

Adaptive Neuro-Fuzzy Inference System for Modeling Magnetic Hysteresis

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Abstract: The accurate characterization and modeling of magnetic material are critical in simulating the performance analysis of electrical circuits incorporating magnetic components. In this study, a new approach for modeling hysteresis loop of ferromagnetic material based on Adaptive Neuro-fuzzy Inference System (ANFIS) was presented. The proposed ANFIS model combined the neural network adaptive capabilities and the fuzzy logic qualitative approach can restored the hysteresis curve with a little RMS error. The applicability of the developed method is illustrated in figures. Simulation tests and results will be presented in the following.

Key words: ANFIS modeling technique, magnetic hysteresis, jiles-atherton model, ferromagnetic core

INTRODUCTION

Simulation of hysteretic characteristics of magnetic materials needs to be implemented into electromagnetic field simulation software tools to predict the behavior of different type of magnetic equipments. However, there exist many approaches to develop a mathematical model to describe the hysteretic relationship between the magnetization M and the magnetic field H . the first approach was the hysteresis model of invented Preisach (1935). The second is the Jiles-Atherton (JA) model, compared to other models, the JA model has some advantages: it is formulated in terms of differential equation and it uses only five parameters whose identification is performed from a single measured hysteresis loop (Jiles and Atherton, 1986). Artificial intelligence has also been applied to the modeling of magnetic hysteresis and parameters identification of these models such as neural network and genetic algorithm (Wilson *et al.*, 2001; Salvin and Riganti, 2002; Salvini and Coltell, 2001; Vecchio and Salvini, 2000; Xu and Refsum, 1997; Saliah and Lowther, 1997; Dimitre *et al.*, 2001, 2003; Saghafifar and Nafalski, 2002; Kuczmann and Ivanyi, 2002;). Like neural networks, fuzzy systems (Zadeh and Fuzzy, 1965) can be conveniently used to approximate arbitrary functions (Buckley and Hayashi, 1994; Kosko, 1992). Neural networks can learn from data, but knowledge learned can be difficult to understand. Models based on fuzzy logic are easy to understand, but they do not have learning algorithms; learning has to be adapted from other technologies. A Neuro-Fuzzy model can be defined as a model built using a combination of fuzzy logic and neural networks. Recently, there has been a remarkable advance in the development of Neuro-Fuzzy models, as it is

described in (Yen and Langari, 1999; Jang *et al.*, 1997; Abraham, 2001). One of the most popular and well documented Neuro-Fuzzy systems is ANFIS, which has a good software support ((The Math works, 1998). Jang (1992, 1993, 1995, 1997) present the ANFIS architecture and application examples in modeling a nonlinear function, dynamic system identification and a chaotic time series prediction. Given its potential in building fuzzy models with good prediction capabilities, the ANFIS architecture was chosen for modeling magnetic hysteresis in this work. In the following sections information is given about adaptive Neuro-Fuzzy modeling, the JA model for magnetic material testing system, the selection of ways to modeling the hysteresis phenomena with Neuro-Fuzzy modeling, results and conclusions.

Jiles-Atherton hysteresis model:

Formulation: The Jiles-Atherton model is a physically based model that includes the different mechanisms that take place at magnetization of a ferromagnetic material. The magnetization M is represented as the sum of the irreversible magnetization M_{irr} due to domain wall displacement and the reversible magnetization M_{rev} due to domain wall bending (Jiles and Atherton, 1986). The rate of change of the irreversible part of the magnetization is given by.

$$\frac{dM_{irr}}{dH} = \frac{(M_{an} - M)}{\frac{k}{\mu_0} \delta - \alpha(M_{an} - M)} \quad (1)$$

The anhysteretic magnetization M_{an} in (1) follows the Langevin function (Wilson *et al.*, 2001), which is a nonlinear function of the effective field:

$$H_e = H + \alpha M \quad (2)$$

$$M_{an} = M_s \left(\coth \left(\frac{H_e}{a} \right) - \frac{a}{H_e} \right) \quad (3)$$

