Asian Journal of Information Technology 15 (14): 2343-2346, 2016
ISSN: 1682-3915

Isomorphism and Anti-Isomorphism in Interval Valued of Q-Fuzzy Subhemirings of a Hemiring

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Abstract: In this study, we study some of the properties of interval valued Q-fuzzy subhemiring of a hemiring under is omorphism and anti-is omorphism. Also prove that the homomorphic image and anti-homomorphic image of an interval valued Q-fuzzy normal subhemiring is an interval valued Q-fuzzy subhemiring.

Key words: Interval valued fuzzy subset, interval valued Q-fuzzy subhemiring, interval valued Q-fuzzy normal subhemiring, homomorphic

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R, +, .). Some of them in particular, near rings and several kinds of semirings have been proven very useful. Semirings (called also half rings) are algebras (R ,+ ,.) share the same properties as a ring except that (R, +) is assumed to be a semi group rather than a commutative group. Semi rings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R,+ ,.) is said to be a semi ring (R, +) and (R ,.) are semi groups satisfying a.(b+c)=a.b+a.c and (b+c). a = b.a+c.a for all a,b and c in R. A semi ring R is said to be additively commutative if a+b = b+a for all a, b and c in R. A semi ring R may have an identity 1, defined by 1.a = a = a.1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0. a for all a in R. A semi ring R is said to be a hemi ring if it is an additively commutative with zero. Interval valued fuzzy sets were introduced independently by Zadeh (1975), Guinness (1976), Jahn, in the seventies, in the same year, an Interval Valued Fuzzy set (IVF) is defined by an interval-valued membership function. Jun and Kim (2002). defined an interval valued fuzzy R-subgroups of nearrings. Akram and Dar (2005) have introduced the notions of fuzzy algebras and d-ideals in d-algebras. The concept of anti fuzzy sub groups of groups, lower level subgroups was introduced by Sebastian (1994). Solairaju and Nagarajan (2009a, b). defined the characterization of interval valued Anti fuzzy Left h-ideals over hemirings. The properties of interval valued fuzzy Rw open maps,closed maps and Rw homeomorphism in an interval valued topological spaces were discussed in Indira *et al*. (2013) and Palaniappan (2007). Rosenfeld (1971) defined fuzzy groups. Kazanci *et al*. (2007) has introduced the Notion of intuitionistic Q-fuzzification of N-subgroups (subnear rings) in a near-ring and investigated some related properties. Solairaju and Nagarajan, have given a new structure in the construction of Q-fuzzy groups and subgroups (Solairaju and Nagarajan, 2009a ,b). In this study, we introduced the concept of interval valued Q-fuzzy subhemiring of a hemiring under isomorphism and antiisomorphism and established some results.

Preliminaries

Definition: Let X be any nonempty set. A mapping is called an interval valued fuzzy subset(briefly, IVFS) of X, where D[0,1] denoted the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$ for all x in X, where M^- and $M^+$and are fuzzy subsets of X such that, $M^- \leq M^+(x)$ for all x in X. Thus [M](x) is an interval (a closed subset of [0,1] and not number from the interval [0,1] as in the case of fuzzy subset. Note that [0] = [0,0] and [1] = [1,1].

Remark: Let D^X be the set of all interval valued fuzzy subsets of X, where D means D [0,1].

Definition: Let the $[M] = \{\langle x,[M^-(x),M^+(x)]\rangle / x \in X\}$,$[N] = \{\langle x,[N^-(x),N^+(x)]\rangle / x \in X\}$ be any two interval valued fuzzy subsets of x. We define the following relations and operations: $[M] \subseteq [N]$ if and only if $M^-(x) \leq N^-(x)$ and $M^+(x) \leq N^+(x)$, for all x in X. [M] = [N] if and only if $M^-(x) = N^-(x)$ and $M^+(x) = N^+(x)$ for all x in X.

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$[M] = \cap [N] = \{\langle x, [\min\{M^-(x), N^-(x)\}, \min\{M^+(x), N^+(x)\}]\rangle / x \in X\}$.

$[M] \cup = [N] = \{\langle x, [\max\{M^-(x), N^-(x)\}, \max\{M^+(x), N^+(x)\}]\rangle / x \in X\}$.

