

A Novel DOA Estimation Based upon Recursive Unitary Music Algorithm

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Abstract: In many communication systems, one is interested in point to point communication and for this, a highly directive beam of radiation is required. In this study, the performance of DOA estimation based upon (recursive unitary MUSIC algorithm) RUMA was proposed. The basic of RUMA is dealing with real-valued matrices and BiSVD subspace tracking method to carry out iterative DOA estimation without heavy (eigen-value decomposition) EVD or (singular-value decomposition) SVD. To investigate the computational time and accuracy of DOA estimation proposed method, first develop the antenna array model for DOA estimation then modified recursive unitary MUSIC matrix data. Simulation result shows the accuracy of DOA estimation better under no mutual coupling effects between the antenna's by adding an unitary type of algorithm and computational time improved even with the increase of number of array elements.

Key words: Antenna arrays, direction of arrival estimation, music algorithm, BiSVD

INTRODUCTION

The growth of wireless technology has received great interest worldwide in recent years. It is foreseen that in the future an enormous increase in track will be experienced for mobile and personal communication systems. This is due both to an increased number of users and to new high bit rate data services being introduced (Strobach, 1997). To overcome this situation a good system performance is needed. To improve the system performance so that it can adapt with future development, it is necessary to have high resolution for the Direction of Arrival (DOA) estimation in order to distinguish the desired signal with interference when the demand for its high performance are increasing (Nobuyoshi, 2003).

Also the DOA estimates are used in adaptive beamforming for interference suppression, a critical component in cellular systems. Interference suppression reduces the Multiple Access Interference (MAI), which lowers the required transmit power. The interference suppression capability directly influences the cellular system capacity, i.e., the number of active mobile subscribers per cell, Beamforming, tracking and DOA estimation are current research topics with various technical approaches. Least mean square estimation, Kalman filtering and neural networks (Nobuyoshi, 2003; Strobach, 1997, 1998) have been successfully applied to these problems. Many approaches have been developed for calculating the DOA; three techniques based on signal subspace decomposition are ESPRIT, MUSIC and Root-MUSIC (Haardt and Nosssek, 1995).

In this study, we propose the recursive Unitary MUSIC to improve the computation efficiency toward real-time DOA estimation.

The basic algorithm of this is Unitary MUSIC which deals with real-valued matrices and in addition BiSVD subspace tracking method is employed to carry out iterative DOA estimation without heavy EVD or SVD. Through computer simulation, the proposed algorithm (Unitary MUSIC algorithm) compared with the recursive Standard MUSIC (complex-valued version).

MATERIALS AND METHODS

Principle background of DOA estimation

Receiving system and signal model: We develop the signal model for DOA estimation by MUSIC algorithm. Receiving system is a K-element equispaced linear array as shown in Fig. 1.

Assuming that there is no mutual coupling between the antennas then we have L incoming signals to the array, the directions of which are θ_l ($l = 1, 2, \dots, L$), respectively. Then, snapshot vector of the array at time instant t can be expressed as follows:

$$x(t) = As(t) + n(t) \quad (1)$$

$$A = [a(\theta_1), \dots, a(\theta_L)] \quad (2)$$

$$s(t) = [s_1(t), s_2(t), \dots, s_L(t)]^T \quad (3)$$

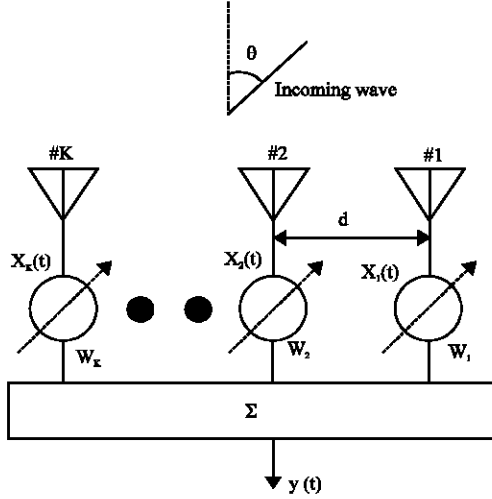


Fig. 1: K-element linear array antenna

Where:

- $s_i(t)$ = The i th incoming signal
- $n(t)$ = An internal noise vector
- $a(\theta_i)$ = The array response vector (mode vector) of the i th signal
- A = The matrix of array response (mode matrix)

From the above input data expression, it is possible to estimate DOA by recursive Unitary MUSIC algorithm (Strobach, 1997).

