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A New Algorithmforsolving Fuzzy Transportation Problems with Triangularfuzzy Numbers

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Abstract: In real world problems, optimization techniques are useful for solving problems like, project schedules, assignment problems and network flow analysis. The main aspect of this study is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Here, we are proposing a new algorithm for solving fuzzy transportation problem, where fuzzy demand and supply all are in the form of triangular fuzzy numbers. Where fuzzy demand and supply all are in the form of triangular fuzzy numbers. So, the proposed approach is very easy to understand and to apply on real life transportation problems for the decision makers.

Key words: Project schedules, assignment problems, network flow analysis, triangular fuzzy numbers, fuzzy transportation problem, ranking technique

INTRODUCTION

The transportation problem is a special type of linear programming problem which deals with the distribution of single product (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. Effective algorithms has been used for solving the transportation problems when all the decision parameters, i.e., the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way. But in real life, there are many diverse situations due to uncertainty in one or more decision parameters and hence they may not be expressed in a precise way. This is due to measurement inaccuracy, lack of evidence, computational errors, high information cost, whether conditions etc. Hence we cannot apply the traditional classical methods to solve the transportation problems successfully. Therefore the use of Fuzzy transportation problems is more appropriate to model and solve the real world problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand are fuzzy quantities.

Bellman and Zadeh (1970) and Zadeh (1965) proposed the concept of decision making in Fuzzy environment. After this pioneering research, several authors such as Shiang-Tai Liu and Chiang Kao (Shiv *et al*., 2011; Kaur and Kumar, 2011; Chanas *et al*., 1984; Pandian and Natarajan, 2010a, b; Liu and Kao, 2004) proposed different methods for the solution of Fuzzy transportation problems. Chanas and Kuchta. (1996) proposed the concept of the optimal solution for the transportation with Fuzzy coefficient expressed as fuzzy numbers. Chanas, Kolodziejckzy, Machaj (Chanas *et al*., 1984) presented a fuzzy linear programming model for solving Transportation problem. Liu and Kao (2004) described a method to solve a fuzzy transportation problem based on extension principle. Lin introduced a genetic algorithm to solve Transportation with fuzzy objective functions.

Gani and Razak (2006) and Pandian and Natrajan (2010a, b) obtained a fuzzy solution for a two stage cost minimizing fuzzy transportation problem in which supplies and demands are triangular fuzzy numbers. A. NagoorGani, Edward Samuel and Anuradha Edward; Gani *et al*. (2011) used Arsham and Kahn (1989)'s algorithm to solve a fuzzy transportation problem. Pandian

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and Natarajan (2010a, b) proposed a Fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problem where all parameters are triangular fuzzy numbers.

In this study, a new algorithm is proposed for solving a specialtype of fuzzy transportation problems. In the proposed algorithm transportation cost represented as triangular fuzzy numbers. The method is to rank the fuzzy objective values of the objective function by some ranking method for numbers to find the best alternative. On the basis of this idea the ranking method with the help of α solution has been adopted a transform the fuzzy transportation problem. To illustrate the proposed algorithm a numerical example is solved and the obtained results are compared with the results of existing approaches. So the proposed approach is very easy to understand and to apply on real life transportation problems for the decision makers.

MATERIALS AND METHODS

Terminology: In this study some basic definitions of fuzzy set theory arereviewed.

Definition: The characteristic function $\mu_A(x)$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu\tilde{A}(x)$ such that the value as signed to the element of the universal set X fall within a specified range i.e., $\mu\tilde{A}(x)$ The assigned value indicates the membership grade of the element in the set A. The function $\mu\tilde{A}(x)$ is called the membership function and the set $\tilde{A} = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0,1]\}$ is called a fuzzy set.

Definition: A fuzzy set A, defined on the set of real numbers R is said to be a fuzzy number if its membership function μ_A:R→[0, 1] has the following characteristics
A is normal. It means that there exists an x_∈ R such that $\mu_A(x) = 1$ A is convex. It means that for every $x_1, x_2 \in R$ μ_A is upper semi-continuous.

$$\mu A(\lambda X_1 + (1-\lambda)x_2) \geq \min \{\mu_A(x_1), \mu_A(x_2)\}, \lambda \in [0,1]$$

supp (A) is bounded in R.

Definition: A fuzzy number A is said to be non-negative fuzzy number if and only $\mu\tilde{A}(x) = 0, \forall x < 0$

Definition: A fuzzy number A in R is said to be a triangular fuzzy number if its membership function μ_A: R→[0,1] has the following characteristics.