$[M]^c = [1]-[M] = \{\langle x, [1-M^+(x), 1-M^-(x)\rangle / x \in X\}$

Definition: Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is function

Definition: Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subset [M] of R is said to be an interval valued Q-fuzzy subhemiring (IVFSHR) of R if the following conditions are satisified:

- [M] $(x+y,q) \geq \min([M](x,q), [M](y,q)$
- [M] $(xy,q) \geq \min([M](x,q), [M](y,q))$, for all x and y in R and q in Q

Definition: Let (R, +, .) be a hemiring. A interval valued Q-fuzzy subhemiring [A] of R is said to be an interval valued Q-fuzzy normal subhemiring (IVFNSHR) of R if [A] (xy,q) = [A] (yx,q) for all x and y in R and q in Q

Definition: Let X and x' be any two sets. Let f: X→X' be any function and [A] be an interval valued Q-fuzzy subset in X, [V] be an interval valued Q-fuzzy subset in, defined by for all x in X and y in X' and q in Q. Then [A] is called a pre-image of [V] under and is denoted by $f^{-1}([V])$

Definition: Let (R, +, .) and (R', +,.) (be any two hemirings. Then the function f: R→R' is called a hemiring homomorphism if it satisfies the following axioms:

- f (x+y) = f(x)+f(y)
- f (x+y) = f(x)+f(y) for all x and y in R

Definition: Let (R, +,.) and (be any two hemirings. Then the function is called a hemiring anti-homomorphism if it satisfies the following axioms:

- f (x+y) = f (y)+f (x)
- f (xy) = f (x)+f (y) for all x and y in R

Definition: Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f: R→R' be a hemiring homomorphism. If f is one-to-one and onto, then f is called a hemiring isomorphism.

Definition: Let (R, +, .) and (R', +, .) be any two hemirings. Then the function f:→R' be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a hemiring anti-isomorphism.

PROPERTIES OF INTERVAL VALUED Q-FUZZY SUBHEMIRINGS

Theorem: Let [A] be an interval valued Q-fuzzy subhemiring of hemiring H and f is an isomorphism from a hemiring Ronto H. Then [A] f is an interval valued Q-fuzzy subhemiring of R.

Proof: Let x and y in rand [A] be an interval valued Q-fuzzy subhemiring of a hemiring H. Then we have, $([A]\circ f)(x+y,q) = [A](f(x+y),q)) = [A](f(x),q)+(f(y),q)) \geq \min\{([A](f(x),q), [A](f(y),q)\} = \min\{([A]\circ f(x,q), ([A]\circ f)(y,q)\}$. which implies that $([A]\circ f)(x+y,q) \geq \min\{([A]\circ f)(x,q), ([A\circ f])(y,q)$ and $([A]\circ f)(xy,q) = [A](f(xy),q) = [A](f(x),q)(f(y),q)) \geq \min\{([A](f(x),q), [A](f(y),q)\} = \min\{([A]\circ f)(x,q), ([A]\circ f)(x,q), ([A]\circ f)(y,q)\}$. which implies that $([A]\circ f)(xy,q) \geq \min\{([A]\circ f)(x,q)([A]\circ f)(x,q), ([A]\circ f)(y,q)\}$. Which implies that $([A]\circ f)(xy,q) \geq \min\{([A]\circ f)(y,q)\}$. Therefore $([A]\circ f)$ is an interval valued Q-fuzzy subhemiring of R.

Theorem: Let [A] be an interval valued Q-fuzzy subhemiring of hemiring H and f is an anti-isomorphism from a hemiring R onto H. Then $[A]\circ f$ is an interval valued Q-fuzzy subhemiring of R.

Proof: Let x and y in R. Then we have, $([A]\circ f)(x+y,q) = [A](f(x+y),q)) = [A](f(y,q)+f(x,q)) \geq \min\{([A](f(x),q)[A](f(y),q))\} = \min\{([A\circ f)(x,q), ([A]\circ f)(y,q)\}$. which implies that. $([A]\circ f)(x+y),q) \geq \min[([A]\circ f)(y,q)\}$. And, $([A]\circ f)(xy,q) = [A](f(yx),q)) = [A](f(y),q)f(x),q)) \geq \min\{([A](f(x),q), [A](f(y),q)\} = \min\{([A]\circ f)(x,q), ([A]\circ f)(y,q)\}$. which implies that $([A]\circ f)(xy,q) \geq$ Therefore $([A]\circ f)$ is an interval valued Q-fuzzy subhemiring of the hemiring R.

Theorem: Let (R, +,.) and (R',+, .) be any two hemirings. The homomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of R'

Proof: Let (R, +,.) and (R', +, .) be any two hemirings. Let f R→R' be a homomorphism. Let [A] is an interval valued Q-fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of (R) = R'

Now, for f(x), f(y) in R and q inQ. Clearly, [V] is an interval valued Q fuzzy subhemiring of f (R) = R'. Since [A] isan interval valued Q-fuzzy subhemiring of R. Now, $[V](f(x),q)(f(y),q)) = [V](f(xy),q) \geq [A](xy,q) = [A](yx,q) \leq [V]\{f(yx),q)\}$ = Hence [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring.

Theorem: Let (R, +,.) and (R', +,.) be any two hemirings. The homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of is an interval valued Q-fuzzy normal subhemiring of R.

