

## Analysis and Synthesis of Algorithms of Solving Inverse Problems by Methods of Classical and Modern Automatic Control Theory

Danil Shaykhutdinov, Dmitriy Shurygin, Grayr Aleksanyan, Irina Grushko, Roman Leukhin, Igor Stetsenko, Roman Oganyan, Viacheslav Bondarenko and Vadim Leukhin  
Department of Information and Measurement Systems and Technologies,  
Platov South Russia State Polytechnic University (NPI), Novocherkassk, Russia

**Abstract:** This study presents the results of a new approach to the analysis of the process of finding the characteristics of materials and synthesis of algorithms for solving inverse problems based on the methods of automatic control theory. The method of full-scale model tests to solve the problem is used. It is shown that the existing techniques for inverse problems can be presented in the form of regulation elements of automatic control systems. The process of finding solutions of inverse problems can be present in the form of an automatic control system. Thus, it becomes possible to synthesize new algorithms for solving inverse problems based on the methods of automatic control theory, in particular control systems with adaptive parameters of regulation elements.

**Key words:** Algorithm, inverse problems, method of full-scale model tests, adaptive regulation elements, magnetic measurement

---

### INTRODUCTION

Inverse problems a class of problems in which it is necessary to set up an adequate process model or object using experimental results. These problems are widespread in the areas of science as geophysics (Solovyev *et al.*, 2014), medical imaging (Aleksanyan *et al.*, 2015a, b; Lankin *et al.*, 2015), computed tomography (Shaykhutdinov *et al.*, 2015a; Grayr *et al.*, 2014), tasks of non-destructive testing (Bulgakov *et al.*, 2015, Gorbatenko *et al.*, 2011, 2015; Shaikhutdinov *et al.*, 2015; Shaykhutdinov *et al.*, 2015b-h), tasks of diagnostic of complex technical systems (Dubrov *et al.*, 2015a,b; Lankin *et al.*, 2015), etc. Inverse problems are so-called “incorrect” problems. Problems will be correct if three conditions was met: the existence of solutions, uniqueness of the solution and its stability. “Correctness” of inverse problems is disturbed by any of these criteria, however, most often on the criterion of “sustainability” solutions.

In order to solve the inverse problem regularization methods is used. The iterative methods for solving inverse problems is most common. An example is the method of full-scale-model experiment. In general view, the method of full-scale-model experiment is realized in the form of:

$$y^{(i)} = y^{(i-1)} + \Delta y^{(i)} \quad (1)$$

Where:

$y^{(i)}$  = The value of adjustable parameter of the model on iteration step #i

$\Delta y^{(i)}$  = Correction of the value of adjustable parameter of the model on iteration step #i

Moreover, to determine the value of a adjustable parameter at the iteration step  $i = 0$  the result of a physical experiment on the measurement of a parameter  $x^*$ , correlated with the desired parameter  $y$  is used:

$$y^{(0)} = f(x^*) \quad (2)$$

As the model, in this case is a description of the physical experiment on the measurement of a parameter  $x^*$ , taking into account the basic conditions of this experiment. The most frequently as for the construction of such a model the finite element method is used.

The correction values  $\Delta y^{(i)}$  is found as a function  $f'$  of the difference between the result of a physical experiment on the measurement of a parameter  $x^*$  and the result of a model experiment to determine the parameter  $x^{(i)}$  by substituting the values of the custom model parameter  $y^{(i)}$  in the model:

$$\Delta y^{(i)} = f'(x^* - x^{(i)}) \quad (3)$$

An example of the implementation of this approach is the algorithm of natural-model testing of electrical steel (Shaykhutdinov *et al.*, 2013). The algorithm for determining the magnetic characteristics of the material B(H), in particular magnetic field strength in the sample H, the following system of expression, corresponding to the expressions (1-3) is used:

$$\begin{cases} H^{(0)} = \frac{I^* w}{l}; \\ \Delta H^{(i)} = \frac{(I^* - I^{(i)}) w}{l}; \\ H^{(i)} = H^{(i-1)} + \Delta H^{(i)}, \end{cases} \quad (4)$$

Where:

