

Equipment Selection by Numerical Resolution of the Hessian Matrix and Topsis Algorithm

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Abstract: The methods of selection are classified in : (1) Elementary methods; (2) Methods of mathematical optimization; (3) Methods of Multi Criterion Decision Making aid (MCDM). The problem of Equipment Selection (ES) is of multi objective nature (minimize the fuel consumption, minimize the machine mass, maximize the machine power etc.). These factors are both qualitative and quantitative according to structure of the selection. It is thus important to synthesize the multi objective approaches in order to clarify their operation and to seize their interest and their difference. We thus present a method of multicriterion aggregation for the Equipment Selection. The Equipment Selection of appropriate machines is one of the most critical decisions in the design and development of a successful production environment in construction and surface mining. Considering the specifications related to the criteria of functional requirements, machine mass, energy consumption and the machine power and the number of available alternative machine in the market, the equipment selection procedure can be quite complicated and time consuming. In this study, a new weighting method of decision matrix based on Hessian matrix is developed. The scores for each criterion and the loader-truck system have been determined for the real conditions of Ben Azzouz quarry, located eastern part of Algeria. These calculated scores are used successful in the study of equipment selection using TOPSIS algorithm. Results obtained for the Ben Azzouz quarry show interest of our approach in the decision-making of the choice of the loader-truck system.

Key words: MCDM, TOPSIS, criteria weights, hessian matrix, equipment selection, quarry

INTRODUCTION

Equipment selection is one of the most important factor that affect surface mining design (pit slopes, bench high, block sizes and geometries, ramp layout as well as excavation sequences and open-pit layout) and production planning. Further, equipment selection also effects economic considerations in open-pit design, specifically overburden, waste rock and ore mining costs and cost escalation parameters as a function of plan location and depth. Mining costs are a function of site conditions, operating scale and equipment. The purpose of equipment selection is to select optimum equipment with minimum cost (Lizote, 1988). We have used Technique for Order Preference by Similarity to Ideal Solution TOPSIS (Lai *et al.*, 1994; Olsen, 2004) approach

for the equipment selection in final decision. MCDM deals with the problem of choosing an alternative from a set of alternatives which are characterised in terms of their attributes. Usually MCDM consists of a single goal, but this may be of two different type. The first is where the goal is to select an alternative from a set of scored ones based on the values and importance of the attributes of each alternative. The second type of goal is to classify alternatives, using a kind of role model or similar cases. The use of past cases to deduce answers or explanations is a recent field of research, termed Case-Based Reasoning. Both type of goals require information about the preferences among the instances of an attribute and the preferences across the existing attributes. The assessment of these preferences is either provided directly by the decision maker or based on past choices.

The general formalisation is: let $A_1, A_2, A_3 \dots A_n$ be set of alternatives to be assessed by criteria $C_1, C_2, C_3 \dots C_n$. Let R_{ij} be the numerical rating of alternative A_i for criteria C_j . Then the general decision function is: $D(A) = (R_{11}o, R_{12}o, R_{13}o, \dots, R_{jn}o)$ for $j = 1, 2, 3, \dots, n$ and o represents the aggregation.

Further, the decision maker might express or define a ranking for the criteria as importance/weights. There are many forms for expressing these importance, but the most common are: (a) Utility preference functions; (b) The Analytical Hierarchy Process (AHP) (c) The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and (d) A fuzzy version of the classical linear weighted average. In this study, it is proposed developing the decision support system that is possible to evaluate all parameters together and assess the weights of criteria and objectives for equipment selection problem. The TOPSIS technique is used to develop the decision support system. The method uses the Hessian matrix for structuring the criteria and objectives of a system, hierarchical in a multiple attribute framework.