The rate of change of the reversible component is proportional to the rate of the difference between the hysteretic component and the total magnetization (Salvini and Raganti, 2002). Consequently, the differential of the reversible magnetization is:

$$\frac{dM_{rev}}{dH} = c \left(\frac{dM_{an}}{dH} - \frac{dM}{dH} \right) \quad (4)$$

Combining the irreversible and reversible components of magnetization, the differential equation for the rate of change of the total magnetization is given by:

$$\frac{dM}{dH} = \frac{1}{1+c} \frac{(M_{an} - M)}{\mu_0} + \frac{c}{c+1} \frac{dM_{an}}{dH} \quad (5)$$

Before using the J-A model, five parameters must be determined. The first parameter, α , is a mean field parameter defining the magnetic coupling between domains in the material and is required to calculate the effective magnetic field, H_e (2) composed by the applied external field and the internal magnetization. The model also needs an equation describing the anhysteretic curve (3) suggests the use of Langevin function. Two parameters are to be specified and included in the function, the saturation value of magnetization M_s and a Langevin parameter, a . Hysteresis is added by including pinning of domain walls, this is done by parameter, k , defining the pinning site density, in J-A the pinning of domain wall motion is assumed to be the major contribution to hysteresis. At this stage only irreversible magnetization is considered, the last parameter, c defines the amount of reversible magnetization, due to wall bowing and reversal rotation, included in the magnetization process; δ is a directional parameter and takes the value +1 for increasing field ($dH/dt > 0$), -1 for decreasing field ($dH/dt < 0$).

Parameter identification:

Anhysteretic susceptibility: The anhysteretic susceptibility at the origin, can be used to define a relationship between M_s , a and α

$$\chi_{an} = \left(\frac{dM_{an}}{dH} \right)_{M=0, H=0} \quad (6)$$

$$a = \frac{M_s}{3} \left(\frac{1}{\chi_{an}} + \alpha \right) \quad (7)$$

Initial susceptibility: The reversible magnetization component is expressed via the parameter c in the hysteresis Eq. (4) defined by:

$$\chi_{ini} = \left(\frac{dM}{dH} \right)_{H=0, M=0} = \frac{c M_s}{3 \alpha} \quad (8)$$

Coercivity: The hysteresis loss parameter k can be determined from the coercivity H_c and the differential susceptibility at the coercive point $\chi(H_c)$.

$$k = \frac{M_{an}(H_c)}{1-c} \left[\alpha + \frac{1}{\chi(H_c) - \left(\frac{c}{1-c} \right) \frac{dM}{dH}} \right] \quad (9)$$

Remanence: The coupling parameter α can be determined independently if a is known by using the remanence magnetization M_r and the differential susceptibility at remanence,

$$M_r = M_{an}(M_r) + \frac{k}{\frac{a}{1-c} + \frac{1}{\chi(M_r) - c dM/dH}} \quad (10)$$

Adaptive Neuro-Fuzzy Inference System (ANFIS): An adaptive Neuro-Fuzzy inference system is a cross between an artificial neural network and a fuzzy inference system. An artificial neural network is designed to mimic the characteristics of the human brain and consists of a collection of artificial neurons. An adaptive network is a multi-layer feed-forward network in which each node (neuron) performs a particular function on incoming signals. The form of the node functions may vary from node to node. In an adaptive network, there are two types of nodes: Adaptive and fixed. The function and the grouping of the neurons are dependent on the overall function of the network. Based on the ability of an ANFIS to learn from training data, it is possible to create an ANFIS structure from an extremely limited mathematical representation of the system.

Architecture of ANFIS: The ANFIS is a fuzzy Sugeno model put in the framework of adaptive systems to facilitate learning and adaptation (Jang, 1997). Such framework makes the ANFIS modeling more systematic and less reliant on expert knowledge. To present the ANFIS architecture, we suppose that there are two input

linguistic variables X and Y and each variable has two fuzzy sets A1, A2, B1 and B2 as is indicated in Fig.1, in which a circle indicates a fixed node, whereas a square indicates an adaptive node.