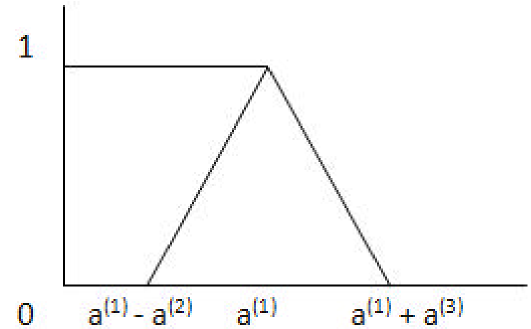

Fig. 1: Membership function of triangular fuzzy number

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

It is denoted by A = $(a^{(1)}), (a^{(2)}), (a^{(3)})$ where $(a^{(1)})$ is Core (A), $(a^{(2)})$ is left width and $(a^{(3)})$ is right width. The geometric representation of Triangular Fuzzy number is shown in Fig. 1. Since, the shape of the triangular fuzzy number A is usually in triangle it is called so. The Parametric form of a triangular fuzzy number is represented by A = $[(a^{(1)})-(a^{(2)})(1-r), (a^{(1)})+(a^{(3)})(1-r)]$

Ranking of triangular fuzzy number: Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. That is, for every A = $((a^{(1)}), (a^{(2)}), (a^{(3)}) \in F(R)$, the ranking function $\Re$: F(R) → R by graded mean is defined as:

$$\Re(A) = \left(\frac{a_1 + 4a_2 + a_3}{6}\right)$$
$$(\because a_2 = a_3)$$

For any two fuzzy triangular Fuzzy numbers A = $((a^{(1)}), (a^{(2)}), (a^{(3)}))$ and B = $(b^{(1)}, (b^{(2)}), (b^{(3)}))$ in F(R), we have the following comparison:

A<B If and only if $\Re$ (A)<$\Re$(B)
A>B If and only if $\Re$ (A)<$\Re$(B)
A≈B If and only if $\Re$ (A) = $\Re$(B)
A-B If and only if $\Re$ (A)-$\Re$(B) = 0

A-triangular fuzzy number A = $((a^{(1)}), (a^{(2)}), (a^{(3)})$ in F(R) is said to be positive if $\Re$(A)>0 and denoted by

A>0. Also if $\Re$ (A>0), then A>0 and if $\Re$(A) = 0, then A≈0. If $\Re$(B), then the triangular numbers A and B are said to be equivalent and is denoted by A≈B.

Mathematical formulation of a fuzzy transportation problem: Mathematically a transportation problem can be stated as follows:

Minimize

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Subject to

$$\left.\begin{aligned} &\sum_{j=1}^{n} x_{ij} = a_i \qquad j = 1, 2, \ldots, n \\ &\sum_{i=1}^{m} x_{ij} = b_j \qquad i = 1, 2, \ldots, m \\ &x_{ij} \geq 0 \qquad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \end{aligned}\right\} \tag{2}$$

where C_{ij} is the cost of transportation of a unit from the ith source to the jth destination and the quantity x_{ij} is to be some positive integer or zero which is to be transported from the i^{th} origin to j^{th} destination. A obvious necessary and sufficient condition for the linear programming problem given in Eq. 1 to have a solution is that:

$$\sum_{i=1}^{n} a_i = \sum_{j=1}^{m} b_j \tag{3}$$

(i.e) assume that total available is equal to the total required. If it is not true, a fictitious source or destination can be added. It should be noted that the problem has feasible solution if and only if the condition (2) satisfied.

Now, the problem is to determine x_{ij}, in such a way that the total transportation cost is minimum. Mathematically a fuzzy transportation problem can be stated as follows:

Minimize:

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{4}$$

Subject to:

Table 1: Fuzzy linear programming problem

Variable	1	....	n	Supply
1	~	....	$\tilde{c}11$	$\tilde{\alpha}1$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
m	$\tilde{c}m1$	....	$\tilde{c}mn$	$\tilde{\alpha}mn$
Demand	$\tilde{b}1$	....	$\tilde{b}n$	

$$\left.\begin{aligned} &\sum_{j=1}^{n} x_{ij} = \tilde{a}_i \qquad j = 1, 2, \ldots, n \\ &\sum_{i=1}^{m} x_{ij} = \tilde{b}_j \qquad i = 1, 2, \ldots, m \\ &x_{ij} \geq 0 \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \end{aligned}\right\} \tag{5}$$

