

## Improved MMSE Scheme in Cooperative Base Station Systems

A. Hameed and Ali Oudah

Faculty of Manufacturing Engineering, Universiti Malaysia Pahang, 26600 Pekan, Malaysia

---

**Abstract:** This study addresses minimum mean square error scheme used with asynchronous interference mitigation in cooperative base stations. The asynchronous transmission is considered owing to the dissimilar transmission times between the base station and mobile stations. Furthermore, the channel quantization errors due to channel quantization are taken into account. The suggested model is resilient to asynchronous interference and channel quantization errors associated with base station cooperative systems. The developed scheme outperforms the conventional ones found in literature as demonstrated.

**Key words:** MMSE, asynchronous interference, cooperative base stations, error, mobile

---

### INTRODUCTION

While the capacity gains in point to point (Foschini and Gans, 1998; Huaiyu *et al.*, 2004) and multiuser (Catreux *et al.*, 2001) Multiple Input and Multiple Output (MIMO) wireless systems are significant. But, they are very limited due to intra and inter-cell co-channel interference (Huaiyu *et al.*, 2004; Catreux *et al.*, 2001) in cellular network. To mitigate this limitation on the cellular downlink and achieve MIMO capacity gains, recently, it has been shown that Base Station (BS) cooperation (Ng *et al.*, 2008) in which different BSs together transmit signal for different Mobile Station (MSs) can significantly improve spectral efficiency. One of the key strategies in cooperative cells is to design the precoding using the CSI for mitigating the interference and maximizing the sum rate.

The theoretical analyses of BS cooperation often assume that the multiple BSs can be modeled as a single giant BS with more antennas. The optimal sum capacity of MIMO broadcast channel is given by Dirty Paper Coding (DPC) (Costa, 1983). Due to high complexity of DPC, more practical precoding techniques are MMSE (Li *et al.*, 2009), ZF and BD precoder which mitigates the interference between each user (Spencer *et al.*, 2004; Pan *et al.*, 2004). Most of the literatures on multi-cell cooperation assume that full CSI of all the active users is available at the transmitter side. Considering, the more practical system, the perfect CSI at the transmitter is impossible. So, the limited feedback has much effect on performance of system. Meanwhile, limited feedback is a key factor for precoding design. The performance of zero-forcing based on limited feedback in a downlink channel (multiple transmit antennas, single antenna per receiver) is studied (Jindal, 2006). Similar to the Jindal (2006), the performance

of BD with limited feedback was studied for multiple receiver antennas (Ravindran and Jindal, 2008). There are other literatures that study the delay limited feedback caused by the channel estimation, quantization and feedback process (Shi *et al.*, 2011; Bhagavatula and Heath, 2011). Research on practical cooperative BSs has obtained more attentions.

Asynchronous interference should not be ignored in cooperative BSs systems. When dealing with multi-cell environments, a crucial assumption often made is that both the desired and interfering signals arrive synchronously at each user. However, the interference is inherently asynchronous (Zhang *et al.*, 2008). Perfect timing advance mechanisms can ensure that the signals from the BSs arrive at their intended recipients synchronously. However, the BSs cannot also align all the interfering signals at each user MS because of the different propagation times between the BSs and MSs. So, it is unrealizable that both of desired and interfering signals arrived at MSs simultaneously. To the best of the knowledge no robust precoding has been designed for cooperative BSs with asynchronous interference.

In this study, researchers design a robust precoder for the the multi-user in cooperative BSs systems with asynchronous interference and limited feedback simultaneously. This new design is based on MMSE for mitigating asynchronous interference. Researchers first calculate the expected MSE cost function conditioned on asynchronous interference and limited feedback. Then, by minimizing the MSE cost function, the new robust MMSE in closed form is obtained.

**Notation:**  $A^H$  is the conjugate transpose of matrix  $A$ .  $\mathbb{E}(\cdot)$ ,  $\text{tr}(A)$   $\|A\|_F$ , stands for the expectation operator, respectively.

**SYSTEM MODEL**

**Asynchronous Interference Model:** Considering the downlink of a multi-user MIMO in cooperative multi-cell system with B BSs, each BS has  $N_t$  transmitter antennas and K active users with  $N_r$  receiver antennas. Each user receives  $L_k$  data streams simultaneously where  $L_k \leq \min\{BN_t, N_r\}$ . For simplicity, it is assumed that all the MSs have the same number of data stream equal to  $N_r$ . Let,  $s_k$  the data streams of user k and  $\mathbb{E}[s_k s_k^H] = I_{N_r}$ . The data vectors of a user k are independent with unit average power. Moreover, the data vectors of a user k are independent over time. As for the data vectors of different user, they are also independent.

