

Denoising of Biological Signals Using Different Wavelet Based Methods and Their Comparison

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Abstract: Denoising of EEG signals using different wavelet shrinkage methods is proposed in this study. We applied these methods to denoise EEG signal contaminated with additive Gaussian noise. In these methods Visu Shrink, minimizing the False Discovery Rate (minFDR), Top, Hypothesis Testing thresholding rules and Hard, Soft thresholding filters are considered. The performances of these methods are evaluated and the results are compared using Mean Square Error (MSE) and Signal to Noise Ratio (SNR). Experiments revealed that minFDR and Hypothesis Testing rules with Hard thresholding filter and Top rule with Soft thresholding filter perform superior to other combinations of thresholding rules and filters.

Key words: EEG, wavelet transform, wavelet shrinkage, thresholding, denoising

INTRODUCTION

With sensors becoming ubiquitous and computers becoming powerful, there has been a phenomenal growth in the collection of signals or data. During signal acquisition or transmission, it is often contaminated with noise. Removing noise from the signal is the first step in data analysis. This is applicable to biological signals also. The random noises uncorrelated with biological signals can be approximated by additive white Gaussian noise. Many techniques have been proposed for denoising the signals.

Wavelet shrinkage denoising methods are very popular for denoising biological signals (Donoho and Johnstone, 1994a, 1995b; Bruce and Gao, 1996; Ogden, 1997; Vidakovic, 1999). In this study the performance evaluation of these methods is done by using EEG signal contaminated with varying levels of additive white Gaussian noise. Results are compared using MSE and SNR.

MATERIALS AND METHODS

Denoising using wavelet shrinkage: Wavelet shrinkage denoising methods remove the noise present in the signal

while preserving the signal characteristics (Carl Taswell, 2000). In these methods noisy biological signal is decomposed into wavelet coefficients by applying wavelet transform. After fixing the threshold using a thresholding rule, the coefficients are modified by using a thresholding filter. Denoised signal estimate is obtained by applying inverse wavelet transform on the modified coefficients. We have to select a wavelet for forward and inverse transformations (Daubechies, 1992; Graps, 1995). Wavelet Symmlet 8 is considered here. The shrinkage methods differ in the choice of thresholding rules and thresholding filters. We can obtain different denoising methods by considering different thresholding rules and filters. In this study Visu Shrink, minFDR, Top, Hypothesis Testing thresholding rules and Hard, Soft thresholding filters are used in these methods.

Visu shrink: Universal threshold for a signal of length N is given by

$$\hat{\sigma} \sqrt{2 \log N}$$

where, $\hat{\sigma}$ is the estimate of noise standard deviation (Donoho and Johnstone, 1994). Visu Shrink is thresholding performed by applying this threshold. This

is global thresholding scheme for one dimensional signals and threshold is determined independently of the thresholding filter.

Minimizing the false discovery rate: The threshold obtained by applying this thresholding rule is same for all thresholding filters.

Calculation of threshold: The minFDR rule (Vidakovic, 1999) determines the threshold by keeping the expected value of the fraction of coefficients erroneously included in the reconstruction below a given fraction s . Let N be the no. of wavelet coefficients $\{T_n, n = 1, 2, \dots, N\}$ then for each wavelet coefficient first compute the r -values, given by

$$r_n = 2 \left[1 - \Phi \left(\frac{|\omega_n|}{\hat{\sigma}} \right) \right]$$

where, $N(\cdot)$ is the cumulative distribution function of the standard normal distribution and $\hat{\sigma}$ is an estimate of the noise standard deviation. Then r_n values are ordered as $r_{(1)} \# r_{(2)} \# \dots \# r_{(N)}$. Starting with $n = 1$, let q be the largest index n such that

$$r_{(n)} \leq \frac{n}{N} s$$

The threshold is then given by

$$\lambda = \hat{\sigma} \varphi^{-1} \left(1 - \frac{r_q}{2} \right)$$

Top: Given b as the fraction of the largest coefficients to keep, the threshold δ is set to be the $(1-b)$ th quantile of the empirical distribution of the absolute values of the wavelet coefficients. The threshold obtained using this rule can be used with any thresholding filter (Bruce and Gao, 1996).