In which the transportation costs, $\tilde{c}_{ij}$ supply $\tilde{\alpha}_i$ and demand quantities are fuzzy quantities. The $\tilde{b}_j$ An obvious necessary and sufficient condition for the fuzzy linear programming problem give in Eq. 4-5 to have a solution is that:

$$\sum_{i=1}^{n} \tilde{a}_i \,\square\, \sum_{j=1}^{m} \tilde{b}_j \tag{6}$$

The problem can also be represented as follows Table 1:

Proposedalgorithm for solving transportation problem: These are the steps to solve Transportation Problem

Step 1: Select the first row (source) and verify which column (destination) minimum unit has cost. Write that source under column 1 and corresponding destination under column 2. Continue this process for each source. However if there is any source more than one same minimum value in different destination then write all these destination under column 2.

Step 2: Select those rows under column-1 which have unique destination. For example, under column-1, sources are S1, S2, S3 have minimum unit cost which represents the destination D1, D1, D3 written under column 2. Here D3 is unique and hence allocate cell (S3, D3) a minimum of demand and supply. For an example if corresponding to that cell supply is 8 and demand is 6, then allocate a value 6 for that cell. However, if destinations are not unique then follow step 3. Next delete that row/column where supply/demand exhausted.

Step 3: If destination under column-2 is not unique then select those sources where destinations are identical. Next

Table 2: Fuzzy transportation problem

Variable	FD_1	FD_2	FD_3	FD_4	Fuzzy capacity
FO_1	[5,10,15]	[5,10,20]	[5,15,20]	[5,10,15]	[10,15,20]
FO_2	[5,10,20]	[5,15,20]	[5,15,20]	[10,15,20]	[5,10,15]
FO_3	[5,10,20]	[10,15,20]	[10,15,20]	[5,10,15]	[20,30,40]
FO_4	[10,15,25]	[5,10,15]	[10,20,30]	[10,15,25]	[15,20,25]
Fuzzy demand	[25,30,35]	[10,15,20]	[5,15,20]	[10,15,25]	

find the difference between minimum and next minimum unit cost for all those sources where destinations are identical.

Step 4 : Correct the source which has maximum difference. Select that source and allocate a minimum of supply and demand to the corresponding destination. Delete that row/column where supply/demand exhausted.

Remark 1: For two or more than two sources, if the maximum difference happens to be same then in that case, find the difference between minimum and next to next minimum unit cost for those sources and selects the source having maximum difference. Allocate a minimum of supply and demand to that cell. Next delete that row/column where supply/demand exhausted.

Step 5: Continue steps 3 and 4 for remaining sources and destinations till (m+n-1) cells are allocated.

Step 6 : Total cost is calculated as sum of the product of cost and corresponding allocated value of supply/ demand. That is,

$$\text{Total Cost} = \sum\sum C_{ij} X_{ij}$$

Numerical example: A company has four sources O_1, O_2, O_3 and O_4 and four destinations D_1, D_2, D_3 and D_4, the fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} where

$$[C_{ij}]_{4\times 4} = \begin{pmatrix} (5,10,15) & (5,10,20) & (5,15,20) & (5,10,15) \\ (5,10,20) & (5,15,20) & (5,10,15) & (10,15,20) \\ (5,10,20) & (10,15,20) & (10,15,20) & (5,10,15) \\ (10,15,25) & (5,10,15) & (10,20,30) & (10,15,25) \end{pmatrix}$$

and fuzzy availability of the product at source are ((10,15,20) (5,10,15) (20,30,40) (15,20,25)) and the fuzzy demand of the product at destinations are ((25,30,35) (10,15,20) (5,15,20) (10,15,25)) respectively. The fuzzy transportation problems are (Table 2)

RESULTS AND DISCUSSION

Solution: In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form Min Z = R(5,10,15)x_{11}+ R(5,10,20)x_{12}+R(5,15,20) x_{13}+ R (5,10,15) x_{14}+R (5,10,20) x_{21}+R5,15,20)x_{22}+R(5,15,20)x_{23}+R(10,15,20)x_{24}+R(5,10,20) x_{31}+R(10,15,20) x_{32}+R (10,15,20) x_{33}+R (5,10,15) x_{34}+ R (10,15,25)x_{41}+R(5,10,15)x_{42} +R(10,20,30)x_{43} +R(10,15,25)x_{44}

$$R(5,10,15) = \frac{5+4*10+15}{6} = 10$$

Similarly

R (5,10,20) = 11.25, R (5,15,20) = 13.75, R (5,10,15) = 10, R (5,10,20) = 11.25,
R(5,15,20) = 13.75, R (5,10,15) = 10, R (10,15,20) = 15, R(5,10,20) = 11.25
R(10,15,20) = 15, R(10,15,20) = 15, R(5,10,15) = 10,R (10,15,25) = 16.25
R(5,10,15) = 10 , R(10,20,30) = 20, R(10,15,25) = 16.25

