

## Modified Max-Log-MAP Turbo Decoding Algorithm for MIMO-OFDM System

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**Abstract:** This study proposes a new Scaling Factor (SF) approach for reducing the over estimation of reliability values for Max-Log-MAP (MLMAP) turbo decoding algorithm. Modified Max-Log-MAP (M-MLMAP) algorithm is smeared by fixing an indiscriminate SF for inner decoder  $S_1$  and an optimized, adaptive SF for the outer decoder  $S_2$ . The performance of various scaling factors is compared and optimized scaling factor is obtained which is influenced by  $E_b/N_0$  with least Bit Error Rate (BER). Suitable mathematical relationship between scaling factor and  $E_b/N_0$  is also given. The effect of the suggested algorithm for a range of system parameters is considered in a systematic fashion, in order to gauge their performance implications. The numerical results show that the proposed M-MLMAP algorithm improves the performance of turbo decoding over Additive White Gaussian Noise (AWGN) and rayleigh fading channels. The proposed M-MLMAP is superior to MLMAP algorithm in both performance and complexity. The performance of the proposed algorithm is also analysed in Multiple Input Multiple Output-Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system for suitable applications in WIMAX and LTE.

**Key words:** Log-MAP, Max-Log-MAP, scaling factor, turbo codes, OFDM

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### INTRODUCTION

Major development in the channel coding area was presented by Berrou *et al.* (1993) by the advent of Turbo codes. Turbo codes have revealed the best Forward Error Correction (FEC) performance known up to now. Turbo codes are essential in the sense that they allow reliable data transmission within a half decibel of the Shannon's Limit. A huge amount of research effort has been performed to expedite the energy efficiency of turbo codes. As a result, turbo codes have been integrated into many standards used by the NASA Consultative Committee for Space Data Systems (CCSDS), Digital Video Broadcasting (DVB), both Third Generation Partnership which requires throughputs from  $2 \text{ Mbsec}^{-1}$  to several  $100 \text{ Mbsec}^{-1}$  in Project (3GPP) standards for IMT-2000, Wideband CDMA, 4G and WIMAX (Wang *et al.*, 2009).

When the turbo decoding standard was first presented by Berrou *et al.* (1993), the maximum a posteriori probability (MAP) algorithm was used in the decoding. Using the MAP algorithm is ideal with regard to the BER performance. However, it is difficult to implement. In practice, a modified version of this algorithm, called the Log-MAP algorithm is implemented in the logarithmic domain in order to avoid numerical computation problems. The Max-Log-MAP algorithm, as will be shown later is resulted from the Log-MAP

algorithm with an approximation. Another well-known practical algorithm for turbo decoding is the Soft Output Viterbi Algorithm (SOVA) which has a reasonable complexity but sustains some performance loss.

**Literature review:** In recent years, some research works have been carried out to enhance the performance of these practical turbo decoding algorithms, especially the one based on SOVA because they have wide spread applications (Siegel *et al.*, 2001). As a result, various modifying techniques have been presented and developed. In particular, Scaling Factor (SF) methods that improve the SOVA based turbo decoder have gained a lot of consideration because of their ease and efficiency. It was observed that the standard SOVA overrates the reliability values. To deal with this problem, an SF-based normalisation of the extrinsic information was first presented in (Papke *et al.*, 1996) where the soft output of SOVA is assumed to follow a Gaussian distribution. In (Blazek and Bhargava, 1998), a SF was used on the extrinsic information which rises linearly with the number of decoding iterations. However, the optimal SF was proved to be a constant from a different perspective in (Colavolpe *et al.*, 2001). In (Stirling, 2000), The method studied by Blazek and Bhargava (1998) and the Gaussian assumption from Papke *et al.* (1996) were applied to one decoder output whereas the other decoder output was normalised by a constant factor (Stirling-Gaacher, 2000).

The optimized SF for one decoder output is determined by trial and error method whereas the other decoder output was normalized by a constant factor. Moreover in (Gnanasekaran *et al.*, 2012), SF concept is applied to Log-MAP and SOVA decoding algorithms.

In the SF was quantised to different levels and another normalization method was proposed which is based on the pseudo median filtering technique (Wang and Parhi, 2003). This SF method leads to a very powerful SOVA turbo decoder. It was also applied to improve the performance of the turbo decoder employing a bi-directional SOVA (BSOVA). The BSOVA considered by Chang and Lain (2005) was originally introduced by Chen *et al.* (2000) and is especially, suitable for decoding turbo codes built from convolutional codes. Different from the BSOVA proposed by Chen *et al.* (2000) there is another BSOVA that was proposed by Li and Vucetic (1995) and is more suitable for turbo codes built from block codes (i.e., Block Turbo Codes (BTC)) (Hagenauer *et al.*, 1996; Pyndiah, 1998). Another approach was reported by Ghayeb and Huang (2005, 2006) which is based on a similar method by Papke *et al.* (1996) where two attenuators are used with either mathematical expressions or fixed values. The use of these attenuators decrease the correlation between the intrinsic and extrinsic information and thus provide a considerable performance enhancement. Based on the revised two-step SF approach for the SOVA decoder's extrinsic information, it was shown by Papaharalabos *et al.* (2006) that the error floor can be suggestively reduced. Moreover, the SF approach has also been considered and functional for the MLMAP algorithm (Vogt and Finger, 2000; Douillard and Berrou, 2005). Nevertheless, there exists a drawback in all of the above-stated SF techniques. The theoretical analysis was largely based on the postulation that the extrinsic information is Gaussian distributed. In general, the extrinsic information is not strictly Gaussian, especially when the data frame is comparatively short (Ghayeb and Huang, 2005).