Proof: Let (R, +,.) and (R', +,.) be any two hemirings. Let f: R→R' be a homomorphism. Let [V] is an interval valued Q-fuzzy normal subhemiring of f (R) = We have to prove that [A] is an interval valued Q-fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly, [A] is an interval valued Q-fuzzy subhemiring of the hemiring R. Now, [A] (xy,q) = [V] (f(xy),q) = [V] {(f(x),q) (f(y),q)} = [V](f(y),q) (f(x), q) = [V] (f(yx),q) = [A] (yx,q) which implies that [A] (xy,q) = [A] (yx, q), for all x and y in R and q in Q. Hence [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

Theorem: Let (R, +,.) and (R', +, .) be any two hemirings. The anti-homomorphic image of an interval valued Q-fuzzy normal subhemiring of R is an interval valued Q-fuzzy subhemiring of

Proof: Let (R, +,.) and (R', +, .) be any two hemirings. Let f: R→R' be a anti homomorphism. Let [A] is an interval valued Q-fuzzy normal subhemiring of R. We have to prove that [V] is an interval valued Q-fuzzy normal subhemiring of f(R) = R'. Now, for f(x), f(y) in R' and q in Q. Clearly, [V] is an iterval valued Q-fuzzy subhemiring of R'. Since [A] is an intervalued Q-fuzzy subhemiring of R. Now, [V] (f(x),q) (f(y),q) = [V] (f (yx), q)≥[A] (yx,q) = [A] (xy,q)≤[V] {f(xy),q)} = [V] (f(y),q) (f(x),q) which implies that [V] ((f(x),q) (f(y),q) = [V] ((f (y),q) (f(x), q. Hence [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring .R'

Theorem: Let (R, +,.) and (R', +,.) be any two hemirings. The anti homomorphic preimage of an interval valued Q-fuzzy normal subhemiring of is interval valued Q-fuzzy normal subhemiring of R'

Proof: Let (R, +,.) and (R', +,.) be any two hemirings. Let f: R→R' be anti homomorphism. Let [V] is an interval valued Q-fuzzy normal subhemiring of f (R) = R' We have to prove that [A] is an interval valued Q-fuzzy normal subhemiring of R. Let x and y in R and q in Q. Then clearly

[A] is an interval valued Q-fuzzy subhemiring of the hemiring R. since [V] is an interval valued Q-fuzzy normal subhemiring of the hemiring.R' Now, [A] (xy,q) = [V] {(f(xy),q) (f(x),q)} = [V] ((f(x),q) (f(y), q))= [V] (f(yx), yx,q) = [A] (yx,q) which implies that [A] (xy,q) = [A] (yx,q), for all x and y R and q in Q. Hence [A] is an interval valued Q-fuzzy normal subhemiring of the hemiring R.

Theorem: Let [A] be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an isomorphism from a hemiring R onto H. Then [A∘] f is an interval valued Q-fuzzy normal subhemiring of R.

Proof: Let x and y in R and [A] be an interval valued Q-fuzzy normal subhemiring of a hemiring H. Then clearly, ([A]∘ f) is an interval valued Q-fuzzy subhemiring of the hemiring R. Then we have ([A]∘ f) (xy,q) = [A] (f (xy),q)) = [A]((f(x),q) (f(y),q)) = [A] ((f(y),q) (f(x),q) = [A] (f(yx),q) = ([A]∘ f) (yx,q) which implies that ([A]∘ f) (xy,q) = ([A]∘f) (yx,q) therefor ([A]∘ f) (yx,q) Therefore ([A]∘ f) is an interval valued Q-fuzzy normal subhemiring of R.

Theorem: Let [A] be an interval valued Q-fuzzy normal subhemiring of hemiring H and f is an anti- isomorphism from a hemiring R onto H. Then [A]∘ f is an interval valued Q-fuzzy normal subhemiring of R.

Proof: Let x and y in R and [A] be an interval valued Q-fuzzy normal subhemiring of a hemiring H. Then clearly ([A]∘ f) is an interval valued Q-fuzzy subhemiring of the hemiring R. ([A]∘ f) ((f(x),q) (f(y), q)) = [A] (f(xy),q) = [A] (f(y),q)) (f(x),q) = [A] ((f(x),q) (f(y),q)) =[A] (f (yx),q) = ([A]∘ f) (yx,q) which implies that ([A]∘ f) (xy,q) = ([A]∘ f) (xy,q) = ([A]∘ f) (yx,q), for all x and y in R and q in Q. Hence ([A]∘ f) is an interval valued Q-fuzzy normal subhemiring of hemiring R.

CONCLUSION

In this study, some properties of interval valued Q-fuzzy subhemiring of a hemiring under isomorphism and anti-isomorphism were discussed. Some results of Interval Valued Q-Fuzzy Subhemirings of a hemirings were proved.

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