MATERIALS AND METHODS

Multi-Criteria Decision Making approaches is concerned with the selection of the optimum choice of action amongst alternatives based on multiple discrete attributes or decision criteria. Example: The selection of the optimum system to purchase from three alternatives, A_1, A_2 or A_3 . A survey of it managers has highlighted the following decision making criteria: masse (C_1), fuel consumption (C_2) and power (C_3). The percentages of managers who selected one of the C_1, C_2 and C_3 criteria as the most important are %17, %33 and %50 respectively. The scored criteria for the examined systems are given in Table 1:

Which of the alternatives to select?

Steps in utilising MCDM methods: Identify the alternatives or scenarios available; identify the criteria for the selection process; identify the impact that each alternative has on the selected criteria; develop the MCDM matrix and analyse. Some analytical tools used are: -WSM: Weighted Sum Method-WPM: Weighted Product Method -AHP: Analytical Hierarchy Process -AHP (Revised): Revised Analytical Hierarchy Process -ELECTRE: Elimination and Choice Translating Reality -TOPSIS: Technique for Order Preferences by Similarity to Ideal Solution. Types of MCDM problems are: Deterministic, stochastic, fuzzy, single, group.

Aggregation procedures are referred to how the notion of relative importance of criteria is taken into account and the way in which inter-criteria preference

Table 1 Scored criteria

	C_1 17%	C_2 33%	C_3 50%
A_1	10	25	10
A_2	30	20	30
A_3	30	15	20

information needs to be assessed in each approach. The important conclusion can be expressed as follows: there is no sense and it is theoretically incorrect, to specify measures of importance for the criteria outside of the context of the specific overall evaluation model to be used, that is to say, without having defined the type of mathematical aggregation rule which is to be used in deriving comprehensive preferences. In fact, the notion of relative importance is understood in significantly different fashions by different aggregation procedures. In this context, the distinction between compensatory and non-compensatory aggregation procedures is particularly relevant.

In compensatory approaches, such as multi-attribute value and utility measurement, tradeoffs or substitution rates are assessed in order to derive values for the parameters (weights) included in the aggregation rule. These parameters are in fact scaling constants needed for the cardinal criteria-functions to be commensurate in some way. Thus, in these approaches, weights have no absolute or intrinsic meaning and there is no sense in attempting to derive them without reference to the criterion-functions.

The bibliographical study (Olsen, 2004), it proves that the choices of the criteria and their weights constitute a crucial stage in the process of decision-making aid and that the final decision, that the decision maker will take, depends on it. It is to be stressed that in the technical literature relating to the field of the multi criteria decision analysis, there are some methods of determination of the weights of the criteria. The most relevant methods are the following ones. Direct Evaluation Methods : method of simple classification (Kendal, 1970), method of probabilistic evaluation (Rietveld, 1982; Rietveld and Owersloot, 1984), method of Vansnick (Vansnick, 1986), ratio method (Van and Edwards, 1986). Method of successive comparisons (Churchmann and Ackoff, 1954; Knoll and Engelberg, 1978). Methods of eigenvalues : Eigenvalues method of Klee (1971), Analytic Hierarchy Process (AHP) of Saaty (1980). Entropy method of Zeleny (1982) Comparison of the actions method: Swing method of Von Winterfeldt and Edwards (1986), WTA Additive Utility of Jacquet-Lagrange and Siskos (1982), LINMAP method (Srinivasan and Schocker, 1973a,b), Zionts method (Zoits, 1981) and Improved LINMAP method of Pekelman and Sen (1974). Well which represent an interest proven in the multicriteria decision analysis,

the methods of comparison of the actions, in order to determine the weights of the criteria, did not find an echo compared to the other methods of the evaluation of the weights of the criteria. For the simple reason that, the decision in multi criteria analysis seeks the weights for the criteria in order to determine the best actions. In conclusion, one deduces that the evaluation of the weights of the criteria is a field open to research and the investigation if one wants to improve quality of the decisions to be taken in real situations. In our study, the selected decision criteria are: machine power, energy consumption, machine mass. These three gathered criteria train a homogeneous and coherent family of criteria which one calls the mechanical performance of a motorized mechanical site machine. Our step is dictated by the following triptych: a machine less heavy, less consuming energy, more powerful equalize a better performance.