Proposed in this study is a unique SF approach based on optimisation which can be applied for a general turbo decoder's extrinsic information. The use of SFs here aims not only at reducing the overestimation of reliability values but also proposes mathematical relationship between SF and  $E_b/N_0$ . The results show that the turbo decoding algorithm that employs the proposed SFs can achieve a better performance than the one without the SFs. The performance of the proposed algorithm is also analysed in Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) system for suitable applications in WIMAX and LTE.

**Turbo decoding algorithms:** Turbo decoder uses any one of the decoding algorithm, MAP or SOVA (Wood and

Hanzo, 2000) because, it yields error correction close to Shannon's limit (Shannon, 1948). In a classic Turbo decoding system, two decoders function iteratively and pass their decisions to each other after each iteration. These decoders yield soft-outputs to increase the decoding performance. Such a decoder is called a SISO decoder. Each decoder operates not only on its own input but also on the other decoder's partly decoded output which is similar to the operation principle of turbo engines. This correlation between the operation of the turbo decoder and the turbo engine gives this coding method its name, "Turbo codes".

Encoded information sequence  $X_k$  is transmitted over an Additive White Gaussian Noise (AWGN) channel and a noisy received sequence  $Y_k$  is obtained. Each decoder computes the Log Likelihood Ratio (LLR) for the k-th data bit  $d_k$ , as:

$$L(d_k) = \log \left[ \frac{P(d_k = 1|Y)}{P(d_k = 0|Y)} \right] \quad (1)$$

LLR can be decomposed into 3 independent terms, as:

$$L(d_k) = L_{apn}(d_k) + L_c(d_k) + L_e(d_k) \quad (2)$$

Where:

- $L_{apn}(d_k)$  = The a-priori information of  $d_k$
- $L_c(d_k)$  = The channel measurement
- $L_e(d_k)$  = The extrinsic information

Extrinsic information of one decoder becomes the a-priori information for the other decoder at the next decoding level. LLRs can be calculated by two diverse SISO algorithms (Pyndiah, 1998) SOVA and MAP Algorithm. In this study, Max-Log-MAP algorithm is considered.

**The Max-Log-MAP algorithm:** The correction function  $f_c = \log(1 + e^{-|x|})$  in the  $\max \times (\cdot)$  operation can be implemented in different ways. The Max-Log-MAP algorithm (Vogt and Finger, 2000) simply neglects the correction term and approximates the  $\max \times (\cdot)$  operator as:

$$\ln(e^x + e^y) \approx \max(x, y) \quad (3)$$

at the cost of some performance deprivation. Because of this approximation Max-Log-MAP algorithm provides sub-optimal performance.

## MATERIALS AND METHODS

**SF approach for the proposed M-MLMAP decoding algorithm:** Max-Log-MAP algorithm undergoes two

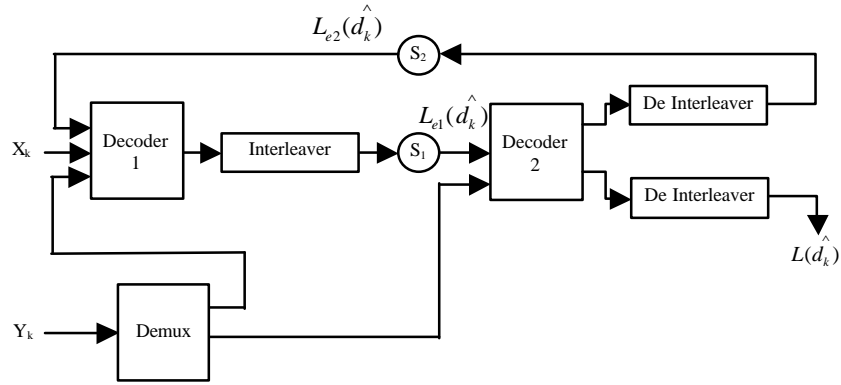


Fig. 1: Proposed turbo decoder with scaling factors

distortions (Chaikalis and Noras, 2002); over estimation of reliability values and correlation between the intrinsic and extrinsic information (Colavolpe *et al.*, 2001). The performance is degraded significantly due to first of these distortions and marginally due to the sec. The first type of distortion which depends on  $E_b/N_0$ , is considered. The reparation co-efficient is calculated. The reparation of  $L_c(d_k)$  is possible with a scaling factor. Turbo decoder with SF is shown in Fig.1. For sub optimal MLMAP algorithm LLR can be calculated by:

$$L(\hat{d}_k) = L_c(d_k) + L_{\text{apri}}(\hat{d}_k) + L_e(\hat{d}_k) \quad (4)$$