Due to different transmission delay between BSs for user k, the asynchronous interference must exist in cooperative BSs System. With the perfect CSI at the transmitter side and interference signal asynchronous, the multi-user interference is not annulled completely and the received signal of user k is:

$$y_k = \underbrace{\sum_{b=1}^B H_k^{(b)} T_k^{(b)} s_k}_{\text{Desired signal}} + \underbrace{\sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} T_j^{(b)} i_{jk}^{(b)}}_{\text{Intere-user interference}=0} + n_k \quad (1)$$

$$= H_k T_k s_k + \sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} T_j^{(b)} i_{jk}^{(b)} + n_k$$

$H_k^{(b)} \in \mathbb{C}^{N_r \times N_t}$  whose elements are complex Gaussian random variables with unit variance is channel matrix from BS b to user k.  $s_k$  is transmitted signal for user k.  $T_k^{(b)}$  is precoding with perfect CSI for the user k in BS b.  $n_k$  is the complex Gaussian noise with  $\mathbb{E}[n_k n_k^H] = I_{N_r}$ .

$$i_{jk}^{(b)} = \rho(\delta_{jk}^{(b)} - T_s) s_j(m_{jk}^{(b)}) + \rho(\delta_{jk}^{(b)}) s_j(m_{jk}^{(b)} + 1)$$

denotes the asynchronous interference at MS k from the signal transmitted by BS b for MS j.  $m_{jk}^{(b)}$  and  $m_{jk}^{(b)}+1$  are the two consecutive symbols. m and n are discrete time indices. In general, unless required, researchers will drop the time index.

$$\rho(\tau) = \int_0^{T_s} g(t)g(t-\tau) dt$$

$g(t)$  is the matched filter,  $\delta_{jk}^{(b)} = \tau_{jk}^{(b)} \bmod T_s$ ,  $T_s$  is the symbol period,  $\tau_{jk}^{(b)}$  represents the difference between the time when user k receives its intended signal and the time when interference from BS b while transmitting to user j arrives at user k. The correlation between  $i_{jk}^{(b)}$  and  $i_{jk}^{(b)}$  is:

$$\mathbb{E}[i_{jk}^{(b1)} i_{jk}^{(b2)H}] = \beta_{jk}^{(b1, b2)} I_{N_r}$$

and the detail of property of  $\beta_{jk}^{(b1, b2)}$  can be seen in (Zhang *et al.*, 2008).

**Limited Feedback Model:** In this study, researchers introduce the Quantization Channel Model for MIMO system with limited feedback. According to Ravindran and Jindal (2008), the quantized CSI  $\hat{H}_k$  can be decomposed as:

$$\tilde{H}_k = \hat{H}_k + X_k Y_k + S_k Z_k \quad (2)$$

where,  $X_k \in \mathbb{C}^{N_r \times N_r}$  is unitary matrix.  $Z_k \in \mathbb{C}^{N_r \times N_r}$  denotes an upper triangular matrix with positive diagonal entries and satisfies  $\text{tr}(Z_k^H Z_k) = d^2(H_k \hat{H}_k)$ .  $Y_k \in \mathbb{C}^{N_r \times N_r}$  is upper triangular matrix with positive diagonal elements and satisfies  $Y_k^H Y_k = I_{N_r} - Z_k Z_k^H$ .  $S_k \in \mathbb{C}^{N_r \times N_r}$  is an orthonormal basis for an isotropically distributed  $N_r$  dimensional plan in the  $N_r \cdot N_r$  dimensional left nullspace of  $\hat{H}_k$ . So, the broadcast channel  $H_k$  can be decomposed by:

$$H_k = (\hat{H}_k (I_{N_r} - Z_k^H Z_k)^{1/2} + S_k Z_k), R_k = \hat{H}_k A_k + S_k B_k \quad (3)$$

where,  $R_k = \text{diag}(\sqrt{\Lambda_k})$ ,  $A_k$  is a diagonal matrix consists of  $N_r$  non-zero unordered eigen values of:

$$H_k H_k^H \cdot A_k = (I_{N_r} - Z_k^H Z_k)^{1/2} R_k, B_k = Z_k R_k$$

Assume a random codebook of size  $2^Q$  (Ravindran and Jindal, 2008), i.e.,  $W = W_1, \dots, W_{2^Q}$  which is known at both the transmitter and the receiver. Each user quantizes its channel to one of the vectors in the codebook where Q is the number of feedback bits. The quantization of a channel matrix  $H_k$ , i.e.,  $\hat{H}_k$  is chosen from the codebook W according to the minimum distance criterion:

$$D = \mathbb{E} \left[ \min d^2(\hat{H}_k, W) \right]$$

Each user feeds back the index of W to the transmitter. For large Q (Ravindran and Jindal, 2008), the distance D has an upper bound  $\bar{D}$ , i.e.,

$$D \leq \bar{D} = \frac{\Gamma(1/T)}{T(C_{N_r, N_r})^{-1/T} 2^{-Q/T}}$$

where,  $\Gamma(\cdot)$  is the gamma function,  $T = N_r(N_r - N_r)$  and  $C_{N_r, N_r}$  is given by:

$$\frac{1}{T!} \prod_{i=1}^{N_r} \frac{(N_r - i)!}{(N_r - i)!}$$

Researchers consider the interference signal asynchronous and the imperfect CSI at the transmitter side, the signal of user k can be expressed by:

$$y_k = \underbrace{\sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k}_{\text{Desired signal}} + \underbrace{\sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} \hat{T}_j^{(b)} i_{jk}^{(b)}}_{\text{Multuser asynchronous interference signal}} + n_k = H_k \hat{T}_k x_k + \sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} \hat{T}_j^{(b)} i_{jk}^{(b)} + n_k \quad (4)$$

$$\sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} \hat{T}_j^{(b)} i_{jk}^{(b)}$$

represents the multi-user asynchronous interference with limited feedback.  $\hat{T}_k^{(b)}$  is precoding with imperfect CSI for the user in BS.