Then a Takagi-Sugeno-type fuzzy if-then rule could be set up as:

Rule i: If (x is A_i) and (y is B_i) then (f_i = p_ix + q_iy + r_i)

f_i are the outputs within the fuzzy region specified by the fuzzy rule. p_i, q_i and r_i are the design parameters that are determined during the training process.

Some layers of ANFIS have the same number of nodes and nodes in the same layer have similar functions. Output of nodes in layer-1 is denoted as O_i¹, where 1 is the layer number and i is neuron number of the next layer. The function of each layer is described as follows:

Layer 1: In this layer, all the nodes are adaptive nodes. The outputs of layer 1 are the fuzzy membership grade of the inputs, which are given by:

$$O_i^1 = \mu_{A_i}(x) \quad i = 1, 2, \quad (11)$$

$$O_i^1 = \mu_{B_{i-2}}(y) \quad i = 3, 4 \quad (12)$$

Where μ_{A_i}(x), μ_{B_{i-2}}(y) can adopt any fuzzy membership function. For example, if the bellshaped membership function is employed μ_{A_i}(x), is given by:

$$\mu_{A_i}(x) = \frac{1}{1 + \left\{ \left(\frac{x - c_i}{a_i} \right)^2 \right\}^{b_i}} \quad (13)$$

where a_i, b_i and c_i are the parameters of the membership function, governing the bell shaped functions accordingly.

Layer 2: Each node computes the firing strengths of the associated rules. The output of nodes in this layer can be presented as:

$$O_i^2 = \omega_i = \mu_{A_i}(x)\mu_{B_i}(y) \quad i = 1, 2 \quad (14)$$

Layer 3: In this third layer, the nodes are also fixed nodes. They play a normalization role to the firing strengths from the previous layer. The outputs of this layer can be represented as:

$$O_i^3 = \bar{\omega}_i = \frac{\omega_i}{\omega_1 + \omega_2} \quad i = 1, 2 \quad (15)$$

Which are the so-called normalized firing levels.

Layer 4: The output of each adaptive node in this layer is simply the product of the normalized firing level and a first order polynomial (for a first order Sugeno model). Thus, the outputs of this layer are given by:

$$O_i^4 = \bar{\omega}_i f_i = \bar{\omega}_i (p_i x + q_i y + r_i) \quad i = 1, 2 \quad (16)$$

Layer 5: Finally, layer five, consisting of circle node labeled with S. is the summation of all incoming signals. Hence, the overall output of the model is given by:

$$O_i^5 = \sum_{i=1}^2 \bar{\omega}_i f_i = \frac{\sum_{i=1}^2 \omega_i f_i}{\omega_1 + \omega_2} \quad (17)$$

From the architecture of ANFIS, we can observe that there are two adaptive layers the first and the fourth. In the first layer, there are three modifiable parameters {a_i, b_i, c_i}, which are related to the input membership functions. These parameters are the so-called premise parameters. In the fourth layer, there are also three modifiable parameters {p_i, q_i, r_i}, pertaining to the first order polynomial. These parameters are so-called consequent parameters (Jang, 1992, 1993, 1995).

Learning algorithm of ANFIS: The learning algorithm for ANFIS is a hybrid algorithm, which is a combination between gradient descent and least squares method (Jang, 1997). For simplicity, the adaptive network has only one output and is assumed to be

$$\text{Output} = F(\bar{I}, S) \quad (18)$$

Where \bar{I} the set of input variables and S is the set of parameters. If there exists a function H such that the composite function H ◦ F is linear in some of the elements of S, then these elements can be identified by the least squares method. More formally, if the parameter set S can be decomposed into two sets

$$S = S_1 \oplus S_2 \quad (19)$$

Where \oplus represents direct sum. Such that H ◦ F is linear in the element S₂, then upon applying H to Eq. (18), we have

$$H(\text{Output}) = H \circ F(\bar{I}, S) \quad (20)$$

Which is linear in the elements of S_2 . Now we given values of elements of S_1 , the P training data can be plugged into Eq. (19) and obtain a matrix equation

$$A.X = B \quad (21)$$

Where X is an unknown vector whose elements are parameters in S_2 . Let $|S_2| = M$, then the dimensions of A, X and B are $P \times M$, $M \times 1$ and $P \times 1$, respectively. Since the number of training data pairs (P) is usually greater than the number of linear parameters (M), this is an over-determined problem and generally there is no exact solution to Eq. (21). Instead, a least squares estimate of X can be sought that minimizes the squared error $\|AX-B\|^2$.