- $I^*$  = The value of the current in the magnetizing coil of the magnetic system, measured during physical experiment
- $w$  = The number of turns of the magnetizing coil of the magnetic system
- $l$  = Equivalent length of the magnetic lines of the magnet system
- $I$  = Current in the magnetizing coil of the magnetic system, defined by using a model

We will write the third expression of the system Eq. 4 in the form of:

$$\begin{cases} H^{(0)} = \frac{I^* w}{l}; \\ H^{(1)} = \frac{(I^* - I^{(1)}) w}{l} + \frac{(I^* - I^{(1)}) w}{l}; \\ H^{(2)} = \frac{(I^* - I^{(1)}) w}{l} + \frac{(I^* - I^{(1)}) w}{l} + \frac{(I^* - I^{(2)}) w}{l}; \\ \dots \end{cases} \quad (5)$$

The system of Eq. 5 can be written as the integral equations:

$$H^{(n)} = \frac{w}{l} \sum_{i=0}^n \Delta I^{(i)} \Delta i \otimes \frac{w}{l} \int_0^n dI di$$

Where:

- $\Delta i$  = The difference between the serial number of iterations
- $\Delta I^{(i)} = I^* - I^{(i)}$
- $\Delta I^{(i)}$  = Difference between values of currents determined experimentally  $I^*$  and  $I^{(i)}$

### MATERIALS AND METHODS

On the basis of assumptions about the possibility of submitting of iterative process setting an adequate model in the process of solving the inverse problem by the full-scale-model experiment in the form of the integral equation, it can be concluded that the process can be presented in form of block diagram. This block diagram is a graphical representation of a system of automatic control of a adjustable parameter of the model y (Fig. 1).

Using the techniques of classical and modern automatic control theory it can be assumed that the algorithm of model setting can be implemented as a standard or adaptive adjusting element and can be implemented in the form:

- Proportional-integral-derivative adjusting element
- Adaptive adjusting element

For example, the algorithm of settings adequate model of the magnetic system (Shaykhutdinov *et al.*, 2013) based on the integral-differential adjusting element as the magnetic induction B, the second magnetic parameter of B(H) magnetic characteristic of tested material:

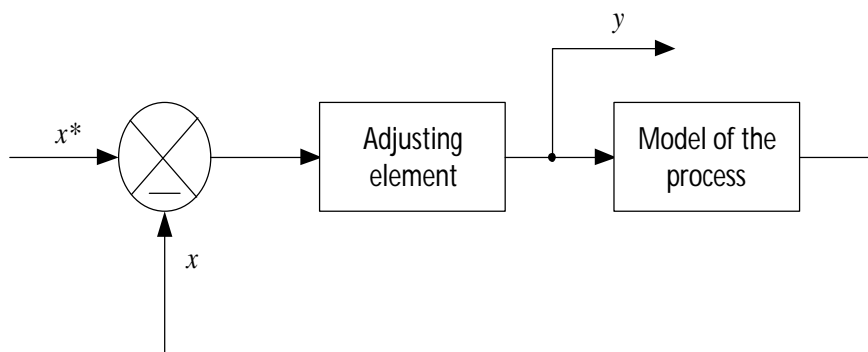


Fig. 1: The block diagram of algorithm for determining the BH curve using a single primary measuring converter based on the method of full-scale model tests

Table 1: The results of the numerical experiment to determine the effectiveness of the algorithms for solving inverse problems

$\delta_s$ (%)	Relative units (k)	Relative units (k')	t, s	$\delta$ (%)
10.0	1	0	132	6.7
	1	0.1	127	4.5
	1	0.2	126	2.3
5.0	1	0	169	3.4
	1	0.1	127	4.5
	1	0.2	126	2.3
1.0	1	0	294	0.8
	1	0.1	379	1.65 (n/r $\delta_s$ )
	1	0.2	1297	4.0 (n/r $\delta_s$ )

$$B^{(n)} = \frac{1}{S} \left( k \int_0^n d\Psi di + k' \times \frac{d\Psi}{di} \right) \rightarrow \frac{1}{S} \left( k \sum_{i=0}^n \Delta\Psi^{(i)} \Delta i + k' \frac{\Psi}{\Delta i} \right)$$

Where:

$\Delta\Psi$  = The difference in values of magnetic flux linkage determined experimentally  $\Psi^*$  and by the model  $\Psi$

$$\Delta\Psi^{(i)} = \Psi^* - \Psi^{(i)}$$

k = The k' some empirical coefficients

Table 1 presents the results of research of the proposed algorithm for different values of the empirical coefficients k, k'. The essence of the experiment was a solve the inverse problem of determining the magnetic characteristics of the sample of the electrical steel sheet based on the measured of flux-current characteristic of the magnetic system “magnetic core, test sample”. The algorithm of the solving of inverse problem has been implemented as a lua-script for FEMM4.2 program. Estimated experiment was carried out on a PC with the following technical specifications:

- CPU-Intel CORE i7-3630QM CPU@2.40 GHz
- RAM – DDR3, 8.00 GB

The following notation is used in Table 1, t-time implementation of the algorithm;  $\delta$  reached a value model error,  $\delta^{(i)} = \Psi^* - \Psi^{(i)} / \Psi^* \times 100$ ,  $\delta_s$  a predetermined value of the relative error in which the model is considered adequate.

### RESULTS

As seen from Table 1, the possibility of solving the inverse problem with the highest required accuracy (1%) is achieved only for  $k' = 0$  using the integral adjustable element. In solving the problem, with an accuracy of 5% or more adjustment algorithms, that include differential component provide a higher rate of convergence, the algorithm has decided to reverse the task in less time t.

### DISCUSSION

The results of the numerical experiment showed that the process of finding an adequate model of the object or process in the course of solving the inverse problem has a similar view, as used in automatic control theory: the use of the integral law when setting up the model provides the greatest convergence of the solution of the inverse problem; the introduction of the derivative component in the formula for calculating the next iteration increases the speed of solving the inverse problem, however, increases the “oscillation” of the solution process; analysis of the inverse problem solution is possible using the approaches of automatic control theory; synthesis of algorithms for solving the inverse problem is possible on the basis of the methods of automatic control theory.

### CONCLUSION

The approach to the analysis of the process of solving the inverse problem and synthesis algorithms of finding decisions and rules of changing of adjustable parameter is designed. The greatest prospects contained in the current approaches to the synthesis of adjusting elements, adaptive regulation rules for calculation of values of the empirical coefficients (Например, k, k') based on the results of solving of inverse problem in the first iteration.

### ACKNOWLEDGEMENTS

This study is prepared based on the results of the projects No. SP 4108.2015.1, implemented under the “Scholarship provided by the President of the Russian Federation for young scientists and graduate students engaged in advanced research and development in priority areas of modernization of the Russian economy” program using equipment center for collective use “Diagnosis and energy-efficient electrical equipment” (NPI).

### REFERENCES

- Aleksanyan, G.K., N.I. Gorbatenko, A.I. Kucher, K.M. Shirokov and C.N. Phong, 2015a. Developing principles and functioning algorithms of the hardware-software complex for electrical impedance tomography of biological objects. Biosci. Biotechnol. Res. Asia, 12: 709-718.

