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Numerical Simulation of a Permanent Magnet Synchronous Machine

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Abstract: In this research, project a permanent magnet synchronous machine is simulated using the developed mathematical models. In the synchronous motor model we have simulated its operation under a variable load. Its apparent power is 6.2 kva and its maximum speed is 30000 tr mn^{-1}, its power supply was 500 V. The synchronous motor has four poles and permanent magnets with Br =0.82T. The rotor mean radius is 39.5 mm with an active length of 50 mm.Two different methods have been used: In the first approach the unknown variables are vector potential nodal values and mesh currents. In the second approach only the potential vector is kept as an unknown variable and expressing the electrical values in function of the same vector.

Key words: Synchronous motor, simulation, discretisation implicit type crank-nicholson back-ward, euler, mathematical models

INTRODUCTION

In the mathematical electromagnetic system model in the bi-dimensional approximation case leads to the resolution of the well known equation:

$$\sigma\frac{\partial A}{\partial t} + \mathrm{rot}(\upsilon.\mathrm{rot}.A) - J = 0 \text{ in the domain } \Omega \subset R^2 \quad (1)$$

by taking into account the boundary conditions. In this expression σ represents the conductivity A ≡ (0,0,A) In the case of moving systems, as rotating machines i.e: taking into account the displacement can be done by resolving Eq. 1 in two different references. In these conditions the term σ(υ∧ rot. A) do not appear explicitly (Piriou and Razek, 1983). Modelling the interface between these two references, different techniques could be used: Displacement strip, macro-element, sliding line.

The current density in Eq. 1 can be expressed in the form:

$$J = J_0 + \sum_{k=1}^{n_p} i_k.\Psi_k \quad (2)$$

Where J_0 represents an imposed current density (a given current source or equivalent current density in the case of permanent magnet). i_k is the phase or branch (k) current, n_p is the number of phases included in the Ω domain and Ψka function of $L^2(\Omega)$ defined as being the current density in space, corresponding to a current of one Ampere in phase (k) (obviously Ψk ≡ 0 outside phase (k) branch). The electric equations could be written for the branches (k), $1 \le k \le np$, with boundaries k1 and k2 under the form:

$$(V_{k1} - V_{k2}) = R_k i_k + \frac{d\Phi_k}{dt} + l_k \frac{di_k}{dt} \quad (3a)$$

and for the other branches q,$np<q\le n_b$ of the electric circuit that could contain resistances, inductances, voltage generators and non-linear devices such as diodes:

$$(V_{q1} - V_{q2}) = e_q + R_q i_q + l_q \frac{di_q}{dt} + \gamma(i_q) \quad (3b)$$

Where n_b represents the electric circuit total number of branches, $(V_{k1}-V_{k2})$ or $(V_{q1}-V_{q2})$ the potential difference across branch k or q, Rk or Rq resistances, lk or lq inductances not taken into account in the magnetic equations, Φk the magnetic flux in a coil and $\gamma(i_q)$ the potential difference across a diode. As it is well known the transient current time in a diode is very short and negligible compared to the system time constants. In these conditions, the diode could be modelled by an equivalent circuit made-up of a resistance Rd and a thresh hold voltage Es, more accurately it can be written:

$$\gamma(i_q) = V_a - V_b = (i_q)i_q + E_s \quad (3c)$$

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With Rd(iq)={ R if iq<0; r if not

With R high resistance value and r small resistance value. The relations (1)-(3) show how the magnetic and electric equations are coupled (the flux Φk in (3a) could be expressed in function of vector A).

Discretisation: To resolve the problem numerically, we could use a space Discretisation using the finite elements method (P1) and a time discretisation with a typical implicit scheme (Crank-Nicholson, backward Euler, Gear……). The Eq. 1 in the case of Euler backward, expressed under the simplified form will be (Piriou and Razek, 1988).

$A_{i+\Delta t} \in \vartheta$, solution for the equation:

$$\begin{aligned}&\frac{1}{\Delta t}\int_\Omega \sigma.A_{t+\Delta t}.\omega.dx + \\ &\int_\Omega \nu(x,[abs(rotA_{t+\Delta t})]^2)rotA_{t+\Delta t}.rot\omega dx = \\ &\int_\Omega \left[J_0 + \left(\sum_{k=1}^{n_p} i_k\psi_k\right)\right].\omega.dx + \frac{1}{\Delta t}\int_\Omega \sigma.A_t.\omega.dx \\ &\forall\omega\in\vartheta\end{aligned} \tag{4}$$

with ϑ adequate functional space (i_k is considered at instant t +Δ*t*) the discretisation can be done to the previous equations in their shown form or even converted to matrix form by the use of classical methods of discretisation.