TOPSIS method: The acronym TOPSIS stands for technique for preference by similarity to the ideal solution. TOPSIS (Lai *et al.*, 1994) is attractive in that limited subjective input is needed from decision makers (Olsen, 2004). The only subjective input needed is weights. In this study, we consider weights obtained by new method. This method Consist to minimize a function cost obtained by a mathematical model. TOPSIS has been applied to a number of applications. This method considers three types of attributes or criteria: Qualitative benefit attributes/criteria; quantitative benefit attributes; cost attributes or criteria. In this method two artificial alternatives are hypothesized. Ideal alternative: the one which has the best level for all attributes considered. Negative ideal alternative: the one which has the worst attribute values. TOPSIS selects the alternative that is the closest to the ideal solution and farthest from negative ideal alternative. TOPSIS assumes that we have m alternatives (options) and n attributes/criteria and we have the score of each option with respect to each criterion. Let x_{ij} score of option i with respect to criterion j . We have a matrix $X = (x_{ij})_{m \times n}$ matrix. Let J be the set of benefit attributes or criteria (more is better). Let J' be the set of negative attributes or criteria (less is better). The idea of TOPSIS can be expressed in a series of steps.

Step 1: Obtain performance data for n alternatives over k criteria. Raw measurements are usually standardised, converting raw measures x_{ij} into standardised mesures s_{ij} . Construct normalized decision matrix. This step transforms various attribute dimensions into non-dimensional attributes, which allows comparisons across criteria. Normalize scores or data as follows:

$$r_{ij} = \frac{x_{ij}}{\sum_i x_{ij}^2} \text{ for } i = 1, \dots, m; j = 1, \dots, n$$

Step 2: Develop a set of importance weights w_{kj} for each of the criteria. The basis for these weights can be anything, but usually, is ad hoc reflective of relative importance. Scale is not an issue if standardising was accomplished in step 1. Construct the weighted normalized decision matrix. Assume we have a set of weights for each criteria w_j for $j = 1, \dots, n$. Multiply each column of the normalized decision matrix by its associated weight. An element of the new matrix is:

$$v_{ij} = w_j r_{ij}$$

Step 3: Determine the ideal and negative ideal solutions.

Ideal solution:

$$A^* = \{v_1^*, \dots, v_n^*\}, \text{ where}$$

$$v_j^* = \begin{cases} \max_i(v_{ij}) & \text{if } j \in J; \\ \min_i(v_{ij}) & \text{if } j \in J' \end{cases}$$

Negative ideal solution:

$$A' = \{v_1', \dots, v_n'\}, \text{ where}$$

$$v_j' = \begin{cases} \min_i(v_{ij}) & \text{if } j \in J; \\ \max_i(v_{ij}) & \text{if } j \in J' \end{cases}$$

Step 4: Calculate the separation measures for each alternative. The separation from the ideal alternative is:

$$S_i^* = \left[\sum_j (v_j^* - v_{ij})^2 \right]^{\frac{1}{2}} \text{ } i = 1, \dots, m$$

Similarly, the separation from the negative ideal alternative is:

$$S_i' = \left[\sum_j (v_j' - v_{ij})^2 \right]^{\frac{1}{2}} \text{ } i = 1, \dots, m$$

Step 5: Calculate the relative closeness to the ideal solution C_i^*

$$C_i^* = \frac{S_i'}{S_i^* + S_i'} \quad 0 < C_i^* < 1$$

Step 6: Rank order alternatives by maximising the ratio in Step 5. Select the option with C_i^* closest to 1.

