Asian Journal of Information Technology 15 (1): 73-75, 2016
ISSN: 1682-3915

On α^m-Connectedness in Topological Spaces

[1]R. Parimelazhagan and [2]Milby Mathew
[1]Department of Mathematics, RVS Technical Campus,
641402 Coimbatore, Tamil Nadu, India
[2]Department of Mathematics, Karpagam University, Coimbatore, Tamil Nadu, India

Abstract: In this study, introduce α^m-homeomorphisms by using α^m-continuous functions and α^m-irresolute functions and introduce the concept of α^m-connectedness by utilizing α^m-open sets in topological spaces and study the characterizations and their properties.

Key words: α^m-homeomorphisms, α^m-continuous functions, α^m-irresolute functions, α^m-connectedness, α^m-open sets

INTRODUCTION

In the mathematical study, Njastad (1965) introduced and defined an α-open/closed set. After the works of Njastad (1965) on α-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, α-open sets. The concept of g-closed (Levine, 1970), s-open (Levine, 1963) and α-open sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians (Balachandran *et al*., 1991; Maki *et al*., 1993). In 1970, Levine generalized the concept of closed sets to generalized closed sets. After that, there is a vast progress occurred in the field of generalized open sets (complement of respective closed sets). In topological spaces, it is well known that normality is preserved under closed continuous surjections. Many researchers tried to weaken the condition "closed" in this theorem. In Long and Herrington (1978), used almost closedness due to Singal and Singal (1968). Malghan in 1982, used g-closedness. Gsesnwooo and Reilly (1986) used a closedness due to Mashhour *et al.* (1982).

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map $f: X \to Y$ when both f and f^{-1} are continuous. It is well known that as Janich in 1980 says correctly: homeomorphisms play the same role in topology that linear isomorphisms play in linear algebra or that biholomorphic maps play in function theory or group isomorphisms in group theory or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Maki *et al.* (1991) introduced g-homeomorphisms and gc-homeomorphisms in topological spaces. Recently, Devi *et al.* (1995) studied semi-generalized homeomorphisms and generalized semi homeomorphisms.

PRELIMINARIES

Throughout this study (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 1: A subset A of a topological space (X, τ) is called:

- A preopen set if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$ (Mashhour *et al*., 1982)
- A semiopen set if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$ (Levine, 1963)
- An α-open set if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$ (Njastad, 1965)
- A semi-preopen set (β open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $int(cl(int(A))) \subseteq A$ (Andrijevic, 1986)
- An closed set if $int(cl(A)) \subseteq U$, whenever $A \subseteq U$ and U is aopen (Mathew and Parimelazhagan, 2014)

The complement of α^m-closed set is called an α^m-open set.

Definition 2: A function $f:(X, \tau) \to (Y, \sigma)$ is called:

- An α^m-continuous if $f^{-1}(V)$ is α^m-closed in (X, τ) for every closed set V of (Y, σ)

Corresponding Author: R. Parimelazhagan, Department of Mathematics, RVS Technical Campus, 641402 Coimbatore, Tamil Nadu, India

- An α^m-irresolute if f^{-1} (V) is α^m-closed in (X, τ) for every α^m-closed set V of (Y, σ)
- An α-irresolute if f^{-1} (V) is aclosed in (X, τ) for every aclosed set V of (Y, σ) (Njastad, 1965)

On α^m-HOMEOMORPHISMS

Definition: A function f: (X, τ_1)$\rightarrow$(Y, σ_2) is said to be an α^m-homeomorphism if both f and f^{-1} are α^m-irresolute. It is denoted that family of all α^m-homeomorphisms of a topological space (X, τ_X) onto itself by α^m-h(X,τ_1).

Proposition: Let f:(X, τ_1)$\rightarrow$(Y, τ_2) and g:(Y, τ_2)$\rightarrow$(Z, τ_3) are α^m-homeomorphisms, then their composition g$\circ$f: (X, τ_1)$\rightarrow$(Z, τ_3) is also α^m-homeomorphism.

Proof: Let U be an α^m-open set in (Z, τ_3). Since, g is α^m-irresolute, g^{-1}(U) is α^m-open in (Y, τ_2). Since, f is α^m-irresolute, $f^{-1}(g^{-1}(V)) = (g\circ f)^{-1}(V)$ is α^m-open set in (X, τ_1). Therefore, g$\circ$f is α^m-irresolute.

Also, for an α^m-open set G in (X, τ_1) and has (g$\circ$f) (G) = g(f(G)) = g(W), where, W = f(G). By hypothesis, f(G) is α^m-open in (Y, τ_2) and so again by hypothesis, g(f(G)) is an α^m-open set in (Z, τ_3). That is (g$\circ$f)(G) is an α^m-open set in (Z, τ_3) and therefore $(g\circ f)^{-1}$is α^m-irresolute. Also, g$\circ$f is a bijection. Hence, g$\circ$f is α^m-homeomorphism.