MATERIALS AND METHODS

First approach: Using the finite elements mesh method and replacing γ(i) by $R_d(i)+E_s$ From the discretised equations found we can write:

$$\begin{aligned}&\frac{\Phi(m)}{\Delta t}\Big|_{t+\Delta t} + \left(R + \frac{L}{\Delta t}\right)..\Im(m)\Big|_{t+\Delta t} = \\ &E_{(m)} + \frac{\Phi_{(m)}}{\Delta t}\Big|_t + \frac{L}{\Delta t}\Im_{(m)}\Big|_t\end{aligned} \tag{5}$$

The correspondence matrix allows the expression of branch currents in function of mesh currents (Feliachi, 1981). We can write:

$$F = F_0 + D\Im_{(m)} \tag{6}$$

with

$$D = \sum_{k=1}^{n_p} F_k(CNT)_k = F\overline{CNT} \tag{7}$$

$(CNT)_k$ represents the kth line of the matrix CNT and $\overline{CNT}$ is the reduced matrix to np first lines of CNT.

In the same manner the flux Φ(m) in Eq. 5 can be written in function of potential vector nodal values under the form:

$$\Phi(m)=G.A \tag{8}$$

With

$$G= h.\sum_{k=1}^{n_p} F_k(CNT)_k^T.F_k^T = h.\overline{CNT}^T.F^T \tag{9}$$

From all the previous equations we can deduce and solve the Matrix system: $[P] = [M].[M]^T$.

Second approach: By considering the previously found electric equations under their discrete forms, it can be noticed that the electric circuit is composed of np internal branches (in direct relation with the magnetic circuit numbered from 1 to np) to which the external branches are added numbered from (1+np) to nb. When the connections are executed, we get a number of nodes nc. By taking as unknowns in the electric circuit branch currents $\Im_{(n_p)} = (i_k)_k = 1.n_b$ and the nodes potentials $V_{(n_c)} = (V_l)_{l=1,n_c}$. Using Ohm-Kirchoff laws at the nodes and the meshes and if the states of the diodes is given: the electrical circuit is linear and by the use of general linear algebra we can write the internal branches currents of the machine as (Glowinski *et al*., 1981).

$$\Im_{(n_p)} = (i_k)_k = 1.n_p$$

in function of electro-motiveforces:

$$E_{(n_p)} = (em_j)_{j=1,n_p} = -\left(\frac{d\Phi_j}{dt}\right)_{j=1,n_p}$$

By a relation of the type:

$$\Im_{(n_p)} = YE_{(n_p)} + I_0$$

(Y) is a matrix np by np which in the present case is positive and symmetric (Admittance matrix in electrical network theory). The non-linearity is processed with a compensator-predictor circuit.

From the use of previous equations we can find:

$$\frac{1}{\Delta t}\int_{\Omega} \sigma.A_{t+\Delta t}.\omega.dx + \int_{\Omega} \nu\left(x,[abs(rotA_{t+\Delta t})]^2.rotA_{t+\Delta t}\right) .rot\omega.dx + \frac{h}{\Delta t}\sum_{i,j=1}^{n_p} y_{i,j}\left(\int_{\Omega} \Psi_i.\omega.dx\right).\left(\int_{\Omega} \Psi_j.A_{t+\Delta t}.dx\right) = \frac{h}{\Delta t}\sum_{i,j=1}^{n_p} y_{i,j}\left(\int_{\Omega} \Psi_i.\omega.dx\right).\left(\int_{\Omega} \Psi_j.A_t.dx\right) + \int_{\Omega}\left(J_0+\sum_{i=1}^{n_p} I_{0,i}\Psi_i\right).\omega.dx + \frac{1}{\Delta t}\int_{\Omega} \sigma.A_t.\omega.dx \quad \forall \omega \, \varepsilon \, \vartheta \tag{10}$$