Rank of all supply: R (10,15,20) = 15, R (5,10,15) = 10, R (20,30,40) = 30 , R (15,20,25) = 20

Rank of all demand: R(25,30,35) = 30, R (10,15,20) = 15, R (5,15,20) = 13.75, R (10,15,25) = 16.25

Table 3: Remark of all demand

Variable	D1	D2	D3	D4	Supply
O1	10	11.25	13.75	10	15
O2	11.25	13.75	10	15	10
O3	11.25	15	15	10	30
O4	16.25	10	20	16.25	20
Demand	30	15	13.75	16.25	

Table 4: Table after ranking

Variable	D1	D2	D3	D4	Supply
O1	11.25 10	11.25	3.75 13.75	10	15
O2	11.25	13.75	10 10	15	10
O3	13.75 11.25	15	15	16.25 10	30
O4	5 16.25	15 10	20	16.25	20
D	30	15	13.75	16.25	

Table after ranking: Table 3 with the help of the proposed algorithm, we can get the below solution.

Hence (4+4-1)=7 cells are allocated and hence we got our feasible soln. Next we calculate total cost and its corresponding allocated value of supply and demand represented in Table 4.

TotalCost(10×11.25)+(13.75×3.75)+(10×10)+(11.25×13.75) +(10×16.25)+(16.25×5)+(10×5) = 712.5

This is a basic feasible solution. The solution obtained using NCM, LCM, VAM and MODI/Stepping stone methods respectively. Hence the basic feasible solution obtained from new method is optional soln.

Our solution is same as that of optional solution obtained by using LCM, VAM, MODI/Stepping stone methods. Thus, the method also gives optional soln.

CONCLUSION

In this study, the transportation costs are considered as imprecise numbers described by fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy transportation problem of triangular fuzzy numbers has been transformed into crisp transportation problem using ranking indices. Numerical examples show that by this method we can have the optimal solution as well as the crisp and fuzzy optimal total cost. By using ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy problems can be obtained by our proposed method effectively. This technique can also be used in solving other types of problems like, project schedules, assignment problems and network flow problems.

REFERENCES

Arsham, H. and A.B. Kahn, 1989. A simplex-type algorithm for general transportation problems: An alternative to stepping-stone. J. Oper. Res. Soc., 40: 581-590.

Bellman, R.E. and L.A. Zadeh, 1970. Decision making in a fuzzy environment. Manage. Sci., 17: 141-164.

Chanas, S. and D. Kuchta, 1996. A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. Fuzzy Set Syst., 82: 299-305.

Chanas, S., W. Kolodziejczyk and A. Machaj, 1984. A fuzzy approach to the transportation problem. Fuzzy Set Syst., 13: 211-221.

Gani, A N. and K.A. Razak, 2006. Two stage fuzzy transportation problem. J. Phys. Sci., 10: 63-69.

Gani, A.N., A.E. Samuel and D. Anuradha, 2011. Simplex type algorithm for solving fuzzy transportation problem. Tamsui Oxf. J. Inf. Math Sci., 27: 89-98.

Kaur, A. and A. Kumar, 2011. A new method for solving fuzzy transportation problems using ranking function. Appl. Math. Modell., 35: 5652-5661.

Liu, S.T. and C. Kao, 2004. Solving fuzzy transportation problems based on extension principle. Eur. J. Oper. Res., 153: 661-674.

Pandian, P. and G. Natarajan, 2010a. A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Appl. Math. Sci., 4: 79-90.

Pandian, P. and G. Natarajan, 2010b. An optimal more-for-less solution to fuzzy transportation problems with mixed constraints. Appl. Math. Sci., 4: 1405-1415.

Shiv, K.K., L. InduBhusan and S.P. Varma, 2011. An alternative method for obtaining initial feasible solution to a transportation problem and test for optimality. Int. J. Comput. Sci. Communications, 2: 455-457.

Zadeh, L., 1965. Fuzzy Sets, Information and Control. Vol. 8, CRC Press, New York, pp: 338-353.