Since, extrinsic information from one decoder becomes a-priori information for the second decoder:

$$L_{\text{apri}}(\hat{d}_k) = L_{e2}(\hat{d}_k) \quad (5)$$

$$L_e(\hat{d}_k) = L_{e1}(\hat{d}_k) \quad (6)$$

Where  $L_{e1}(\hat{d}_k)$  and  $L_{e2}(\hat{d}_k)$  are the extrinsic information corresponding to decoder 1 and decoder 2 respectively. Here,  $L(d_k)$  and  $L(\hat{d}_k)$  are the LLRs of the optimal MAP and sub optimal MLMAP algorithms respectively. In general, the estimation  $L(\hat{d}_k)$  is less reliable than  $L(d_k)$  because  $L_{e1}(\hat{d}_k)$  is less reliable than  $L_{e2}(\hat{d}_k)$  where  $j = 1, 2$ . In order to improve the reliability value  $L(\hat{d}_k)$ , we propose to use two SFs  $S_1$  and  $S_2$  on the extrinsic information  $L_{e1}(\hat{d}_k)$  and  $L_{e2}(\hat{d}_k)$  respectively as follows:

$$L(\hat{d}_k, S_1, S_2) = L_c(d_k) + S_1 L_{e1}(\hat{d}_k) + S_2 L_{e2}(\hat{d}_k) \quad (7)$$

The two SFs  $S_1$  and  $S_2$  shall be derived based on the Minimum Mean-Square Error (MMSE) criterion (Papoulis, 1991) as follows. For a fixed channel SNR defined as  $E_b/N_0$  where  $E_b$  is the energy per information bit and  $N_0/2$  is

two-sided power spectral density of AWGN, both the optimal MAP algorithm and the modified practical algorithm with SFs produce two LLR estimations of the information bit at each iteration. Statistically their Mean-Square Difference (MSD):

$$\Phi_j(S_j) = E\{(S_j L_{e_j}(\hat{d}_k) - L_{e_j}(d_k))^2\}, j=1,2 \quad (8)$$

describes the effectiveness of the modified decoding algorithm. Specifically the smaller the MSD value is the better the suboptimal decoding becomes. Therefore, the parameter  $S_j$  should be found to minimize the MSD  $\Phi_j(S_j)$ . So,  $d \Phi_j(S_j) / d(S_j) = 0$  :

$$S_j E(L_{e_j}(\hat{d}_k))^2 = E(L_{e_j}(d_k))^2 \quad (9)$$

$$S_j = \frac{E(L_{e_j}(d_k))^2}{E(L_{e_j}(\hat{d}_k))^2} \quad (10)$$

It should be noted that because of over-optimistic effect of MLMAP,  $E(L_{e_j}(d_k))^2 \leq E(L_{e_j}(\hat{d}_k))^2$ , it follows from the Cauchy-Schwarz inequality (Roussas, 1997) that  $S_j \leq 1$ . The two SF values are computed from equation (10). But, it is observed from computer simulations shown in Table1 that SF  $S_2$  depends on  $E_b/N_0$ . The reason is attributed to the fact as follows: LLR for the MLMAP with SF is given by:

$$L(\hat{d}_k, S_1, S_2) = L_c(d_k) + S_1 L_{e1}(\hat{d}_k) + S_2 L_{e2}(\hat{d}_k) \quad (11)$$

Where the channel reliability value  $L_c(d_k)$  depends on the channel model and SNR:

$$L_c(d_k) \cong b_n \left( \frac{E_b}{N_0} \right) \quad (12) \quad \text{Where:}$$

$$X = \frac{L(\hat{d}_k, S_1, S_2) - S_1 [L_{e1}(\hat{d}_k)]}{L_{e2}(\hat{d}_k)}$$

where  $b_n$  is the fading amplitude for Rayleigh fading channel. For AWGN channel  $b_n=1$ . So:

$$L(\hat{d}_k, S_1, S_2) \cong b_n \left( \frac{E_b}{N_0} \right) + S_2^* [L_{e2}(\hat{d}_k)] + S_1 [L_{e1}(\hat{d}_k)] \quad (13)$$

$$S_2^* \cong \frac{L(\hat{d}_k, S_1, S_2) - b_n \left( \frac{E_b}{N_0} \right) - S_1 [L_{e1}(\hat{d}_k)]}{L_{e2}(\hat{d}_k)} \quad (14)$$

where \* is optimised SF. Since,  $S_1$  is constant,

$$S_2^* \cong x - b_n \left( \frac{E_b}{N_0} \right) \quad (15)$$

It is evident from Eq. 15 that there exists a dependency between SF  $S_2$  and  $E_b/N_0$ . M-MLMAP algorithm is further refined using fixed value for inner decoder ( $S_1$ ) calculated from Eq. 10 and an optimized, adaptive value for the outer decoder ( $S_2$ ) obtained from Eq. 15 which gives lowest BER.

**Methodology to obtain scaling factor for modified MLMAP algorithm:** Methodology for obtaining optimized SF in the proposed algorithm is shown in Fig. 2