Hypothesis testing: The threshold estimation in this method is independent of thresholding filter used. It calculates level dependant thresholds after performing wavelet transformation on the signal (Ogden, 1997).

Calculation of threshold: Let the wavelet coefficients T are N_s in number at a particular level and assume that they are normally distributed. Find α -critical value,

$$v_{N_s}^\alpha = \left\{ \Phi^{-1} \left[\frac{(1-\alpha)^{1/N_s} + 1}{2} \right] \right\}^2$$

where, α is error probability parameter. $N(\cdot)$ is cumulative distribution function of standard normal density. Then

find the largest of the squared wavelet coefficients at that level, denoted by $T_{(N_s-1)}^2$ and compare it to the above value $< v_{N_s}^\alpha$. If

$$\omega_{(N_s)}^2 / \hat{\sigma}^2 > v_{N_s}^\alpha$$

where, $\hat{\sigma}$ is an estimate of the standard deviation of noise, $T_{(N_s)}$ is retained as signal. Next repeat the process with the square of second largest (in absolute value) wavelet coefficient $T_{(N_s-1)}^2$. If

$$\omega_{(N_s-1)}^2 / \hat{\sigma}^2 > v_{N_s-1}^\alpha$$

the procedure continues until at some point the p th largest (in absolute value) coefficient satisfies

$$\omega_{(p)}^2 / \hat{\sigma}^2 \leq v_p^\alpha$$

The threshold at that level is then set as $\delta = *T_{(p)}*$. The recommended value for α is 0.05.

Hard and soft thresholding filters: The popular Hard and Soft thresholding filters are commonly used in these methods. Algorithm for Hard thresholding filter is $H(T, \delta)$ for all $*T* > \delta$ otherwise zero (Marteen Jansen, 2001). Soft thresholding filter (Donoho, 1995) is defined as $S(T, \delta) = \text{sgn}(T) \max(0, *T* - \delta)$ (Donoho, 1995) T represents detail wavelet coefficients, δ represents the threshold.

RESULTS AND DISCUSSION

This section reports the results obtained on denoising of EEG signals using shrinkage denoising methods. EEG signals (Andrzejak *et al.*, 2001) of sample size 2048 contaminated with additive white Gaussian noise of different values of standard deviation (F) are simulated. Wavelet decomposition of EEG signal is made up to three levels using Symmlet 8 (Mallat, 1989). After fixing the threshold using a thresholding rule, the wavelet coefficients are filtered by using a thresholding filter. The inverse wavelet transform is applied on the resultant coefficients and denoised signal estimate is obtained.

MSE and SNR are used as measure of denoising. They are calculated as:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (X(i) - \hat{X}(i))^2$$

$$\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^n X(i)^2}{\sum_{i=1}^n (X(i) - \hat{X}(i))^2} \text{ dB}$$

Table 1: Denoising Results of EEG F057 using Hard Thresholding Filter

	F = 10		F = 20		F = 30	
	MSE	SNR	MSE	SNR	MSE	SNR
Noisy signal	99.94	15.70	399.85	9.68	901.43	6.15
Visu shrink	74.04	17.01	186.22	13.01	336.76	10.43
min FDR	56.55	18.18	156.68	13.75	283.52	11.18
Top	75.99	16.89	306.61	10.83	692.69	7.29
Hyp testing	64.47	17.61	157.27	13.74	268.46	11.42

Table 2: Denoising Results of EEG F057 using Soft Thresholding Filter

	F = 10		F = 20		F = 30	
	MSE	SNR	MSE	SNR	MSE	SNR
Noisy signal	99.94	15.70	399.85	9.68	901.43	6.15
Visu shrink	188.13	12.96	456.16	9.11	764.92	6.87
min FDR	118.71	14.96	320.40	10.65	560.61	8.22
Top	57.14	18.13	149.75	13.95	275.30	11.30
Hyp testing	121.44	14.86	279.47	11.25	466.16	9.02