Theorem 1: The set a^m-h(X,τ_1) is a group under the composition of maps.

Proof: Define a binary operation *:α^m-h(X, τ_1)$\times\alpha^m$-h (X, τ_1)$\rightarrow\alpha^m$-h(X, τ_1) by f*g = g$\circ$f for all f, g$\in\alpha^m$-h(X, τ_1) and $\circ$ is the usual operation of composition of maps g$\circ$f$\in\alpha^m$-h(X, τ_1).

It is known that the composition of maps is associative and the identity map I: (X, τ_1)$\rightarrow$(X, τ_1) belonging to α^m-h(X, τ_1) serves as the identity element. If f$\in\alpha^m$-h(X, τ_1), then $f^{-1}\in\alpha^m$-h(X, τ_1) such that $f\circ f^{-1} = f^{-1}\circ f$ = I and so inverse exists for each element of α^m-h(X, τ_1). Therefore, (α^m-h(X, τ_1), $\circ$) is a group under the operation of composition of maps.

Theorem 2: Let f: (X, τ_1)$\rightarrow$(Y, τ_2) be an α^m-homeomorphism. Then f induces an isomorphism from the group α^m-h(X, τ_1) onto the group τ^m-h(Y, τ_2)

Proof: Using the map f is defined a map Ψ_f: α^m-h(X, τ_1)$\rightarrow\alpha^m$-h(Y, τ_2) by $\Psi_f(h) = f\circ h\circ f^{-1}$ for every h$\in\alpha^m$-h(X, τ_1). Then, Ψ_f is a bijection. Further, for all h_1, $h_2\in\alpha^m$-h(X, τ_1), $\Psi_f(h_1\circ h_2) = f\circ(h_1\circ h_2)\circ f^{-1} = (f\circ h_1\circ f^{-1})\circ(f\circ h_2\circ f^{-1}) = \Psi_f(h_1)\circ\ \Psi_f(h_2)$. Therefore, Ψ_f is a homeomorphism and so, it is an isomorphism induced by f.

ON α^m-CONNECTEDNESS

Definition: A topological space (X, τ_1) is said to be α^mconnected if X cannot be expressed as a disjoint union of two non-empty α^m-open sets. A subset of X is α^mconnected if it is α^m-connected as a subspace.

Theorem: For a topological space (X, $\hat{o}_1$), the following are equivalent:

- (X, τ_1) is α^m-connected
- (X, τ_1) and ϕ are the only subsets of (X, τ_1) which are both α^mopen and α^m-closed
- Each α^mcontinuous map of (X, τ_1) into a discrete space (Y, τ_2) with at least two points is constant map

Proof (a)$\Rightarrow$(b): Suppose (X, τ_1) is α^m-connected. Let S be a proper subset which is both α^m- open and α^m- closed in (X, τ_1). Its complement X/S is also α^m-open and α^m-closed. X = S$\cup$(X/S), a disjoint union of two non empty α^m-open sets which is contradicts (a). Therefore, S = Φ or X.

(b)$\Rightarrow$(a): Suppose that X = A$\cup$B where, A and B are disjoint non empty α^m-open subsets of (X, τ_1). Then, A is both α^m-open and α^m-closed. By assumption A = Φ or X. Therefore X is α^m-connected.

(b)$\Rightarrow$(c): Let f:(X, τ_1)$\rightarrow$(Y, τ_2) be an α^m-continuous map. Then (X, τ_1) is covered by α^m-open and α^m-closed covering {f^{-1}(y):y$\in$Y}. By assumption $f^{-1}(y) = \Phi$ or X for each y$\in$Y. If $f^{-1}(y) = \Phi$ for all y$\in$Y, then f fails to be a map. Then, there exists only one point y$\in$Y such that $f^{-1}(y) \neq \Phi$ and hence f^{-1}(y) = X. This shows that f is a constant map.

(c)$\Rightarrow$(b): Let S be both α^m-open and α^m-closed in X. Suppose S$\neq\Phi$ Let f:(X, τ_1)$\rightarrow$(Y, τ_2) be an α^m-continuous map defined by f(S) = y and f(S^c) = {w} for some distinct points y and win (Y, τ_2). By assumption f is a constant map. Therefore, it has S = X.

Theorem 1: Every α^m-connected space is connected.

Proof: Let (X, τ_1) be α^m-connected. In case X is not connected. Then there exists a proper non-empty subset B of (X, τ_1) which is both open and closed in (X, τ_1). Since, every closed set is α^m-closed, B is a proper non empty subset of (X, τ_1) which is both α^m-open and α^m-closed in (X, τ_1), (X, τ_1) is not α^m-connected. This proves the theorem.