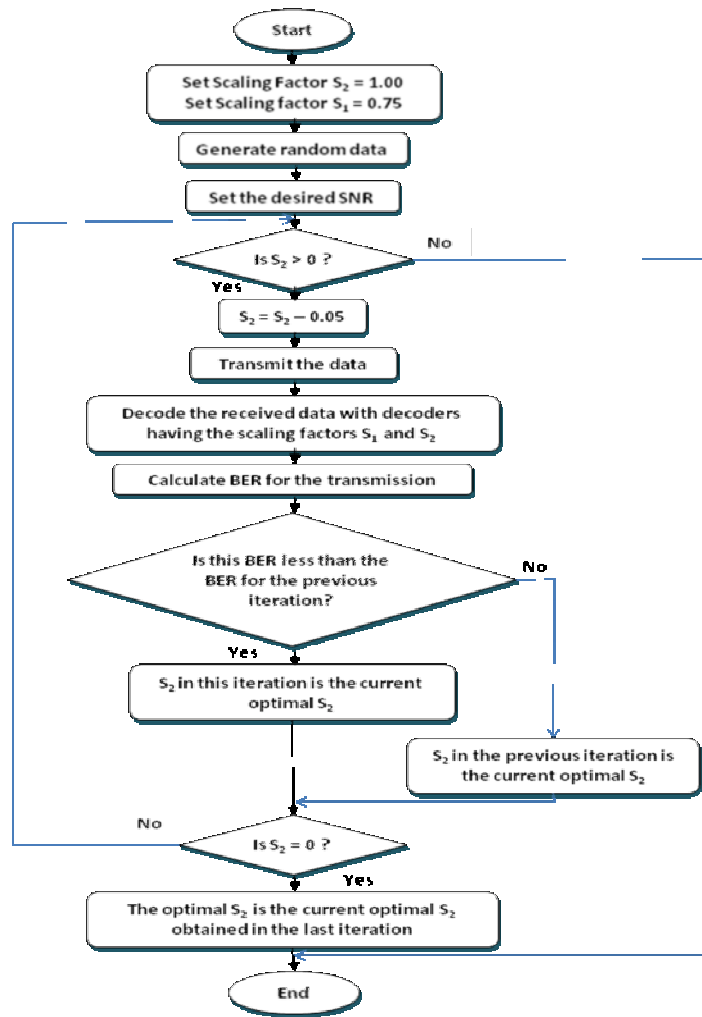


Fig. 2: Flowchart for calculating optimized SF in M-MLMAP Algorithm x

**FEC codes for MIMO-OFDM:** RS-CC is the obligatory channel coding scheme used for MIMO-OFDM (Bahai *et al.*, 2004; Wang *et al.*, 2009). Other possible codes are LDPC and Turbo codes.

**Reed Solomon-Convolutional Code (RS-CC):** In the Mobile WIMAX OFDMA part, the RS-CC is the only obligatory coding scheme (Liu *et al.*, 2009). RS error correction is a coding scheme which works by first building a polynomial from the data symbols to be transmitted and then directing an oversampled version of the polynomial instead of the original symbols themselves. After the RS encoding process, data bits are further encoded by a binary CC which has an intuitive rate of  $\frac{1}{2}$  and a constraint length of 7. The generator polynomials used to develop its two output code bits, denoted X and Y, are stated as:  $G_1 = 171$  (octal) for X and  $G_2 = 133$  (octal) for Y. Its computations rest not only on the present set of input symbols but on some of the past input symbols. A trellis depiction is used for convolution encoding which gives relation how each possible input to the encoder impacts the output in shift register. It uses the Viterbi algorithm for decoding (Blazek and Bhargava, 1998).

**Low Density Parity Check (LDPC) codes:** Low-density parity-check (LDPC) codes are a part of linear block codes (Yang *et al.*, 2004). An  $(n, j, k)$  LDPC code is indicated by a parity check matrix H, having  $n-k$  rows,  $n$  columns and  $j$  1's per column. In this paper  $j = 3$  i.e., all the parity check matrices will have 3 ones per column. Rate  $\frac{1}{2}$  encoder is employed. For decoding simplified log domain belief propagation decoder using sum-product algorithm is used.

**Turbo codes:** Turbo codes are suggested for improved capacity at higher transmission rates due to their higher performance over conventional convolutional codes. The entire Turbo coding scheme consists of recursive systematic encoders, interleavers, puncturing and decoder. In this study, for MIMO-OFDM applications  $(7,5)$ , rate  $\frac{1}{2}$  Turbo code is used. Two decoding algorithms Max-Log-MAP and Modified Max-Log-MAP algorithm are considered.

## RESULTS AND DISCUSSION

**Simulation profile and results for M-MLMAP decoding algorithm:** The suggested scaling factors for the turbo coded system are simulated in AWGN channel.

Transmission of 500 frames with a constant frame length of 2048 bits and random interleaver (Barbulescu and Pietrobon, 1994) is considered to show the influence of the scaling factors on the performance of error correction. Simulation results have been gathered with diverse combinations of scaling factors at different  $E_b/N_0$  to view the minimum BER at the decoder side. The simulation parameters are:

- Channel: AWGN
- Modulation: Quadrature Phase shift Keying (QPSK)
- Component Encoder: Two identical Recursive Convolutional codes (RSCs)
- Rate: 1/2 (punctured)
- Interleaver: 2048 bit random interleaver
- Iteration: 8
- Frame limit: 500

Scaling factors considered range from 0.05 to 0.95 for M-MLMAP algorithm and is shown in Fig. 3. A varied range of scaling factors, the  $E_b/N_0$  and the corresponding BER has been showed. The scaling factor having the lowest BER for a particular  $E_b/N_0$  is considered to be optimized SF. Table 1 shows the optimized scaling factor ( $S_2$ ), having the lowest BER against  $E_b/N_0$ . It is observed from Table 1 that  $S_2$  is found to vary with  $E_b/N_0$  and hence, it is not only optimal but also adaptive with respect to  $E_b/N_0$ .