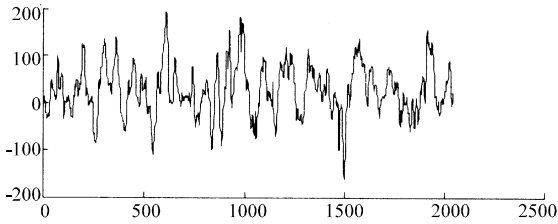


Fig. 1: Original EEG

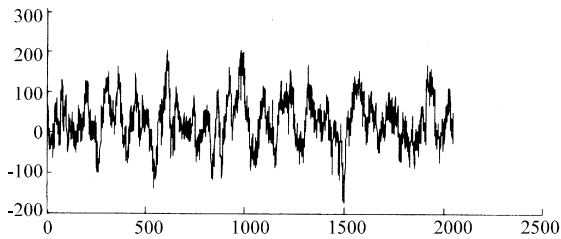


Fig. 2: Noisy EEG

where, n represents no. of samples, $X(i)$ original signal data, $\hat{X}(i)$, denoised signal data

The simulation experiment is repeated 100 times and average values of MSE and SNR are found. These experiments are conducted on 50 numbers of EEG signals and found that the results are same. The simulation is implemented in MATLAB environment. In these experiments the thresholding rules Visu Shrink, minFDR, Top, Hypothesis Testing are applied using Hard and Soft thresholding filters. Table 1 shows the denoising results of EEG signal F057 obtained using Hard thresholding filter with different thresholding rules. The results of denoising of EEG F057 with Soft thresholding filter are reported in Table 2. The original and denoised signals F057 obtained using Hard, Soft filters for $F = 20$ are shown in Fig. 1-4.

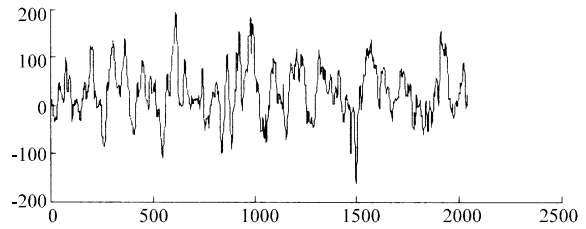


Fig. 3: Denoised EEG using minFDR, Hard Filter

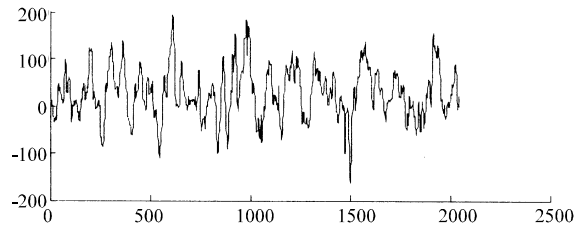


Fig. 4: Denoised EEG using Top rule, Soft Filter

From the results it is observed that with Hard thresholding filter for $F = 10$, MSE of 56.55 and SNR of 18.18 are obtained using minFDR (Table 1) and for $F = 20$, MSE of 149.75 and SNR of 13.95 are obtained using Top rule with Soft thresholding filter (Table 2). It shows that when $F = 10$ min FDR with Hard thresholding filter and when $F = 20$. Top rule with Soft thresholding filter performs superior to other combination of thresholding rules and filters at these values of F . MSE of 268.46 and SNR of 11.42 obtained for $F = 30$ with Hypothesis Testing rule using Hard thresholding filter indicates that it performs better than other thresholding rules and filters at $F = 30$ (Table 1).

CONCLUSION

In this study estimation of EEG signal from noisy environment is made using wavelet shrinkage methods. In these methods Visu Shrink, minFDR, Top, Hypothesis Testing thresholding rules and Hard, Soft thresholding filters are used. MSE and SNR are used as criteria for testing the performances of these methods. We can further extend the denoising of biological signals by applying these shrinkage methods in succession.

ACKNOWLEDGEMENT

The authors place on record their thanks to the authorities of the RVR and JC college of Engineering, Guntur, A P India and KL College of Engineering, Vijayawada, A P India for the facilities provided.

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