Based on the ANFIS architecture shown in the Fig. 1, we observe that the values of the premise parameters are fixed and the overall output can be expressed as a linear combination of the consequent parameters. In symbols, the output f in the Fig. 1 can be rewritten as

$$f = \frac{\omega_1}{\omega_1 + \omega_2} f_1 + \frac{\omega_2}{\omega_1 + \omega_2} f_2 \quad (22)$$

Substituting Eq. (15) into Eq. (22) yields:

$$f = \bar{\omega}_1 f_1 + \bar{\omega}_2 f_2 \quad (23)$$

Substituting the fuzzy if-then rules into Eq. (23), it becomes:

$$f = \bar{\omega}_1 (p_1 x + q_1 y + r_1) + \bar{\omega}_2 (p_2 x + q_2 y + r_2) \quad (24)$$

After rearrangement, the output can be expressed as:

$$f = (\bar{\omega}_1 x).p_1 + (\bar{\omega}_1 y).q_1 + (\bar{\omega}_1).r_1 + (\bar{\omega}_2 x).p_2 + (\bar{\omega}_2 y).q_2 + (\bar{\omega}_2).r_2 \quad (25)$$

Which is a linear combination of the modifiable consequent parameters p_1, q_1, r_1, p_2, q_2 and r_2 . From this observation, we have

- S = Set of total parameters,
- S1 = Set of premise (nonlinear) parameters,
- S2 = Set of consequent (linear) parameters

The learning algorithm for ANFIS is a hybrid algorithm which is a combination between gradient descent and least-squares method. More specifically, in

Table 1: In the backward pass, the error signals propagate backward and the premise parameters are updated by gradient descent

	Forward pass	Backward pass
Premise parameters	Fixed	Gradient descent
Consequent parameters	Least-squares estimator	Fixed
Signals	Node outputs	Error signals

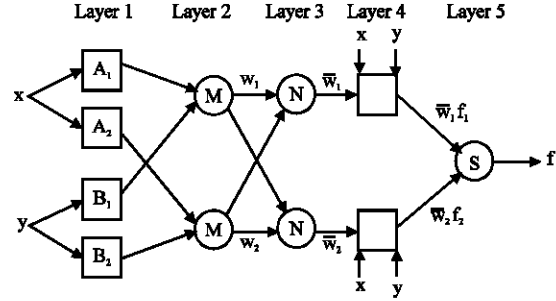


Fig. 1 : Anfis architecture

the forward pass of the hybrid learning algorithm, node outputs go forward until layer 4 and the consequent parameters are identified by the least-squares method. In the backward pass, the error signals propagate backward and the premise parameters are updated by gradient descent. The Table 1 summarizes the activities in each pass.

The consequent parameters are identified optimal under the condition that the premise parameters are fixed. Accordingly, the hybrid approach converges much faster since it reduced the search space dimensions of the original pure backpropagation method.

Approximating magnetic hysteresis

Simulation: The differential Eq. (5), which in its original form has derivatives with respect to H , was reformulated into a differential equation in time by multiplying the left and the right sides by dH/dt , thus resulting in:

$$\frac{dM}{dt} = \frac{1}{1+c} \frac{dH}{dt} \frac{(M_{an} - M)}{\frac{\delta.k}{\mu_0} - \alpha.(M_{an} - M)} + \frac{c}{1+c} \frac{dM_{an}}{dt} \quad (26)$$

This reformulation allows for the determination of magnetization by use of Runge Kutta method in Matlab environment.