Unlike, we have used adaptive SF ( $S_2$ ), rather than fixed SF (Vogt and Finger, 2000). The supremacy of our proposed SF approach over the method by Vogt and Finger (2000) is illustrated in Fig. 4 and 5 for AWGN and Rayleigh fading channels respectively. The BER simulation results between the constant SFs in (Vogt and Finger, 2000) and proposed adaptive SF  $S_2$  are compared for the turbo code constructed from the  $(7, 5)$  code. The performance advantage by using the proposed approach is very clear.

Figure 6 displays the performance of Modified Max-Log-MAP algorithm with the scaling factors  $S_2 = 0.85$  (optimal) and  $S_1 = 0.75$  (arbitrary) is giving improved results comparing with the Max-Log-MAP algorithm without scaling factor. The MLMAP and M-MLMAP algorithms are also compared with the customary Log-MAP decoding algorithm, at  $E_b/N_0$  of 2dB in AWGN channel. This graph substantiates the improved performance of M-MLMAP algorithm in terms of BER. It is noted from the Fig. 6 that Log-MAP which is an optimal algorithm gives better enactment than sub-optimal Max-Log-MAP algorithm. But, with the introduction

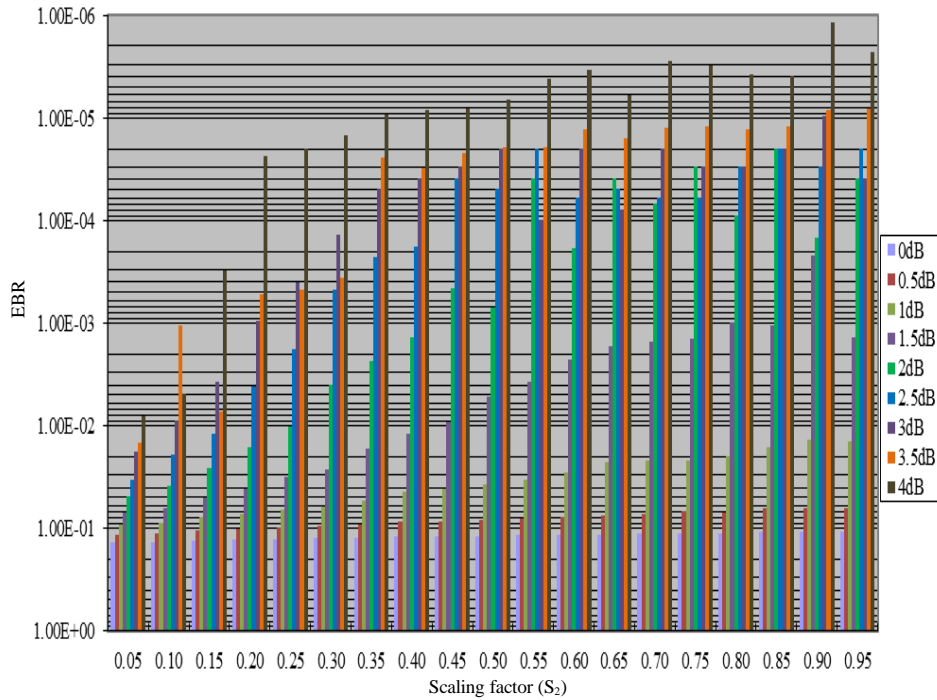


Fig. 3: BER plot of various Scaling Factors and  $E_b/N_0$  with code generator (7,5), punctured for Max-Log-MAP algorithm

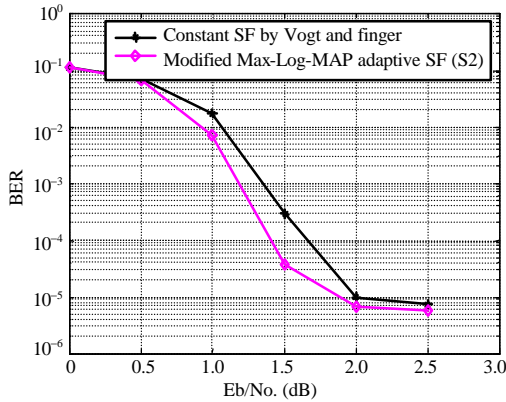


Fig. 4: BER comparison between proposed adaptive scaling factor  $S_2$  and the scaling factor in (Vogt and Finger, 2000) for AWGN channel with 8 iterations

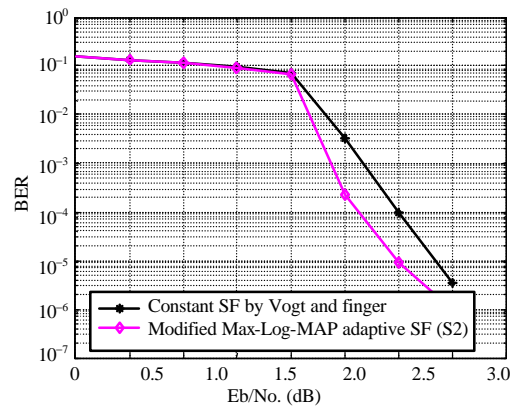


Fig. 5: BER comparison between proposed adaptive scaling factor  $S_2$  and the scaling factor for Rayleigh fading channel with 8 iterations (Vogt and Finger, 2000)