The admittance matrix Y=y(ij) calculated at

$t+\Delta t$ or t

RESULTS AND DISCUSSION

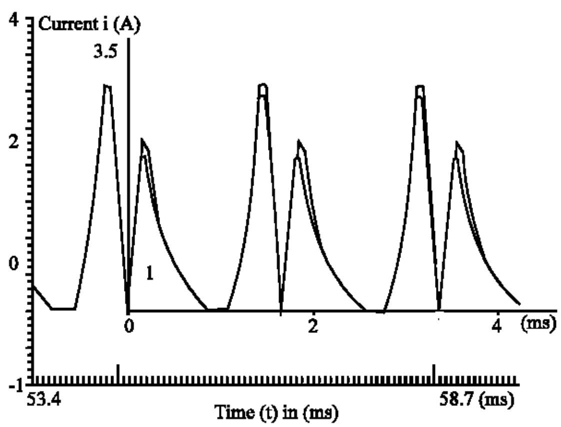

Fig. 1: Waveform of calculated absolute current value and measured absolute current value in the phases

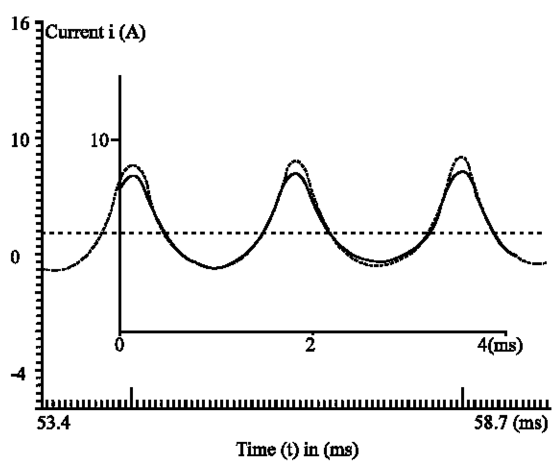

Fig. 2: Waveform of calculated and measured current in the excitation coil circuit

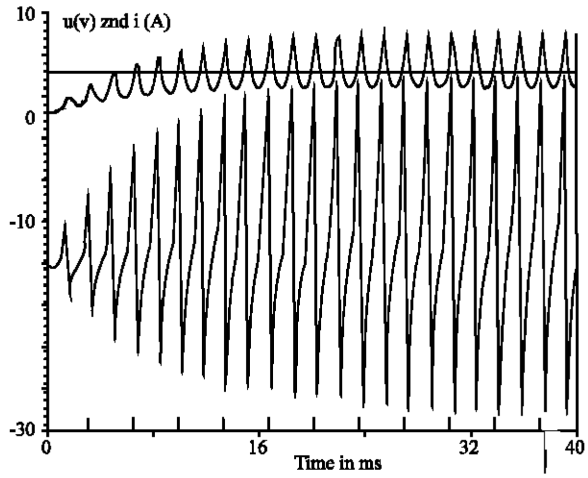

Fig. 3: Numerical simulation of the excitation current and voltage evolution across the circuit

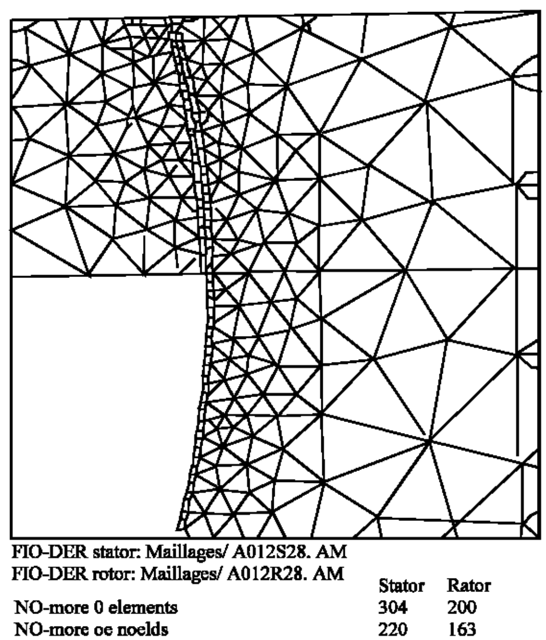

Fig. 4: Zoom round the air Gap meshwork. (case of displacement line)

The Fig. 1-3 show the results obtained from the simulation (Discontinuous curves) and from the experiment (Continuous curves), for, respectively the current in the phases circuit, the current in excation circuit and the evolution current and voltage across the excation circuit power supply.

The calculations have been done by taking Es = 0.8V the thresh hold diode voltage value. The stator thickness was taken to be 50 mm.

The Fig. 4 represents a "Zoom" in the air gap region used in the second approach by taking into account the displacement by the use of the sliding line.

CONCLUSION

A simulator of a synchronous motor was built and tested under mainly Fortran language. The usefulness and effectiveness of the suggested approaches were proved. The new simulator presented has good performance in terms of accuracy compared to the results obtained from the experimental bench.

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