To calculate the magnetic flux density B from M and H, the following constitutive law of the magnetic material property is used.

$$B = \mu.H = \mu_0 \mu_r H = \mu_0 (H + M) \quad (27)$$

Where $\mu_0 = 4.\pi.10^{-7}$ (H/m) is the permeability of free space and μ_r is the relative permeability.

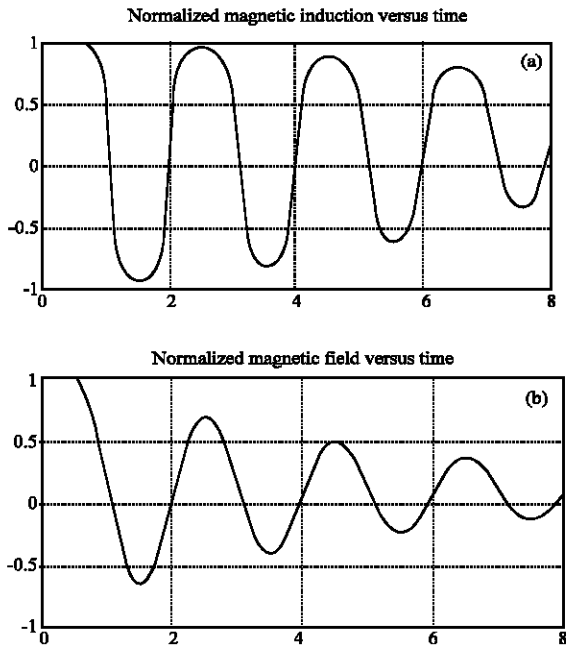


Fig. 2: (a) Normalized magnetic field and magnetic versus time (b) Normalized magnetic induction versus time

The BH curve results of simulation of the Jiles-Atherton model will be used as ‘experimental data’ to be approximate by proposed Neuro-Fuzzy model.

Proposed model: In this study, the learning ability of ANFIS is verified by approximating a hysteresis of magnetic material. The data set used as input/output pairs for Anfis was generated by Jiles Atherton model for ferrite core described in (Emilio *et al.*, 2000) with sinusoidal magnetic field as an input $H(t)$ and magnetic field $B(t)$ as output (Fig. 2a and b).

Our purpose is to predict the magnetic hysteresis cycles using 12 candidate inputs to ANFIS : $B(t-i)$ for $i = 1:5$ and $H(t-j)$ for $j=1:7$. Converted from the original data sets containing 353 $[H(t) B(t)]$ pairs.

In the first time, we suppose that there are two inputs for ANFIS and we have to construct 35 ANFIS models (5x7) with various input combinations and then select the one with the smallest training error for further parameter-level fine tuning. In Table 2 we can see that the ANFIS with B4 and H1 (in red) as inputs has the smallest training error, so it is reasonable to choose this ANFIS for further parameter tuning. Note that each ANFIS has four rules and the training took only one epoch each to identify linear parameters. Let us note that the computing time for selecting the good model is 3.6250s.