Table 1: Optimized scaling factor ( $S_2$ ) and BER for varying  $E_b/N_0$

$E_b/N_0$ (dB)	Optimized scaling factor ( $S_2$ )	Corresponding BER
0	0.95	$1.0720 \times 10^{-1}$
0.5	0.90	$6.2454 \times 10^{-2}$
1	0.90	$1.3669 \times 10^{-2}$
1.5	0.90	$2.1744 \times 10^{-4}$
2	0.85	$1.9767 \times 10^{-5}$
2.5	0.85	$1.9767 \times 10^{-5}$
3	0.90	$9.6767 \times 10^{-6}$
3.5	0.95	$8.2034 \times 10^{-6}$
4	0.90	$1.1836 \times 10^{-6}$

of appropriate scaling factor, the performance of Max-Log-MAP algorithm is improved and is found that the proposed M-MLMAP algorithm gave optimal performance with BER of  $5 \times 10^{-6}$  for iteration 5. The BER of Log-MAP and MLMAP algorithms are  $1 \times 10^{-5}$  and  $2 \times 10^{-5}$  respectively for iteration 5.

It is also detected from Fig. 6 that the BER performance of M-MLMAP algorithm remains constant

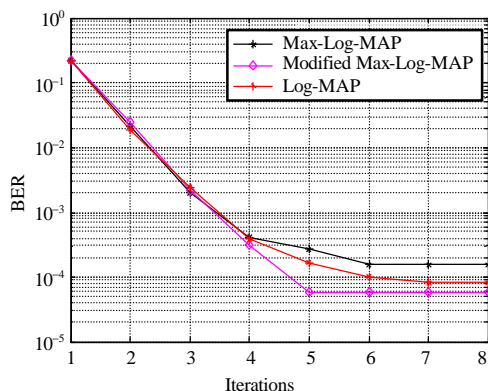


Fig. 6: BER plot of Log-MAP, Max-Log-MAP and Modified Max-Log-MAP decoding algorithms for different iterations. Code generator (7,5), punctured, frame length=2048, frame limit=500, for 2dB in AWGN channel

Table 2: Number of Iterations Required For each Decoding Algorithm

Decoding algorithms	Iteration from which BER is constant	Complexity reduced in (%)	Corresponding BER
Log-MAP	7	12.5	$8.1460 \times 10^{-6}$
MLMAP	6	25	$1.5740 \times 10^{-5}$
M-MLMAP	5	37.5	$5.8887 \times 10^{-6}$

from iteration 5. It is revealed for M-MLMAP algorithm, the efficient BER has been achieved by 5 iterations. So, instead of using 8 iterations, least BER can be obtained in just 5 iterations for M-MLMAP algorithm. Thus, in the proposed M-MLMAP algorithm, complexity has been reduced by 37.5% compared to standard MLMAP algorithm and the BER has been reduced by the order of  $10^{-1}$  compared to MLMAP algorithm. The main design benchmark for any decoding algorithm is to reduce the BER and complexity which is achieved by the proposed M-MLMAP algorithm.

Table 2 presents the summary of the number of iterations, BER and the percentage of reduction in complexity for each decoding algorithms. Compared to Log-MAP algorithm, the complexity of Max-Log-MAP algorithm is reduced but at the cost of BER. But, the proposed M-MLMAP algorithm gives improved performance with least complexity. Analyses have been done to show the performance of decoding algorithms in AWGN and Fading channels with QPSK modulation. Figure 7 shows the performance of Log-MAP, Max-Log-MAP and Modified Max-Log-MAP in AWGN channel. At  $E_b/N_0$  of 1.5dB and above, M-MLMAP algorithm is better than MLMAP with BER of  $5 \times 10^{-6}$  at  $E_b/N_0$  of 2.5dB. The proposed M-MLMAP algorithm attains performance nearer to optimal Log-MAP algorithm. The M-MLMAP gives better performance than MLMAP with a gain of 0.3dB at BER of  $3 \times 10^{-5}$  on the curve.

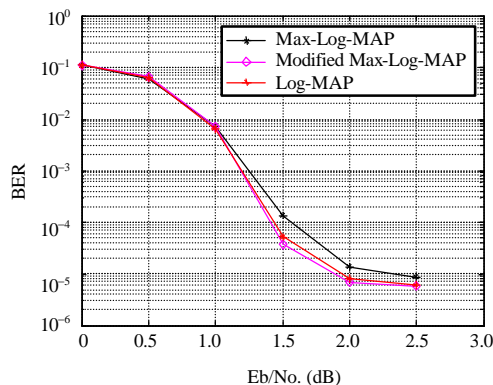


Fig. 7: Performance comparison of log-mAP, max-log-MAP and modified Max-Log-MAP in AWGN channel. code generator (7,5), punctured, frame length = 2048, frame limit = 500.

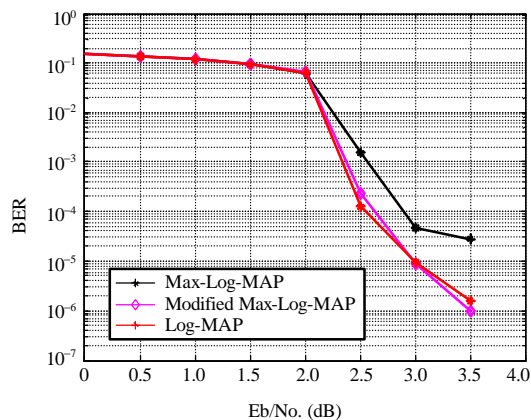


Fig. 8: Performance comparison of Log-MAP, Max-Log-MAP and Modified Max-Log-MAP in Rayleigh Fading channel. Code generator (7,5), punctured, frame length = 2048, frame limit = 500.