Thus, TOPSIS minimises the distance to the ideal alternative while maximising the distance to the nadir. There are a number of specific procedures that can be used for Step 2 (developing weights) and for Step 5 (distance measures). Additionally, different conventions can be applied to defining best performance and worst performance. A number of distance metrics can be applied. Traditional TOPSIS applied the Euclidean norm (minimisation of square root of the sum of squared distances) to ideal and nadir solution, a second power metric (P2). TOPSIS2 is a variant where distance was measured in least absolute value terms, a first power metric (P1). Another commonly used metric is the Tchebychev metric, where the minimum maximum difference is the basis for selection. This coincides with an infinite power-term (P8).

A relative advantage of TOPSIS is the ability to identify the best alternative quickly. TOPSIS has been comparatively tested with a number of other multiattribute methods (Olsen, 2004). The other methods primarily focused on generating weights (Step 2 in the prior description), with one method (ELECTRE) including a different way to combine weights and distance measures. TOPSIS was found to perform almost as well as multiplicative additive weights and better than analytic hierarchy process in matching a base prediction model. When there were few criteria, TOPSIS had proportionately more rank reversals. When there were many criteria, TOPSIS differed more from simple additive weight results and TOPSIS was also affected more with diverse sets of weights. TOPSIS performed less accurately than AHP on both selecting the top ranked alternative and in matching all ranks in this set of simulations.

New numerical resolution method of the determination of the weights of the criteria: For the numerical resolution, one transformed the problem of determination of the weights of the criteria into a problem of optimization of a functional cost whose total minimum is the required solution. Basing on the total minimum obtained, we wrote a Matlab program which consists in seeking all the combinations of the weights of the acceptable criteria while calculating, for each combination, the minimum of the functional cost under the following constraints:

$$\left\{ \sum C_i = C_1 + C_2 + C_3 = 1; C_1 > 0, C_2 > 0, C_3 > 0 \right\}$$

The weights of the criteria to be retained, for the posterior calculation of the best actions which helps the decision maker to make the best decision, are determined by the combination whose functional cost has the smallest minimum among the whole of the minima obtained.

Hessian matrix and procedure to be followed: To consider the criteria importance, one classifies them at least important to most important. To the least important criterion, one gives value 1, with the second value 2 and with the third value 3. Then, one standardizes the values obtained while dividing by the sum. In our study case, the criteria are classified as follows: for the masse criterion $C_1 = 1/6$; for the energy consumption criterion $C_2 = 2/6$; for the machine power criterion $C_3 = 3/6$. Then, one arranges the values of the relative importance of the criteria C_1 , C_2 and C_3 in the diagonal of the Hessian matrix

$$H_j(C_1, C_2, C_3) = \begin{vmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{2}{6} & 0 \\ 0 & 0 & \frac{3}{6} \end{vmatrix}$$

Then the functional cost to be minimized is reconstituted.

$$J(C_1, C_2, C_3) \rightarrow \min$$

$$H_j(C_1, C_2, C_3) = \frac{\partial^2 J}{\partial C_i \partial C_j} \quad i = 1, 2, 3; j = 1, 2, 3$$

$$H_j(C_1, C_2, C_3) = \begin{vmatrix} \frac{\partial^2 J}{\partial C_1^2} & \frac{\partial^2 J}{\partial C_1 \partial C_2} & \frac{\partial^2 J}{\partial C_1 \partial C_3} \\ \frac{\partial^2 J}{\partial C_2 \partial C_1} & \frac{\partial^2 J}{\partial C_2^2} & \frac{\partial^2 J}{\partial C_2 \partial C_3} \\ \frac{\partial^2 J}{\partial C_3 \partial C_1} & \frac{\partial^2 J}{\partial C_3 \partial C_2} & \frac{\partial^2 J}{\partial C_3^2} \end{vmatrix}$$

The functional cost to be minimized will be

$$J(C_1, C_2, C_3) = \frac{1}{12} (C_1^2 + 2C_2^2 + 3C_3^2) \longrightarrow \text{Min}$$