### ROBUST MMSE PRECODING DESIGN

In this study, taking into account asynchronous interference and channel quantization error, researchers propose robust precoding under MMSE criterion. Given the channel model described in study, researchers are interested in designing the precoding at the BS to minimize the following MSE object function:

$$\begin{aligned} \text{MSE}_k &= \mathbb{E} \left[ \|s_k - y_k\|_F^2 \right] \\ &= \mathbb{E} \left[ \left\| s_k - \sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k - \sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} \hat{T}_j^{(b)} i_{jk}^{(b)} - n_k \right\|_F^2 \right] \end{aligned} \quad (5)$$

Let,  $J_k$  denote  $\sum_{b=1}^B \sum_{\substack{j=1 \\ j \neq k}}^K H_k^{(b)} \hat{T}_j^{(b)} i_{jk}^{(b)}$ , Eq. 5 can be changed as:

$$\begin{aligned} \text{MSE}_k &= \mathbb{E} \left\{ \left\| s_k - \sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k - J_k - n_k \right\|_F^2 \right\} = \mathbb{E} \left\{ \|s_k\|_F^2 \right\} + \mathbb{E} \left\{ \|n_k\|_F^2 \right\} + \mathbb{E} \left\{ \left\| \sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k \right\|_F^2 \right\} \\ &\quad + \mathbb{E} \left\{ \|J_k\|_F^2 \right\} - \mathbb{E} \left\{ \text{tr} \left( \sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k s_k^H \right) \right\} - \mathbb{E} \left\{ \text{tr} \left( s_k s_k^H \sum_{b=1}^B \hat{T}_k^{(b)H} H_k^{(b)H} \right) \right\} \end{aligned} \quad (6)$$

Six terms in right side of Eq. 6 is calculated, respectively (Appendix A), researchers obtain the  $\text{MSE}_k$  that be given by:

$$\begin{aligned} \text{MSE}_k &= \mathbb{E} \left\{ \left\| s_k - \beta^{-1} \sum_{b=1}^B H_k^{(b)} \hat{T}_k^{(b)} s_k - \beta^{-1} J_k - \beta^{-1} n_k \right\|_F^2 \right\} \\ &= N_t + \beta^{-2} N_t + \beta^{-2} \left( N_t - \frac{N_t D}{N_r} \right) \underbrace{\text{tr} \left( \sum_{b=1}^B \hat{H}_k^{(b)} \hat{T}_k^{(b)} \hat{T}_k^{(b)H} \hat{H}_k^{(b)H} \right)}_{\langle 1 \rangle} + \beta^{-2} \frac{N_t D}{N_t - N_r} \underbrace{\text{tr} \left( \sum_{b=1}^B \hat{T}_k^{(b)H} \hat{H}_k^{(b)} \right)}_{\langle 2 \rangle} - \\ &\quad \underbrace{\beta^{-2} \frac{N_t N_r D}{N_r (N_t - N_r)} \text{tr} \left( \sum_{b=1}^B \hat{T}_k^{(b)H} \hat{H}_k^{(b)} \hat{H}_k^{(b)H} \hat{T}_k^{(b)} \right)}_{\langle 3 \rangle} + \underbrace{\beta^{-2} \mathbb{E} \left\{ \text{tr} \left( \sum_{b_1=1}^B \sum_{b_2=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_k^{(b_1)} \hat{T}_k^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H} \right) \right\}}_{\langle 4 \rangle} + \\ &\quad \underbrace{\beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j \neq k}^K \sum_{b_1=1}^B \sum_{b_2=1}^B \beta_{jk}^{(b_1, b_2)} (\hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_j^{(b_1)} \hat{T}_j^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H} \right)}_{\langle 5 \rangle} - \underbrace{2\psi \beta^{-1} \text{Re} \left[ \text{tr} \left( \sum_{b=1}^B (\hat{H}_k^{(b)} \hat{T}_k^{(b)}) \right) \right]}_{\langle 6 \rangle} \end{aligned} \quad (7)$$

**Proof (Appendix A):** Then, MSE cost function can be defined as:

$$\{\hat{T}_k^{(b)\text{opt}}\}_{k=1}^K = \arg \min_{\{\hat{T}_k\}_{k=1}^K} \sum_{k=1}^K \text{MSE}_k \text{ s.t. } \text{tr}\{\hat{T}_k^{(b)\text{H}}\hat{T}_k^{(b)}\} = P \quad (8)$$