Similar analysis is being done for the rayleigh fading channel and is shown in Fig. 8. The performance in fading channel is almost similar to the AWGN channel with M-MLMAP and Log-MAP algorithms giving almost identical performances. On comparing MLMAP and proposed M-MLMAP algorithms, later showed a gain of 0.75dB at BER of  $2 \times 10^{-5}$  on the curve which validates the robustness of the proposed algorithm.

The following has been observed from above graphs: Log-MAP algorithm is optimal in enactment but complex; MLMAP algorithm is simple but gives non-optimal performance; the proposed M-MLMAP is both simple and optimal. Hence, M-MLMAP algorithm is best suited for practical applications. Figure 9 is a plot between scaling factor ( $S_2$ ) and  $E_b/N_0$  for M-MLMAP algorithm. It

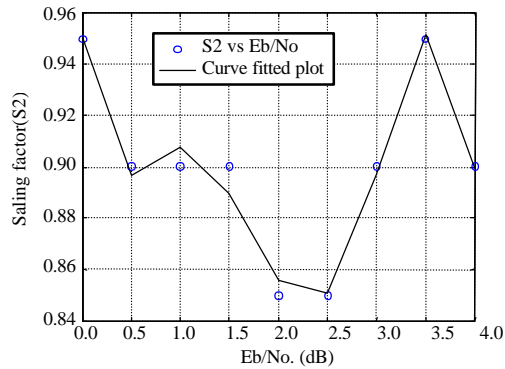


Fig. 9: Curve fitted plot between  $E_b/N_0$  and optimum scaling factor for M-MLMAP

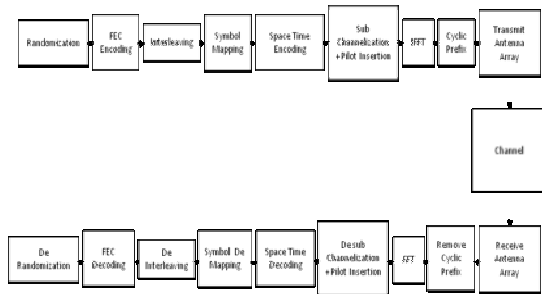


Fig. 10: MIMO-OFDM system

shows the variation of magnitude of scaling factor  $S_2$  with respect to the  $E_b/N_0$  where scaling factor  $S_1$  is kept constant. It can be inferred from the plot that there is a relational dependence between scaling factor  $S_2$  and  $E_b/N_0$ .

The variation between these two parameters is considered to make the Turbo decoder as adaptive. The adaptive decoder, by itself, will set the scaling factor ( $S_2$ ) of the decoder corresponding to the received  $E_b/N_0$ . To make it adaptive, we have obtained an expression by the curve fitting method. Eq. 16 shows a polynomial expression of sixth degree with seven coefficients for proposed M-MLMAP algorithm. The expression is given by:

$$f(x) = 0.0018x^6 - 0.0336x^5 + 0.2111x^4 - 0.5685x^3 + 0.6723x^2 - 0.3255x + 0.9505 \quad (16)$$

Where,  $f(x) = S_2$  and  $x = E_b/N_0$  Vogt and Finger (2000) have reported 0.2-0.4dB gain over the standard Max-Log-MAP algorithm for 3GPP standards. They used a constant scaling factor of 0.7. But in our study, proposed algorithm using adaptive SF, shown in Fig. 9 achieved a gain of 0.3dB and 0.75dB in aWGN and rayleigh fading channels, respectively.

**MIMO-OFDM system simulation model, simulation profile and simulation results:**

The simulation for MIMO-OFDM is done using MATLAB. Each block is independently coded. Simulation model is shown in Fig. 10. It consists of the following:

**Source generator:** The information bits that will be transmitted are produced using MATLAB function “randint”.

**Modulation:** Quadrature Phase Shift Keying (QPSK) modulation is used in the simulation.

**FEC:** Reed Solomon-Convolutional Code (RS-CC), Turbo and Low Density Parity Check (LDPC) codes are used. In Turbo codes, Log MAP, Max-Log-MAP and Modified Max-Log-MAP decoding algorithms are considered for analysis.

**Interleaving:** Serial data after FEC block is passed through an interleaver block which avoids burst errors.

**S/P:** Converts serial data into parallel data and vice versa.

**IFFT:** An inverse Fourier transform converts the frequency domain data set into samples of the corresponding time domain representing OFDM subcarrier. Specifically IFFT is useful for OFDM because it generates samples of a waveform with frequency component satisfying orthogonality condition.

**Cyclic prefix addition:** In this block, numbers of bits falling in  $T_g$  time are added in opening of an OFDM symbol to avoid Inter Symbol Interference (ISI) and Inter Carrier Interference (ICI) (Bahai *et al.*, 2004).