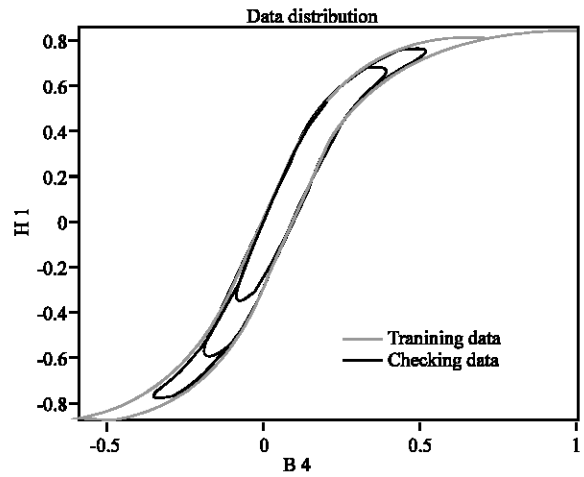


Fig.3 : Data distribution

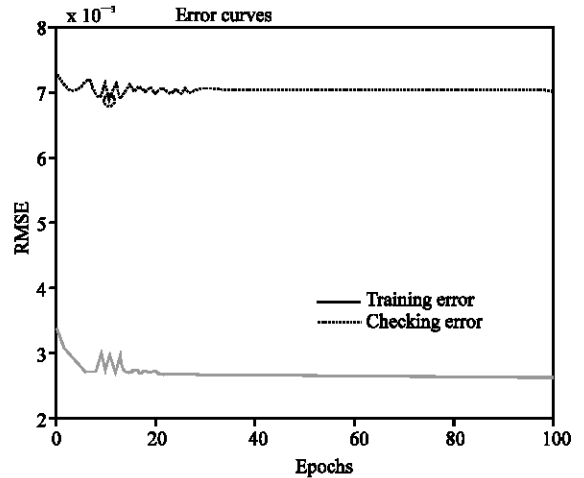


Fig. 4 : Error curves

After selection of the good and adapted model, we made train the network 100 epochs, for this purpose we have used 173 pairs as training data and 173 pairs for checking, shown in Fig. 3.

The number of MFs assigned to each input of the ANFIS was set to two bell type, so the number of rules is 04. The training was run for 100 iterations, the network performance were evaluated on the checking set after every iteration, by calculating the Root-Mean-Square Errors (RMSE).

$$RMSE = \sqrt{\frac{\sum_{k=1}^K (y_k - \hat{y}_k)^2}{K}}$$

Where k is the pattern number, $k = 1, \dots, K$. The RMSE was also evaluated on training data set in every iteration. The

optimal number of iteration was obtained when checking RMSE has reached its minimum value 0.0069 after 11 epochs Fig. 4.

Table 2: The ANFIS with B4 and H1

Model	Training error	Checking error
B1 H1	0.00003501930205	0.00005237387113
B1 H2	0.01007440714157	0.00800225277619
B1 H3	0.01752470640605	0.01198431179800
B1 H4	0.02426970100209	0.01536326214534
B1 H5	0.03081731046969	0.01927371816614
B1 H6	0.03748876601555	0.02457067414985
B1 H7	0.04436386215981	0.03166533456145
B2 H1	0.00003762707899	0.00004451505598
B2 H2	0.01376365775729	0.01540601381771
B2 H3	0.01934001717874	0.01439440856647
B2 H4	0.02538261513892	0.01671041089325
B2 H5	0.03139732428427	0.02005410243223
B2 H6	0.03756279016798	0.02477985332016
B2 H7	0.04401096250169	0.03141151576432
B3 H1	0.00003246300868	0.00003736685372
B3 H2	0.01067960934829	0.01890881385189
B3 H3	0.02674500025744	0.02918359625911
B3 H4	0.02785874502310	0.01994937304432
B3 H5	0.03281732981992	0.02186208656381
B3 H6	0.03831184619918	0.02609472281251
B3 H7	0.04411128584929	0.03227491110643
B4 H1	0.00002571202168	0.00003254290855
B4 H2	0.00948091974762	0.01022982236350
B4 H3	0.02157909185014	0.03958605040899
B4 H4	0.03886365068660	0.04253883911318
B4 H5	0.03576665521093	0.02568298961189
B4 H6	0.04004473971704	0.02826405796018
B4 H7	0.04507868818210	0.03399955457224
B5 H1	0.00003396289474	0.00003924291522
B5 H2	0.00910245284031	0.00666069608720
B5 H3	0.01676944380416	0.01102022047559
B5 H4	0.02386189227013	0.0148317779224
B5 H5	0.03077981552004	0.01932591344086
B5 H6	0.03774316641094	0.02536369036347
B5 H7	0.04473822577434	0.03453095956642