P is the transmission power of per BS. The Lagrange Multiplier Method is used to solve problem and the Lagrangian functions is constructed as:

$$f\left(\{\hat{T}_k^{(b)\text{opt}}\}_{k=1}^K\right) = \text{MSE}_k + \lambda_k \left( \text{tr}\left\{\sum_{b=1}^B \hat{T}_k^{(b)\text{H}}\hat{T}_k^{(b)}\right\} - P \right) \quad (9)$$

where,  $\lambda_k$  is the Lagrange multiplier. Taking the derivative of  $f\left(\{\hat{T}_k^{(b)\text{opt}}\}_{k=1}^K\right)$  with respect to  $\hat{T}_k^{(b)}$ , researchers have:

$$\frac{\partial f}{\partial(\hat{T}_k^{(b)})} = \frac{\partial}{\partial(\hat{T}_k^{(b)})} \text{MSE}_k + \frac{\partial}{\partial(\hat{T}_k^{(b)})} \left( \text{tr}\left\{\sum_{b=1}^B \hat{T}_k^{(b)\text{H}}\hat{T}_k^{(b)}\right\} - P \right) \quad (10)$$

For the first term in Eq. 10, researchers take derivative of each term in Eq. 7. Researchers use the following property of matrix derivative:  $\partial \text{tr}(AX)/\partial(X) = A^H$  and  $\partial \text{tr}(AXBX^H)/\partial(X) = BX^HA$ :

$$\frac{\partial \langle 1 \rangle}{\partial(\hat{T}_k^{(b)})} = \beta^{-2} \left( N_t - \frac{N_t D}{N_r} \right) \hat{H}_k^{(b)\text{H}} \hat{H}_k^{(b)} \hat{T}_k^{(b)} \quad (11)$$

$$\frac{\partial \langle 2 \rangle}{\partial(\hat{T}_k^{(b)})} = \beta^{-2} \frac{N_t D}{N_t - N_r} \hat{T}_k^{(b)} \quad (12)$$

$$\frac{\partial \langle 3 \rangle}{\partial(\hat{T}_k^{(b)})} = \beta^{-2} \frac{N_t N_r D}{N_r (N_t - N_r)} \hat{H}_k^{(b)\text{H}} \hat{H}_k^{(b)} \hat{T}_k^{(b)} \quad (13)$$

$$\frac{\partial \langle 4 \rangle}{\partial(\hat{T}_k^{(b)})} = \frac{\partial}{\partial(\hat{T}_k^{(b)})} \beta^{-2} \mathbb{E} \left\{ \text{tr} \left( \sum_{b1=1}^B \left( \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_k^{(b1)} \hat{T}_k^{(b)\text{H}} A_k^{(b)\text{H}} \hat{H}_k^{(b)\text{H}} \right) \right) \right\} = \beta^{-2} \sum_{b1=1}^B A_k^{(b)\text{H}} \hat{H}_k^{(b)\text{H}} \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_k^{(b1)} \quad (14)$$

According to Ravindran and Jindal (2008),  $\beta_{kk}^{(b1,b2)} = 1$ , so Eq. 14 can be rewritten as:

$$\frac{\partial \langle 4 \rangle}{\partial(\hat{T}_k^{(b)})} = \beta^{-2} \sum_{b1=1}^B \beta_{kk}^{(b,b1)} A_k^{(b)\text{H}} \hat{H}_k^{(b)\text{H}} \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_k^{(b1)} \quad (15)$$

$$\begin{aligned} \frac{\partial \langle 5 \rangle}{\partial(\hat{T}_k^{(b)})} &= \frac{\partial}{\partial(\hat{T}_k^{(b)})} \beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j=1}^K \sum_{b1=1}^B \beta_{jk}^{(b1,b2)} \left( \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_j^{(b1)} \hat{T}_j^{(b)\text{H}} A_k^{(b)\text{H}} \hat{H}_k^{(b)\text{H}} \right) \right) \\ &= \frac{\partial}{\partial(\hat{T}_k^{(b)})} \beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j=1}^K \sum_{b1=1}^B \beta_{kj}^{(b,b2)} \left( \hat{H}_j^{(b1)} A_j^{(b1)} \hat{T}_k^{(b1)} \hat{T}_k^{(b)\text{H}} A_j^{(b)\text{H}} \hat{H}_j^{(b)\text{H}} \right) \right) \\ &= \beta^{-2} \sum_{j=1}^K \sum_{b1=1}^B \beta_{kj}^{(b,b1)} \left( A_k^{(b)\text{H}} \hat{H}_k^{(b)\text{H}} \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_k^{(b1)} \right) \end{aligned} \quad (16)$$