**Transmit and Receive Antenna Array:** Here 2x2 Alamouti Space Time Block Code (STBC) is employed (Alamouti, 1998). The simulation parameters for MIMO-OFDM system is given by: No. of transmit antennas: 2; No. of receive antennas: 2

**Primitive parameters:**

- Carrier frequency: 2.3GHz
- Channel Bandwidth (BW): 5MHz
- FFT size ( $N_{FFT}$ ): 512
- Cyclic Prefix (CP): 1/8
- Oversampling rate (n): 28/25
- Channel: Rayleigh Fading



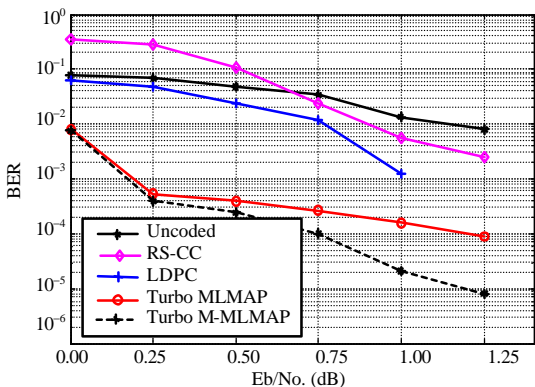


Fig. 11: Performance of M-MLMAP Turbo Coded MIMO-OFDM system with 2x2 STBC in Rayleigh fading channel

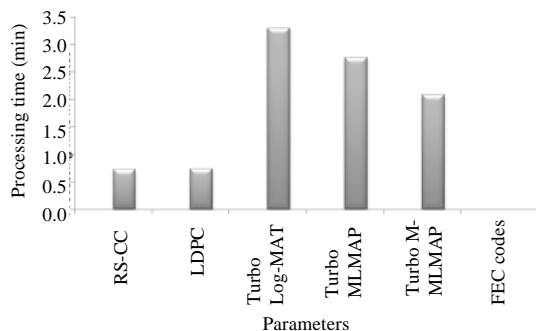


Fig. 12: Processing time of FEC codes in MIMO-OFDM system.

**Derived parameters:**

- Sampling frequency ( $F_s = n \times BW$ ): 5.6MHz
- Subcarrier spacing ( $\Delta f = F_s / N_{FFT}$ ): 10.94KHz
- Useful symbol time ( $T_b = 1 / \Delta f$ ): 91.4 $\mu$ s
- Cyclic Prefix time ( $T_g = CP \times T_b$ ): 11.4 $\mu$ s
- OFDM symbol duration ( $T_s = T_b + T_g$ ): 102.8 $\mu$ s

Figure 11 depicts the evaluation of different FEC codes for MIMO-OFDM system in Rayleigh fading channel. It is observed from Fig.11 that RS-CC code shows poor result for lower  $E_b/N_0$  values performing lesser to uncoded BER. But, for higher values of  $E_b/N_0$ , RS-CC gives 0.25dB performance enhancement than uncoded. LDPC gives further improvement of 0.2dB than RS-CC at BER of  $8 \times 10^{-3}$  over the curve. On comparing LDPC with Turbo, it is found that later shows improvement in performance. On considering the decoding algorithms for Turbo code, the MLMAP algorithm gives superior performance than LDPC for MIMO-OFDM applications.

Hence, the MLMAP decoding algorithm is further modified by the introduction of optimised, adaptive scaling factor to give modified MLMAP (M-MLMAP) and its performance is shown in Fig. 11. It is found that the performance of MLMAP and M-MLMAP are almost identical up to  $E_b/N_0$  of 0.25dB. M-MLMAP do better than MLMAP for higher values of  $E_b/N_0$  with a coding gain of 0.5dB at BER of  $9 \times 10^{-5}$  over the curve. Further the proposed M-MLMAP algorithm achieves the lowest BER of  $8 \times 10^{-6}$  at  $E_b/N_0$  of 1.25dB for MIMO-OFDM solicitations in rayleigh fading channel.

Figure 12 shows the processing time taken for each FEC codes when implemented in MIMO-OFDM system. Though Turbo code gives superior performance than LDPC and RS-CC, its processing time and hence time complexity is very high when used with Log-MAP decoding algorithm. But, the processing time taken is significantly reduced for Turbo codes with proposed M-MLMAP decoding algorithm and hence the time complexity is also reduced.

**CONCLUSION**

Thus, on optimizing the scaling factor  $S_2$  in Max-Log-MAP algorithm, enhancement in performance is attained. The proposed Modified Max-Log-MAP algorithm not only reduces the BER but also the complexity which is the key design criterion for Turbo codes. In AWGN channel, the recommended M-MLMAP algorithm achieves performance nearer to optimal Log-MAP and better performance than MLMAP with a gain of 0.3dB at BER of  $3 \times 10^{-5}$ . The performance in fading channel is almost undistinguishable to that in AWGN channel with M-MLMAP presenting a gain of 0.75dB at BER of  $2 \times 10^{-5}$  which proves the robustness of the proposed algorithm. There exists a reliance between scaling factor  $S_2$  and  $E_b/N_0$ . The analytical expression offers simplification of the selection of the best scaling factor for the received  $E_b/N_0$ . Finally it is found that Turbo code is found to be the appropriate FEC for MIMO-OFDM system than RS-CC and LDPC. On analyzing the decoding algorithms for Turbo codes, M-MLMAP is found to give better results than MLMAP. M-MLMAP algorithm is highly strong, less complex and gives improved BER performance for MIMO-OFDM system.

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