Unitary operation to input data: First, to perform iterative estimation at every time instant t ($t = 1, 2, \dots$), the data matrix is defined as follows:

$$X(t) = [\alpha^{1/2} X(t-1) (1 - \alpha)^{1/2} x(t)] \quad (4)$$

Where:

- $X(t-1)$ = The old data matrix
- $x(t)$ = A current snapshot vector
- α = A forgetting factor ($0 < \alpha < 1$)

The snapshot vector of $x(t)$ is transformed into another vector $y(t)$ with a unitary matrix and it is written as:

$$y(t) = Q^H x(t) \quad (5)$$

Where, Q_k is the unitary matrix and is defined as follows (Nobuyoshi, 1998, 2003) when K is even ($K = 2M$):

$$Q_K = Q_{2M} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_M & jI_M \\ \Pi_M & -j\Pi_M \end{bmatrix} \quad (6)$$

When K is odd $CK = 2M + 1$

$$Q_K = Q_{2M} = \frac{1}{\sqrt{2}} \begin{bmatrix} I_M & 0 & jI_M \\ 0 & \sqrt{2} & 0^T \\ \Pi_M & 0 & -j\Pi_M \end{bmatrix} \quad (7)$$

Where:

- I_M = The identity matrix of the dimension
- M, Π_M = The square matrix of dimension M , which is defined as:

$$\Pi_M = \begin{bmatrix} 0 & & 1 \\ & 1 & \\ 1 & & 0 \end{bmatrix} \quad (8)$$

For Unitary MUSIC the data matrix is modified as follows (Strobach, 1997). The data matrix of Eq. 9 is real-valued.

$$X(t) = \left[\alpha^{1/2} X(t-1) (1 - \alpha)^{1/2} yc(t) \right] \quad (9)$$

$$yc(t) = [\text{Re}[y(t)] \text{Im}[yc(t)]] \quad (10)$$

BiSVD subspace tracking method: In the proposed algorithm, we use BiSVD (Bi-iteration Singular-Value Decomposition) subspace tracking method Strobach (1998) to estimate the signal subspace. BiSVD is based on decomposition of singular values of $X(t)$. It determines recursively the signal subspace matrix $Q_A(t)$ which consists of eigenvectors belong- ing to signal subspace of $X(t) X^T(t)$. Strobach (1998) the BiSVD to $X(t)$ can be expressed as follows:

$$B(t) \triangleq X^T(t) Q_A(t-1) \quad (11)$$

$$B(t) = Q_B(t) R_B(t) : N \times L \quad (12)$$

(QR decomposition)

$$A(t) \triangleq X(t) Q_B(t) \quad (13)$$

$$A(t) = Q_A(t) R_A(t) : K \times L \quad (14)$$

(QR decomposition)

Where, $X(t)$ is supposed to be $(K \times N)$ matrix. Through Eq. 11-14 matrices $B(t)$ and $A(t)$ are decomposed into $\{Q_B(t) \in R^{N \times L}, R_B(t) \in R^{L \times L}\}$ and $\{Q_A(t) \in R^{K \times L}, R_A(t) \in R^{L \times L}\}$, respectively. By BiSVD subspace tracking method, $Q_A(t), R_A(t), R_B(t)$ are recursively determined. For the initial value of $Q_A(t)$, we consider the following two methods (Strobach, 1997).

Initial value A:

$$Q_A(0) = \begin{bmatrix} I_L \\ 0 \end{bmatrix} \times p \in \mathbb{R}^{k \times L}$$

$$R_A(0) = 0_{L \times L} \quad R_B(0) = I_L$$

where, p determines the magnitude of initial value which is in the range $0 < p \leq 1$.

Initial value B: $Q_A(0)$ is derived from the SVD of $X(t)$ with first several snapshots. $R_A(0)$, $R_B(0)$ are in turn determined from $Q_A(0)$ through BiSVD.

RESULTS AND DISCUSSION

Recursive unitary MUSIC algorithm: From unitary operation of Eq. 5, $a(\theta)$ can be transformed into a real-valued vector which is given by:

$$d(\theta) = Q_k^H a(\theta) \tag{15}$$

This is because the components of $a(\theta)$ have property of so called conjugate centrosymmetry with respect to the center component (Nobuyoshi, 2003; Strobach, 1998).

This property is important to create angular spectrum with Unitary MUSIC. From the property, Unitary MUSIC algorithm can compute the angular spectrum by using noise space eigenvectors Nobuyoshi (1998) of the

real-valued correlation matrix. Then, the angular spectrum of recursive Unitary MUSIC algorithm at time instant t can be written as follows (Nobuyoshi, 2003):

$$P_{MU}(\theta; t) = \frac{d^T(\theta)d(\theta)}{d^T(\theta)\{I_L - Q_A(t)Q_A^T(t)\}d(\theta)} \tag{16}$$

Computer simulation: Table 1 shows the simulation conditions used in our simulation. In the simulation, we estimate 3 in-coming waves ($L = 3$), which are uncorrelated with each other. We also examine the performance of recursive Standard MUSIC algorithm for comparison to the proposed algorithm.