Then, researchers combine  $\frac{\partial \langle 4 \rangle}{\partial(\hat{T}_k^{(b)})}$  and  $\frac{\partial \langle 5 \rangle}{\partial(\hat{T}_k^{(b)})}$ , the following equation can be obtained:

$$\frac{\partial \langle 4 \rangle}{\partial (\hat{T}_k^{(b)})} + \frac{\partial \langle 5 \rangle}{\partial (\hat{T}_k^{(b)})} = \beta^{-2} \sum_{j=1}^K \sum_{b1=1}^B \beta_{kj}^{(b, b1)} \left( A_k^{(b)H} \hat{H}_k^{(b)H} \hat{H}_k^{(b1)} A_k^{(b1)} \hat{T}_k^{(b1)} \right) \quad (17)$$

The  $\langle 6 \rangle$  in Eq. 7 can be calculated as:

$$\frac{\partial \langle 6 \rangle}{\partial (\hat{T}_k^{(b)})} = -\psi \beta^{-1} \hat{H}_k^{(b)H} \quad (18)$$

Researchers have finished the derivation of the first term in Eq. 10, the second term in Eq. 10 is calculated as follows:

$$\frac{\partial}{\partial (\hat{T}_k^{(b)})} \lambda_k \left( \text{tr} \left\{ \sum_{b=1}^B \hat{T}_k^{(b)H} \hat{T}_k^{(b)} \right\} - P \right) = \lambda_k \hat{T}_k^{(b)} \quad (19)$$

Finally, substituting Eq. 11, 12, 13, 17 and 18 into Eq. 10, researchers obtain the value of:

$$\frac{\partial}{\partial (\hat{T}_k^{(b)})} f(\hat{T}_k^{\text{opt}})$$

in Eq. 10. Setting:

$$\frac{\partial}{\partial (\hat{T}_k^{(b)})} f(\hat{T}_k^{\text{opt}})$$

to zero, the optimal MMSE precoding  $\hat{T}_k$  under asynchronous interference and limited feedback can be expressed as:

$$\hat{T}_k^{(b)} = \psi \beta \left( \theta_1 \hat{H}_k^{(b)H} \hat{H}_k^{(b)} + \frac{N_t D}{N_t - N_r} I_{N_t} + \lambda_k \beta^2 I_{N_t} + \theta_2 \right)^{-1} \hat{H}_k^{(b)H} \quad (20)$$

Where:

$$\theta_1 = \left( N_t - \frac{N_t D}{N_r} - \frac{N_t N_r D}{N_r (N_t - N_r)} \right),$$

$$\theta_2 = \left[ C_k^{(b,1)} \quad C_k^{(b,2)} \quad \dots \quad C_k^{(b,B)} \right]$$

the sub-matrices  $C_k^{(b1,b2)}$  are given by:

$$C_k^{(b, b2)} = \beta^{-2} \sum_{j=1}^K \beta_{kj}^{(b, b2)} \left( A_j^{(b)H} \hat{H}_j^{(b)H} \hat{H}_j^{(b2)} A_j^{(b2)} \right) \quad (21)$$

By further considering the power constraint in problem Eq. 8 then researchers take  $\partial f / \partial \beta = 0$  and combine with Eq. 20, the optimal  $\beta$  can be obtained as:

$$\beta = \frac{\eta}{\psi \theta_1} \quad (22)$$

where  $\eta$  is the transmit power normalization factor given by:

$$\eta = \sqrt{\frac{P}{\left\| \left( \hat{H}_k^{(b)H} \hat{H}_k^{(b)} + \theta_1 \frac{N_t D}{(N_t - N_r)} I_{N_t} + \theta_1 \frac{N_r}{P} I_{N_t} + \theta_1 \theta_2 \right)^{-1} \hat{H}_k^{(b)H} \right\|_F^2}} \quad (23)$$

So, the optimal MMSE with asynchronous interference and limited feedback can be obtained as:

$$\hat{T}_k^{(b)\text{opt}} = \eta \left( \hat{H}_k^{(b)H} \hat{H}_k^{(b)} + \theta_1 \frac{N_t D}{(N_t - N_r)} I_{N_t} + \theta_1 \frac{N_r}{P} I_{N_t} + \theta_1 \theta_2 \right)^{-1} \hat{H}_k^{(b)H} \quad (24)$$

### NUMERICAL RESULTS

In this study, simulation results are employed to show the proposed MMSE scheme with asynchronous interference and limited feedback on the downlink transmission of coordinated communications. Each terminal is equipped with  $N_r = 2$  receive antennas and BS have antenna  $N_t = 2$ . The symbol period  $T_s$  is 1  $\mu$ sec. There are  $K = 2$  users in the simulations. The number of cooperative BS is  $B = 2$ , the modulation scheme used is BPSK.

Figure 1 presents the sum rate of the proposed scheme and MMSE neglected asynchronous interference versus the average SNR for  $Q = 8$  and 16 bits. Researchers mainly consider two schemes, one is traditional MMSE ignoring asynchronous interference and another is the proposed scheme that MMSE mitigates asynchronous interference. Researchers plot the sum rate of proposed MMSE scheme and traditional ignoring asynchronous interference for compare. It is shown that the proposed MMSE scheme has higher sum rate than traditional MMSE ignoring asynchronous interference with feedback bit  $Q = 8$  and  $Q = 16$ . This is because that if the asynchronous interference is ignored in practical system (but it must be exist), its performance must be worse compare with proposed MMSE which has mitigated asynchronous interference. Researchers also observe that the sum rate increase with increasing number of feedback bits  $Q$ . Because the more feedback bits is the more precise the channel quantization is. Of course, the performance of perfect CSI must has higher sum rate compare with proposed scheme, since it is an ideal condition.