The necessary condition of optimality (with $\min J(C_1, C_2, C_3)$ without constraints) is the gradient

$$\begin{aligned} \nabla J(\bar{C}_1, \bar{C}_2, \bar{C}_3) &= 0 \\ \nabla J(\bar{C}_1, \bar{C}_2, \bar{C}_3) &= \left(\frac{\partial J}{\partial C_1}, \frac{\partial^2 J}{\partial C_2}, \frac{\partial J}{\partial C_3} \right) = 0 \\ \nabla J(\bar{C}_1, \bar{C}_2, \bar{C}_3) &= \left(\frac{2}{12}C_1, \frac{4}{12}C_2, \frac{6}{12}C_3 \right) = \\ (0, 0, 0) &\Rightarrow (\bar{C}_1, \bar{C}_2, \bar{C}_3) = (0, 0, 0) \end{aligned}$$

Like the criticizes point $(\bar{C}_1, \bar{C}_2, \bar{C}_3) = (0, 0, 0)$, we can disturb the functional cost calculus above by modifying it slightly in the following way.

$$\text{If, } J(C_1, C_2, C_3) = \frac{1}{12} (C_1^2 + 2C_2^2 + 3C_3^2) + e_1 C_1 + e_2 C_2 + e_3 C_3$$

wher $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are three real numbers close to zero

$$\text{Thus, } \nabla J(\bar{C}_1, \bar{C}_2, \bar{C}_3) = \left(\begin{array}{l} \frac{2}{12}C_1 + \varepsilon_1, \frac{4}{12}C_2 + \\ \varepsilon_2, \frac{6}{12}C_3 + \varepsilon_3 \end{array} \right) = (0, 0, 0)$$

$$C_1 = -\frac{12}{2}\varepsilon_1 = -6\varepsilon_1$$

$$C_2 = -\frac{12}{4}\varepsilon_2 = -3\varepsilon_2$$

$$C_3 = -\frac{12}{6}\varepsilon_3 = -2\varepsilon_3$$

The Hessian matrix of the disturbed functional cost calculus remains unchanged

$$\begin{aligned} H_J(C_1, C_2, C_3) &= \begin{vmatrix} \frac{\partial^2 J}{\partial C_1^2} & \frac{\partial^2 J}{\partial C_1 \partial C_2} & \frac{\partial^2 J}{\partial C_1 \partial C_3} \\ \frac{\partial^2 J}{\partial C_2 \partial C_1} & \frac{\partial^2 J}{\partial C_2^2} & \frac{\partial^2 J}{\partial C_2 \partial C_3} \\ \frac{\partial^2 J}{\partial C_3 \partial C_1} & \frac{\partial^2 J}{\partial C_3 \partial C_2} & \frac{\partial^2 J}{\partial C_3^2} \end{vmatrix} = \\ \begin{vmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{2}{6} & 0 \\ 0 & 0 & \frac{3}{6} \end{vmatrix} \end{aligned}$$

It is definite positive since its eigenvalues are positive for the three values.

Proof: The polynomial characteristic $\det(H_J - \lambda I_3) = 0$ give the eigenvalues of $H_J(C_1, C_2, C_3)$

where λ -eigenvalues of $H_J(C_1, C_2, C_3)$; I_3 -identity matrix of order 3.

$$\begin{aligned} \det(H_J - \lambda I_3) &= \begin{vmatrix} 1/6 & 0 & 0 \\ 0 & 2/6 & 0 \\ 0 & 0 & 3/6 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \\ \begin{vmatrix} 1/6 - \lambda & 0 & 0 \\ 0 & 2/6 - \lambda & 0 \\ 0 & 0 & 3/6 - \lambda \end{vmatrix} &= 0 \end{aligned}$$

$$\Rightarrow (1/6 - \lambda)(2/6 - \lambda)(3/6 - \lambda) = 0; \Rightarrow \lambda_1 = 1/6, \lambda_2 = 2/6, \lambda_3 = 3/6$$