Figure 2 and 3 show the change of spectrum from $t = 1-12$. In Fig. 2, the recursive Unitary MUSIC and recursive Standard MUSIC are compared in the case of initial value A. On the other hand in Fig. 3, the initial values A and B are compared in the recursive Unitary MUSIC. Five snapshots are used in the calculation of initial value B. From the results, it is found that the

Table 1: Simulation conditions

Conditions	Values
Type of array	Equispaced linear array
Antenna element	Isotropic
Element spacing	0.5λ (λ : wave length)
No. of elements	10
No. of incoming waves	1
SNR	20 days
B forgetting factor: α	0.9

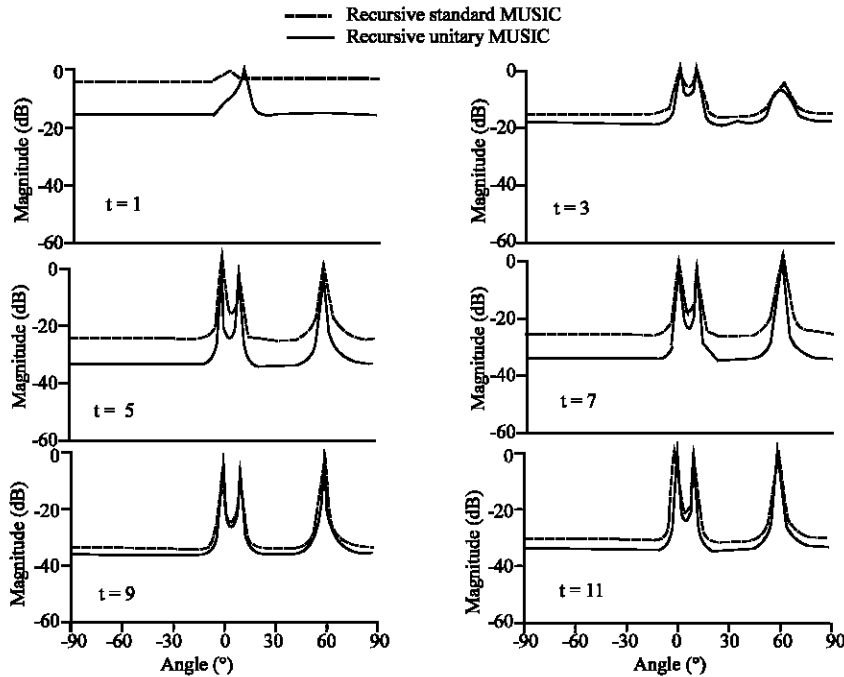


Fig. 2: MUSIC spectrum of recursive MUSIC with initial value A

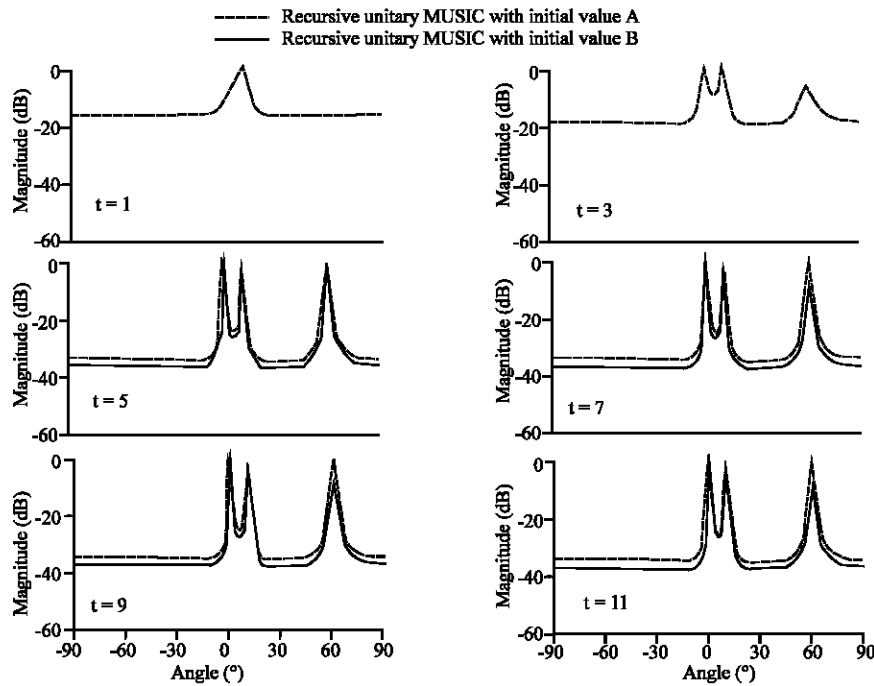


Fig. 3: MUSIC spectrum of recursive Unitary MUSIC with initial values A and B

estimation accuracy of unitary type is better than standard type. Also, the initial value B provides more stable spectrum than the initial value A immediately after $t = 5$.

CONCLUSION

In this study, the novel DOA estimation with an equispaced linear array using recursive Unitary MUSIC algorithm, also, the performance analysis of this algorithm was carried out.

From the simulation results shows that recursive Unitary MUSIC with the initial value determined by SVD of the first several snapshots can provide accurate and stable DOA estimation as well as high computation efficiency.

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