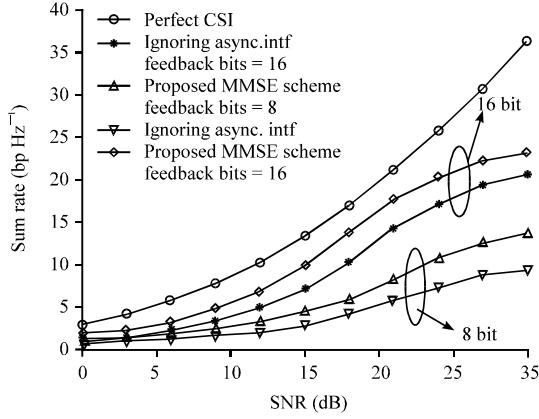


Fig. 1: Sum rate considering asynchronous interference and limited feedback

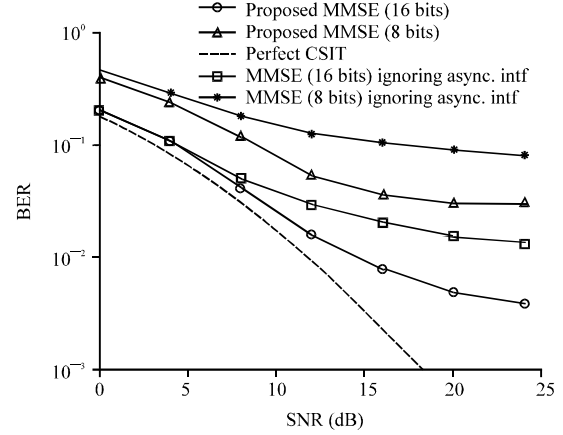


Fig. 2: Uncoded BER considering asynchronous interference and limited feedback

In Fig. 2, it presents average uncoded the Bit Error Rate (BER) as function of SNR with asynchronous interference and  $Q = 8$  and 16 bits. It can be observed that proposed outperforms the traditional MMSE ignoring asynchronous interference. When asynchronous interference is account for or neglected, the BER is decreased with increase the number of feedback bits.

### CONCLUSION

In this study, a novel MMSE precoding scheme for cooperative base stations has been introduced. It has been shown that the new MMSE scheme outperforms its predecessors both in terms of sum rate as well as BER. Moreover, the new scheme proved resilient to asynchronous interference.

### APPENDIX

**Appendix A:** Researchers can easily get that  $\mathbb{E}\{|s_k|^2\} = N$ , and  $\beta^{-2}\mathbb{E}\{|p_k|^2\} = \beta^{-2}N$ . For the third term in Eq. 6, researchers substitute the Channel Quantization Model Eq. 3 into 6:

$$\begin{aligned} \beta^{-2}\mathbb{E}\left\{\left\|\sum_{b=1}^B \hat{H}_k^{(b)} \hat{T}_k^{(b)} s_k\right\|_F^2\right\} &= \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1=1}^B \sum_{b_2=1}^B \hat{H}_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)} \hat{H}_k^{(b_2)H}\right)\right\} \\ &= \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1=1}^B \sum_{b_2=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} + B_k^{(b_1)} S_k^{(b_1)}) \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)} (\hat{H}_k^{(b_2)} A_k^{(b_2)} + B_k^{(b_2)} S_k^{(b_2)})^H\right)\right\} \\ &\stackrel{(a)}{=} \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1=1}^B \sum_{b_2=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H} + B_k^{(b_1)} S_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)H} S_k^{(b_2)H} B_k^{(b_2)H}\right)\right\} \end{aligned} \quad (25)$$

(a) follows that is independent of A and B and  $\mathbb{E}[s] = 0$ . There are two cases in Eq. 25, so it can be further calculated as:

$$\begin{aligned} \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1=1}^B \sum_{b_2=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} + B_k^{(b_1)} S_k^{(b_1)}) \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)} (\hat{H}_k^{(b_2)} A_k^{(b_2)} + B_k^{(b_2)} S_k^{(b_2)})^H\right)\right\} &= \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2}^B \sum_{b_1=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} + B_k^{(b_1)} S_k^{(b_1)}) \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)} (\hat{H}_k^{(b_2)} A_k^{(b_2)} + B_k^{(b_2)} S_k^{(b_2)})^H\right)\right\} + \\ &\quad \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2}^B \sum_{b_1=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} + B_k^{(b_1)} S_k^{(b_1)}) \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)} (\hat{H}_k^{(b_2)} A_k^{(b_2)} + B_k^{(b_2)} S_k^{(b_2)})^H\right)\right\} \\ &= \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b=1}^B (\hat{H}_k^{(b)} A_k^{(b)} \hat{T}_k^{(b)} s_k s_k^H \hat{T}_k^{(b)H} A_k^{(b)H} \hat{H}_k^{(b)H} + B_k^{(b)} S_k^{(b)} \hat{T}_k^{(b)} s_k s_k^H \hat{T}_k^{(b)H} S_k^{(b)H} B_k^{(b)H}\right)\right\} + \\ &\quad \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2}^B \sum_{b_1=1}^B (\hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_k^{(b_1)} \hat{T}_k^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H}\right)\right\} \end{aligned} \quad (26)$$