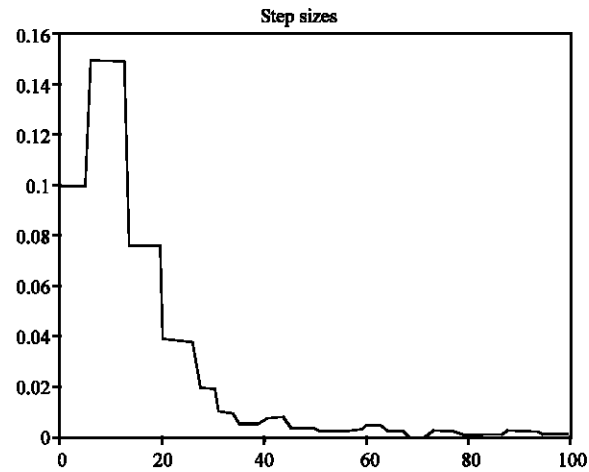


Fig.5: Initial and final generalized bell-shaped membership function of input 1 and 2 for the Best model

Figure 5 depicts the initial and final membership functions for each input variable. The anfis used here contains a total of 24. fitting parameters, of which 12 are premise (nonlinear) parameters and 12 are consequent (linear) parameters. Table 2 summarize all characteristics of the network used.

The ANFIS shown in Fig.1 was implemented by using MATLAB software package (MATLAB version 6.5 with fuzzy logic toolbox), it uses 346 training data in 100 training periods and the step size for parameter adaptation had an initial value of 0.1. The steps of parameter adaptation of the ANFIS are shown in Fig. 5.

The obtained ANFIS network was evaluated on, the complete data set using $T_s = 0.76$ s and resulted in a good prediction with RMSE = 0.0026; Fig. 6.

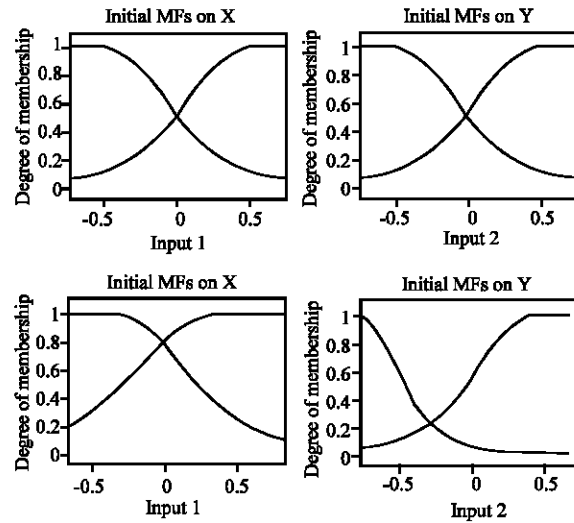


Fig. 6 : Adaptation of parameter steps of ANFIS

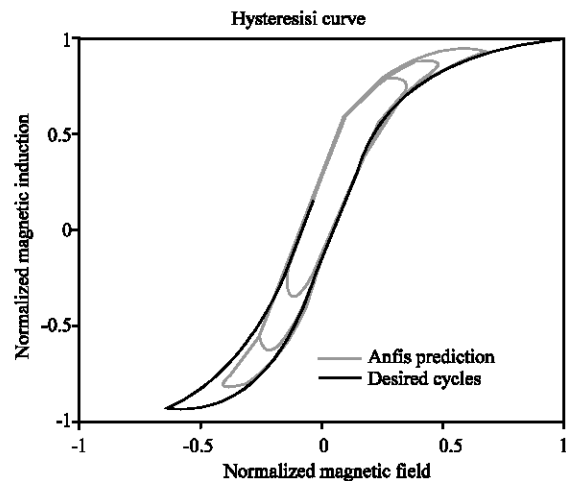


Fig. 7: Hysteresis curves

RESULTS

We have successfully developed, implemented and tested a neurofuzzy system for predicting the magnetic hysteresis of ferromagnetic core. It is clear that the system output closely approximates the required hysteresis output by Jiles-Atherton model show in Fig. 7.

CONCLUSION

The proposed model is an alternative and less complicated approach in determining the magnetic properties of ferromagnetic materials with good accuracy. The collection of well-distributed, sufficient and accurately measured input data is the basic requirement to obtain an accurate model. The adequate functioning of ANFIS depends on the sizes of the training set and test set. Simulation result revealed that neuro-fuzzy model was capable of closely reproducing the optimal performance. In the future studies, we will incorporate this model on the finite element procedure for modeling electromagnetic devices.

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