)}\, 2\, u\, e^{-u^2} du = 2\, \lambda e^{-\lambda t}\ I(t)$$

Systems with reparable components: According to the associated delay nature of FT basic components, we have:

Fixed delay: The F(t) and f(t) of T of the component and of the system when the variables X and Y follow respectively the exponential laws of respective parameters λ and μ, are expressed by Shanthikumar (1986):

$$\tilde{F}_T(S) = \int_0^t e^{-st} dF_T(t)$$

the pdf is:

$$f_T(t) = \lambda \sum_{n=0}^{r-1} (\lambda\mu)^n\ e^{-(n+1)\mu\tau}\ \Phi(t-(n+1)\tau) \qquad \text{pour } t = \tau$$

où $$\Phi(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

Random delay: The F(t) and f(t) depend an usual laws of T and τ where we have:

$$F_T(s) = \frac{c\lambda}{s(s+\lambda+\lambda+\mu+c)+\lambda c} \text{ and}$$

$$F_T(t) = \frac{c\lambda}{s_2 - s_1}\ (e^{s}2^{t} - e^{s}1^{t})$$

Where c is an exponential law parameter, s_1 and s_2 are the following equation roots (Theologou and Limnios, 1990):

$$S^2 + (\lambda+\mu+C)S + \lambda C = 0$$

RESULTS AND DISCUSSION

Application example: We considered a non reparable system with reparable and non reparable components. The default delays are fixed and random. The FT representing the system is shown in Fig. 5.

Table 1: The basic components data

Components	λ (1/h)	μ (1/h)	d (1/h)
1	10^{-4}	10^{-2}	
2	10^{-3}	10^{-1}	2.10^{-1}
3	10^{-4}	10^{-2}	2.10^{-2}
4	10^{-3}	10^{-1}	2.10^{-1}

Where d data column for non reparable component with random delay. Gates delay: τ_1 = 10 th and τ_2 = 5 h

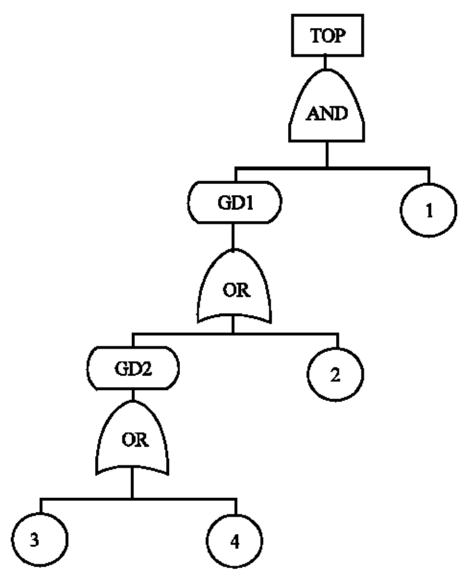

Fig. 5: Fault-tree with délay

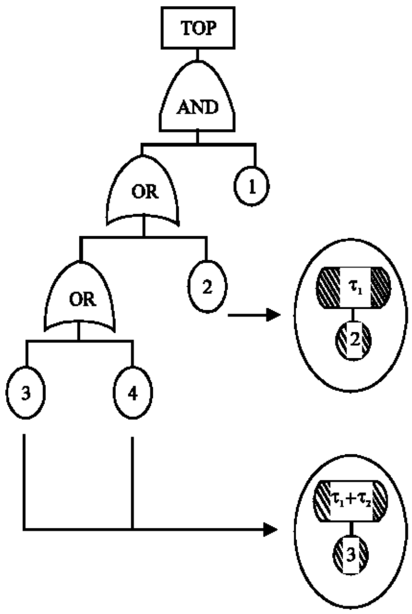

Fig. 6: Basic components (events) delays association

The basic components data are empirical and shown in the following Table 1:

The FT considered processing:

- Tree transformation
- Gates delays domains determination GD1 and GD2
  D (GD1) = {2,3,4}
  D (GD2) = {3,4}
- Basic components (events) delays association for component 2 we have:

$\tau_1 = 10$ h
For components 2, 3 and 4 we have:
$\tau_1 + \tau_2 = 15$ h
The components 2', 3' and 4' become fictitious their data change (Fig. 6).

Occurrences probabilities calculation: We determined the occurrences probabilities of undesirable events of FT. On the basis of repartition functions equations (F(t)) and the probability densities (f(t)) of life expectancy of components and system.

Non reparable components: The basic events occurrence probabilities calculation results with an exploitation duration time t = 100 h, are gathered in the Table 2:

Occurrence probability calculation of the FT top event for the same exploitation time duration (100h) are:
TOP Prob = 1.008299E-02,for random delay
TOP Prob = 1.008024E-02, for fixed delay

Table 2: The basic events occurrence probabilities calculation results

Component	Fixed delay	Random delay	Without delay
1			0.01
2'	$9{,}1393.10^{-2}$	$9{,}04837.10^{-2}$	
3'	$9{,}91536.10^{-3}$	$9{,}9005.10^{-3}$	
4'	$9{,}18512.10^{-2}$	$9{,}04837.10^{-2}$	

Table 3: The basic components occurrence probabilities evaluation

Component	Fixed delay	Random delay	Without delay
1			0.00629
2'	$2{,}1799.10^{-2}$	$6{,}2440.10^{-2}$	
3'	$8{,}5434.10^{-3}$	$6{,}2985.10^{-3}$	
4'	$2{,}1799.10^{-2}$	$6{,}2440.10^{-2}$	

Reparable components: The basic components occurrence probabilities evaluation: The component 1 has no delay, components to 3 and 3 have become fictitious and their occurrence probability is evaluated according to the delay nature.The calculation results are gathered in the following Table 3:

The top events occurrence probabilities for τ fixed and random for an exploitation duration time of 100 h are:
Top Prob = 1.008299E-02 fixed delay
Top Prob = 1.008024E-02 random delay

CONCLUSION

By this research project we have shown that the FT intermediate gates delays transformation consists in their transformation to the basic events level, without changing their output parameters

This process allowed their elimination from the tree representation.

By their delays association with the tree basic events data.

By the integration of F and pdf of components life expectancies for the calculation of the system reliability. It is also possible to have global system model and to make the parameters reliability calculation automatic.

Notation:

GD	:	Delay gate (delay operator)
S:		Set of delay gates
Gdi	:	Delay gate belonging to S
τ	:	Failure time delay
τ_I	:	GDi time delay expressed in hours
T	:	Random variable of system life time
f (pdf)	:	Component probability density function of T
$D(DG)_i$	:	Delay gate domain
I	:	Basic event of FT
$\{i_1,\ldots,i_k\}$	:	Set of D(DG) basic events
I'	:	Fictitious basic event
λ	:	Mean failure rate

μ	:	Mean repair rate
F	:	T repartition function
F	:	Laplace-Stieltjes transform
*	:	Convolution product
S_1	:	Indicating variable (i.v) of output event gate
S_2	:	I.v output of the event of gate AND or OR.
Y	:	I.v input event of delay gate before transformation.
Y_1, Y_2	:	I.v. output events of delay gate after the transformation.
X_1, X_2	:	i.v input gate events.

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