The first term in Eq. 26, using,  $\text{tr}(A+B) = \text{tr}(A)+\text{tr}(B)$ , <7> can be rewritten as:

$$\begin{aligned} \langle 7 \rangle &= \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2}^B \sum_{b_1=1}^B \hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H}\right)\right\} + \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2}^B \sum_{b_1=1}^B B_k^{(b_1)} S_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)H} S_k^{(b_2)H} B_k^{(b_2)H}\right)\right\} \\ &\stackrel{(b)}{=} \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2=1}^B R_k (I_{N_r} - Z_k^H Z_k)^{1/2} R_k^H \hat{H}_k^{(b_1)} \hat{T}_k^{(b_1)} \hat{T}_k^{(b_2)H} \hat{H}_k^{(b_2)H}\right)\right\} + \beta^{-2}\mathbb{E}\left\{\text{tr}\left(\sum_{b_1 \neq b_2=1}^B B_k^{(b_1)} S_k^{(b_1)} \hat{T}_k^{(b_1)} s_k s_k^H \hat{T}_k^{(b_2)H} S_k^{(b_2)H} B_k^{(b_2)H}\right)\right\} \end{aligned} \quad (27)$$

(b) follows that  $A_k = (I_{N_t} - Z_k^H Z_k)^{1/2} R_k$ , it further can be expressed as:

$$\begin{aligned} \langle 7 \rangle &= \beta^{-2} (N_t - \frac{N_t D}{N_t}) \text{tr} \left( \sum_{b_1=b_2=1}^B \hat{H}_k^{(b)} \hat{T}_k^{(b)} \hat{T}_k^{(b)H} \hat{H}_k^{(b)H} \right) + \beta^{-2} \mathbb{E} \left\{ \text{tr} \left( \sum_{b_1=b_2=1}^B B_k^{(b)} S_k^{(b)} \hat{T}_k^{(b)} S_k^{(b)H} \hat{T}_k^{(b)H} S_k^{(b)} B_k^{(b)H} \right) \right\} \\ &= \beta^{-2} (N_t - \frac{N_t D}{N_t}) \text{tr} (\hat{H}_k \hat{T}_k \hat{T}_k^H \hat{H}_k^H) + \beta^{-2} \frac{N_t D}{N_t} \text{tr} (\hat{T}_k^H \hat{T}_k) - \beta^{-2} \frac{N_t N_t D}{N_t (N_t - N_t)} \text{tr} \left( \sum_{b_1=b_2=1}^B \hat{T}_k^{(b)H} \hat{H}_k^{(b)} \hat{H}_k^{(b)H} \hat{T}_k^{(b)} \right) \end{aligned} \quad (28)$$

$\langle 7 \rangle$  follows  $\mathbb{E}\{RR^H\} = N_t I_{N_t}$ , and  $\mathbb{E}\{Z_k Z_k^H\} = \frac{D}{N_t} I_{N_t}$ . The fourth term in Eq. 6 can be calculated by:

$$\begin{aligned} \beta^{-2} \mathbb{E} \left\{ \|J_k\|_{\mathbb{F}}^2 \right\} &= \beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j=k}^K \sum_{b_1=1}^B \sum_{b_2=1}^B \beta_{jk}^{(b_1, b_2)} H_k^{(b_1)} \hat{T}_j^{(b_1)} \hat{T}_j^{(b_2)H} H_k^{(b_2)H} \right) \\ &= \beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j=k}^K \sum_{b_1=1}^B \sum_{b_2=1}^B \beta_{jk}^{(b_1, b_2)} \left( \hat{H}_k^{(b_1)} A_k^{(b_1)} + B_k^{(b_1)} S_k^{(b_1)} \right) \hat{T}_j^{(b_1)} \hat{T}_j^{(b_2)H} \left( \hat{H}_k^{(b_2)} A_k^{(b_2)} + B_k^{(b_2)} S_k^{(b_2)} \right)^H \right) \\ &= \beta^{-2} \mathbb{E} \text{tr} \left( \sum_{j=k}^K \sum_{b_1=1}^B \sum_{b_2=1}^B \beta_{jk}^{(b_1, b_2)} \left( \hat{H}_k^{(b_1)} A_k^{(b_1)} \hat{T}_j^{(b_1)} \hat{T}_j^{(b_2)H} A_k^{(b_2)H} \hat{H}_k^{(b_2)H} \right) \right) \end{aligned} \quad (29)$$