- Aleksanyan, G.K., N.I. Gorbatenko and A.I. Kucher, 2015b. Development and production of multi-layered electrode system for electrical impedance tomography devices. *Int. J. Appl. Eng. Res.*, 19: 40580-40584.
- Bulgakov, A., D. Shaykhutdinov, N. Gorbatenko and S. Akhmedov, 2015. Application of full-scale experiments for structural study of high-rise buildings. *Procedia Eng.*, 123: 94-100.
- Dubrov, V.I., D.V. Shaykhutdinov and K.M. Shirokov, 2015. Information-measuring system for the flow process hydraulic machines testing stand. *Procedia Eng.*, 129: 184-190.
- Dubrov, V.I., D.V. Shaykhutdinov, K.M. Shirokov, S.V. Akhmedov and N.I. Gorbatenko, 2015. Information-measurement system for stand for testing hydraulic products. *Proceedings of the 2015 International Siberian Conference on Control and Communications (SIBCON)*, May 21-23, 2015, IEEE, Omsk, Russia, ISBN: 978-1-4799-7102-2, pp: 1-3.
- Gorbatenko N.I., A.M. Lankin, M.V. Lankin, D.V. Shaykhutdinov, 2015. Determination of weber-ampere characteristic for electrical devices based on the solution of harmonic balance inverse problem. *Int. J. Appl. Eng. Res.*, 10: 6509-6519.
- Gorbatenko, N., M. Lankin, D. Shaykhutdinov, K. Gazarov and A. Kolomiets, 2011. Electromagnetic induction system for testing ferromagnetic shape memory alloys. *Proceedings of the 6th International Forum on Strategic Technology*, Volume 1, August 22-24, 2011, Harbin, Heilongjiang, China, pp: 194-196.
- Gorbatenko, N.I., V.V. Grechikhin and D.V. Shaikhutdinov, 2015. Measuring and actuating devices based on shape memory ferromagnets. *Met. Sci. Heat Treat.*, 56: 609-613.
- Gray, K.A., G. Nikolay and D.T. Alexander, 2014. Development of hardware-software complex for electrical impedance tomography of biological objects. *Res. J. Applied Sci.*, 9: 1030-1033.
- Lankin, A.M., M.V. Lankin, V.V. Grechikhin and D.V. Shaikhutdinov, 2015. Determination of magnetic characteristics of alternative current electrotechnical devices using the method of full-scale-model tests. *Res. J. Appl. Sci.*, 10: 690-695.
- Shaikhutdinov, D.V., N.I. Gorbatenko and K.M. Shirokov, 2015. Facility for measuring magnetic parameters of articles from sheet electrical steel on the basis of national instruments technologies. *Met. Sci. Heat Treat.*, 56: 618-620.
- Shaykhutdinov, D.V., N.I. Gorbatenko, S.V. Akhmedov and R.I. Leukhin, 2015a. Device for control of magnetic properties of electrical steel for industrial production management system. *International Siberian Conference on Control and Communications*, May 21-23, 2015, Omsk, Russia, pp: 1-2.
- Shaykhutdinov, D., N. Gorbatenko, V. Grechikhin, K. Shirokov and V. Dubrov, 2015b. Development of the computer-based stand for research of the voltage generation effect of the magnetic shape memory material. *Res. J. Appl. Sci.*, 10: 170-172.
- Shaykhutdinov, D., N. Gorbatenko, G. Aleksanyan, V. Grechikhin and K. Shirokov *et al.*, 2015c. Development of a computer-based stand for testing algorithms of electrical impedance tomography. *Res. J. Appl. Sci.*, 10: 173-175.
- Shaykhutdinov, D., A. Lankin, N. Narakidze, V. Grechikhin, K. Shirokov and N. Gorbatenko, 2015d. Complex predict fault diagnostics of electromagnetic actuators based on the principle component analyses. *Res. J. Appl. Sci.*, 10: 555-557.
- Shaykhutdinov, D.V., R.I. Leukhin and K.M. Shirokov, 2015e. Automated stand for intelligent actuators regulators setup parameters. *Procedia Eng.*, 129: 749-753.
- Shaykhutdinov, D., A. Lankin, S. Janvarev, N. Gorbatenko and D. Schuchkin, 2015f. Application of tensor methodologies for the description of non-linear processes in electromagnetic drive. *Res. J. Appl. Sci.*, 10: 798-800.
- Shaykhutdinov, D., A. Lankin, S. Janvarev, N. Narakidze and R. Leukhin *et al.*, 2015g. Development multiprocessing mathematical model of electromagnetic AC drive. *Res. J. Appl. Sci.*, 10: 812-814.
- Shaykhutdinov, D., N. Gorbatenko, R. Leukhin, 2015h. Development of a computer-based stand for research of the methods of sensorless control of electromagnets DC. *Res. J. Appl. Sci.*, 10: 845-847.
- Shaykhutdinov, D.V., N.I. Gorbatenko, V. Akhmedov, M.V. Shaykhutdinova and K.M. Shirokov, 2013. Experimental and simulation tests of magnetic characteristics of electrical sheet steel. *Life Sci. J.*, 10: 2698-2702.
- Solovyev, S. and S. Tordeux, 2014. Compute SVD of a very large matrix in the context of geological prospection. *Proceedings of the 6th EAGE Saint Petersburg International Conference and Exhibition on Geosciences-Investing in the Future*, April 7-10, 2014, EAGE, Saint Petersburg, Russia, ISBN: 9781632665393, pp: 1-5.