Moreover, the disturbed functional cost calculus $J(C_1, C_2, C_3)$ is thus convex. Then any local minimum becomes total minimum. Moreover, one satisfies the sufficient conditions of optimality of $[\min J(C_1, C_2, C_3)]$ without constraint]. Thus, the point of total minimum of $\min J(C_1, C_2, C_3)$ is

$$C^* = \begin{vmatrix} C_1^* \\ C_2^* \\ C_3^* \end{vmatrix} = \begin{vmatrix} -6\varepsilon_1 \\ -3\varepsilon_2 \\ -2\varepsilon_3 \end{vmatrix}$$

After having found the point of total minimum C^* , one elaborated a computer programme (see Appendix) who consists in seeking all the combinations of the weights of the acceptable criteria while calculating, for each combination, the minimum of the functional cost calculus under the following constraints:

$$C_1 + C_2 + C_3 = 1; C_1 > 0, C_2 > 0, C_3 > 0$$

RESULTS AND DISCUSSION

Simulation carried on 10^6 combinations of weight of criteria since one varied $\varepsilon_1, \varepsilon_2, \varepsilon_3$ of -1 to 0 with step 0.01. On 10^6 combinations of weight of criteria, that the program treated, only 12 results checking the constraints above are obtained and viewed in Fig. 1.

It is noted that combination 12 gives the best minimum of $J(C_1, C_2, C_3)$ with the following values: $C_1 = 0.06; C_2 = 0.12; C_3 = 0.82; J = -0.1706$ and that among all those obtained.

The results show that the machine power criterion C_3 can be given using the following equation.

$$C_3 = -3.881J + 0.176$$

With: coefficient of correlation $R = -0.97852$; Standard Deviation $SD = 0.02377$; Number of sample $N = 12$ and $p < 0.0001$.

Table 2: Loading machines acceptable to be compared for Ben Azzouz constraints

Model	Case	Fiatallis	Frish	Hanomag	Kaelble	Volvo
Type	760B	FR15	F1400C	55C	SL12C	4500
Power (kW)	162.0	163.0	160.0	151.0	150.0	186.0
Masse (tonne)	14.90	14.60	12.80	14.00	12.30	13.90
Spend energy (L h ⁻¹)	33.90	34.11	33.48	33.60	31.39	38.93

Table 3: The trucks, with the capacity of 15 m³, acceptable to be compared and machines retained

Model	Astra	Euclid	Faun	Kockum	Komatsu	Kaelble	Brimont
Type	BM2	R25	K25-2	425B	HD200-2	KVW30	BB20c
Power (kW)	239	214	256	290	280	320	192
Masse (tonne)	13.30	16.70	16.40	16.40	18.50	22.50	18.20
Spend energy (L h ⁻¹)	43.77	39.19	46.88	53.11	51.27	58.59	35.15

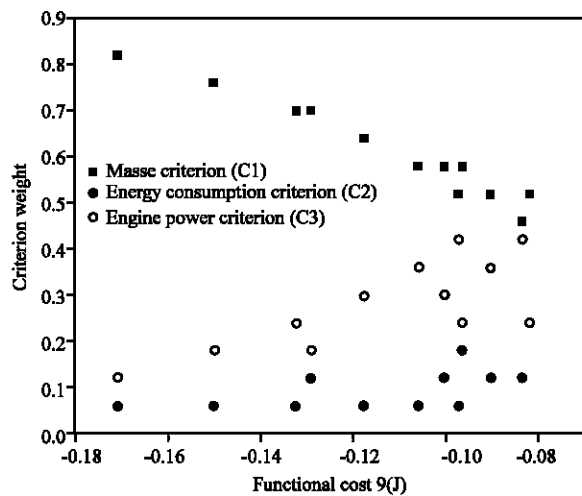


Fig. 1: Criterion weight versus functional cost

In the same way the energy consumption C_2 and the engin masse C_1 can be given versus functional cost using the following exponential equations.