The fifth and sixth in Eq. 6 can be further calculated by:

$$\begin{aligned} -\beta^{-1} \mathbb{E} \left\{ \text{tr} \left( \sum_{b=1}^B \hat{H}_k^{(b)} \hat{T}_k^{(b)} \right) \right\} - \beta^{-1} \mathbb{E} \left\{ \text{tr} \left( \sum_{b=1}^B \hat{T}_k^{(b)H} H_k^{(b)} \right) \right\} &= -\beta^{-1} \left( \mathbb{E} \left\{ \text{tr} \left( \sum_{b=1}^B \left( \hat{H}_k^{(b)} A_k^{(b)} + B_k^{(b)} S_k^{(b)} \right) \hat{T}_k^{(b)} \right) \right\} + \mathbb{E} \text{tr} \left( \sum_{b=1}^B \hat{T}_k^{(b)H} \left( \hat{H}_k^{(b)} A_k^{(b)} + B_k^{(b)} S_k^{(b)} \right)^H \right) \right) \\ &\stackrel{(d)}{=} -\beta^{-1} \left( \mathbb{E} \left\{ \text{tr} \left( \sum_{b=1}^B \left( \hat{H}_k^{(b)} A_k^{(b)} \hat{T}_k^{(b)} \right) \right) \right\} + \mathbb{E} \text{tr} \left( \sum_{b=1}^B \hat{T}_k^{(b)H} A_k^{(b)H} \hat{H}_k^{(b)H} \right) \right) \\ &\stackrel{(e)}{=} -2\eta\beta^{-1} \text{Re} \left[ \text{tr} \left( \sum_{b=1}^B \left( \hat{H}_k^{(b)} \hat{T}_k^{(b)} \right) \right) \right] \end{aligned} \quad (30)$$

(c) follows that  $H_k = \hat{H}_k A_k + S_k B_k$  and (d) comes from S is independent of A and B and  $\mathbb{E}\{S\} = 0$ . (e) holds because  $\mathbb{E}\{A\} = \mathbb{E}\{(I_{N_t} - Z_k^H Z_k) R_k\} = \psi I_{N_t}$ .

### ACKNOWLEDGEMENT

This research is supported by Universiti Malaysia Pahang (UMP) Vot. RDU 130387. Therefore, researchers would like to thank UMP for the continuous support.

### REFERENCES

Bhagavatula, R. and R.W. Heath, 2011. Adaptive bit partitioning for multicell intercell interference nulling with delayed limited feedback. *IEEE Trans. Signal Process.*, 59: 3824-3836.

Catreux, S., P.F. Driessen and L.J. Greenstein, 2001. Attainable throughput of an interference-limited Multiple-Input Multiple-Output (MIMO) cellular system. *IEEE Trans. Commun.*, 49: 1307-1311.

Costa, M.H.M., 1983. Writing on dirty paper (Corresp.). *IEEE Trans. Inform. Theory*, 29: 439-441.

Foschini, G.J. and M.J. Gans, 1998. On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Commun.*, 6: 311-335.

Huaiyu, D., A.F. Molisch and H.V. Poor, 2004. Downlink capacity of interference-limited MIMO Systems with joint detection. *IEEE Trans. Wireless Commun.*, 3: 442-453.

Jindal, N., 2006. MIMO broadcast channels with finite-rate feedback. *IEEE Trans. Inform. Theory*, 52: 5045-5060.

Li, J., I.T. Lu and E. Lu, 2009. Optimum MMSE transceiver designs for the downlink of multicell MIMO systems. *Proceedings of the Military Communications Conference, October 18-21, 2009, Boston, MA., USA.*, pp: 1-7.

Ng, B.L., J.S. Evans, S.V. Hanly and D. Aktas, 2008. Distributed downlink beamforming with cooperative base stations. *IEEE Trans. Inform. Theory*, 54: 5491-5499.

Pan, Z., K.K. Wong and T.S. Ng, 2004. Generalized multiuser orthogonal space-division multiplexing. *IEEE Trans. Wireless Commun.*, 3: 1969-1973.

Ravindran, N. and N. Jindal, 2008. Limited feedback-based block diagonalization for the MIMO broadcast channel. *IEEE J. Selected Areas Commun.*, 26: 1473-1482.

Shi, J., T. Zhang, Y. Zhou, Z. Zeng and Z. Huang, 2011. Rate loss caused by limited feedback and channel delay in coordinated multi-point system. *Proceedings of the IEEE 73rd Vehicular Technology Conference, May 15-18, 2011, Tokyo, Japan*, pp: 1-6.

Spencer, Q.H., A.L. Swindlehurst and M. Haardt, 2004. Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels. *IEEE Trans. Signal Process.*, 52: 461-471.

Zhang, H., N.B. Mehta, A.F. Molisch, J. Zhang and H. Dai, 2008. Asynchronous interference mitigation in cooperative base station systems. *IEEE Trans. Wireless Commun.*, 7: 155-165.