$$C_2 = y_0 + Ae^{\left(\frac{-J}{t}\right)}$$

$\chi^2 = 0.00481$; $R^2 = 0.58043$; $y_0 = 0.54804$;
 $A = -0.09341 \pm 0.48627$; $t = 0.11041 \pm 0.26396$.

$$C_1 = y_0 + Ae^{\frac{J}{t}}$$

$\chi^2 = 0.00202$; $R^2 = 0.50717$; $y_0 = 0.05644 \pm 0.0349$;
 $A = 5.44842 \pm 19.33794$; $t = 0.02119 \pm 0.01982$

Consequently, the machine power criterion C_3 can be obtained with strong decreasing linéare relationship versus functional cost. However, the machine energy consumption C_2 and machine masse C_1 can be obtained with exponential relationship versus functional cost.

Application of the MCDM procedure in the Ben Azzouz quarry with 800 000 t/y of aggregats situated in the North Est of Algeria.

Loading machines acceptable to be compared for Ben Azzouz constraints are given in Table 2 (with the same bucket capacity of 2.5 m³).

The trucks, with the capacity of 15 m³, acceptable to be compared according to the loading machines retained are given in Table 3.

To be able to choose the best couple loader-trucks among the models comparable (even capacity for all) a Matlab program codes with 747 lines was written while basing itself on the weights of the criteria obtained by the resolution of the Hessian matrix applied to the TOPSIS multicriterion algorithm method of choice and loader-truck system requirements.

Results: For 3 loading buckets in the truck, the optimal fleet size is:

Loader: VOLVO 4500, number: 1

Truck: ASTRA BM2-SA, number: 8

For 4 loading buckets in the truck, no machine is rational for Ben Azzouz quarry.

For 5 loading buckets in the truck, the optimal fleet size is:

Loader: VOLVO 4500, number: 1

Truck: KAELBLE K20B, number: 6

For 6 loading buckets in the truck, the optimal fleet size is:

Loader: VOLVO 4500, number: 1

Truck: EUCLID R25, number: 5

CONCLUSIONS

There is a number of equipment selection methods and weighting criteria approaches in the literature. This study presented a new numerical resolution using TOPSIS algorithm on basis of the solution of Hessian matrix to

define the weights of the criteria of decision-making. The results show that our method of aggregation of the criteria with constraints gives the best functional cost in comparison with some other methods. Indeed, our approach of numerical determination of the weights of the criteria includes not only the alternative with the optimal functional cost, but also included the weights determined by other methods. This rational weights as energy consumption, machine masse and machine power determined through TOPSIS method and Hessian matrix can be used effectively in the equipment selection problems. Indeed, the application to Ben Azzouz quarry showed that for an annual production of 800 thousand tonnes per year and for a number of buckets of loader necessary to fill a truck going from 3 to 6, the rational equipment is the loader VOLVO 4500 and 8 trucks ASTRA BM2-SA, the loader VOLVO 4500 and 6 trucks KAELEBLE K20B and the loader VOLVO 4500 and 5 trucks EUCLID R25.

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APPENDIX

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%-MINIMISATION PROGRAM OF FUNCTIONAL COST
BASED ON HESSIAN MATRIX-%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all
clear all
clc
i = 0;
for eps1=-1:0.01:0,
    A=eps1;
    for eps2=-1:0.01:0,
        B =eps2;
        for eps3 =-1:0.01:0,
            C =eps3;
            C1=-6*A;C2=-3*B;C3=-2*eps3;
            CC=C1*C2*C3;
            x=C1+C2+C3;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            if x==1% Constraints verification
                if (CC>0)
                    i=i+1;
                    n(i)=i;
```

```
J(i)=(C1^2+2*C2^2+3*C3^2)/12+A*C1+B*C2+C*C3;%%
Functional cost
a(i)=A; b(i)=B; c(i)=C; %% Eps Tables
c1(i)=C1; c2(i)=C2; c3(i)=C3;%% Decision criteria
J(n)=J;%%Min
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    end
end
Result =[n' a' b' c' c1' c2' c3' J']
```

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