Theorem 2: If f:(X, τ_1)$\rightarrow$(Y, τ_2) is an α^m-continuous and X is α^m-connected, then (Y, τ_2) is connected.

Proof: In case that (Y, τ_2) is not connected. Let $Y = A \cup B$ where, A and B are disjoint non-empty open set in (Y, τ_2). Since, f is α^m-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty α^m-open sets in (X, τ_1). This contradicts the fact that (X, τ_1) is α^m-connected. Hence, Y is connected.

Theorem 3: If f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ is an α^m-irresolute and (X, τ_1) is α^m-connected, then (Y, τ_2) is α^m-connected.

Proof: In case that (Y, τ_2) is not α^m-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty α^m-open sets in (Y, τ_2). Since, f is α^m-irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty α^m-open sets in (X, τ_1). This contradicts the fact that (X, τ_1) is α^m-connected. Hence (Y, τ_2) is α^m-connected.

Theorem 4: Perhaps that (X, τ_1) is T α^m-space then (X, τ_1) is connected if and only if it is α^m-connected.

Proof: In case that (X, τ_1) is connected. Then (X, τ_1) cannot be expressed as dis-joint union of two non-empty proper subsets of (X, τ_1). If (X, τ_1) is not a α^m-connected space. Let A and B be any two α^m-open subsets of (X, τ_1) such that $Y = A \cup B$, where $A \cap B = \Phi$ and $A \subset X$, $B \subset X$. Since, (X, τ_1) is $T\alpha^m$-space and A, B are α^mopen. A, B are open subsets of (X, τ_1), which contradicts that (X, τ_1) is connected. Therefore (X, τ_1) is α^m-connected. onversely, every open set is α -op̄en. Therefore, every α^m-connected space is connected.

Theorem 5: If the α^m-open sets C and D form a separation of (X, τ_1) and if (Y, τ_2) is α^m-connected subspace of (X, τ_1), then (Y, τ_2) lies entirely within C or D.

Proof: Since, C and D are both α^m-open in (X, τ_1), the sets $C \cap Y$ and $D \cap Y$ are α^m-open in (Y, τ_2). These two sets are disjoint and their union is (Y, τ_2). If they were both non-empty, they would constitute a separation of (Y, τ_2). Therefore, one of them is empty. Hence (Y, τ_2) must lie entirely in C or in D.

Theorem 6: Let A be an α^m-connected subspace of (X, τ_1). If $A \subset B \subset C\alpha^m(A)$ then B is also α^m-connected.

Proof: Let A be α^m-connected and let $A \subset B \subset C\alpha^m(A)$. Perhaps that $B = C \cup D$ is a separation of B by α^m-open sets. A must lie entirely in C or in D. In case that $A \subset C$, then $C\alpha^m(A) \subset C\alpha^m(C)$. Since, $C\alpha^m(C)$ and D are disjoint, B cannot intersect D. This contradictsthe fact that D is non-empty subset of B. So, $D = \Phi$ which implies B is α^m-connected.

CONCLUSION

It is recently introduced that the notion of α^m-closed sets which are strictly weaker than aclosed sets. α^m-closed sets are used to define a new class of homeomorphisms called α^m-homeomorphisms. Also, introduce α^m-connectedness in topological spaces. The purpose of the present study is to improve characterizations and properties.

REFERENCES

Andrijevic, D., 1986. Semi-preopen sets. Mat. Vesnik, 38: 24-32.

Balachandran, K., P. Sundaram and H. Maki, 1991. On generalized continuous maps in topological spaces. Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 12: 5-13.

Devi, R., K. Balachandran and H. Maki, 1995. Semi-generalized homeomorphisms and generalized semi-homoeomorphisms in topological spaces. Indian J. Pure Appl. Math., 26: 271-284.

Gsesnwooo, S.I.N.A. and I.L. Reilly, 1986. On feebly closed mappings. Indian J. Pure Appl. Math., 17: 1101-1105.

Levine, N., 1963. Semi-open sets and semi-continuity in topological spaces. Amer. Math. Monthly, 70: 36-41.

Levine, N., 1970. Generalized closed sets in topology. Rend. Circ. Mat. Palermo, 19: 89-96.

Long, P.E. and L.L. Herrington, 1978. Basic properties of regular-closed functions. Rend. Cir. Mat. Palermo., 27: 20-28.

Maki, H., P. Sundaram and K. Balachandran, 1991. On generalized homeomorphisms in topological spaces. Bull. Fukuoka Univ. Ed. III, 40: 13-21.

Maki, H., R. Devi and K. Balachandran, 1993. Semi-generalized closed maps and generalized semi-closed maps. Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 14: 41-54.

Mashhour, A.S., M.A. El-Monsef and S.N. El-Deeb, 1982. On precontinuous and weak precontinuous mappings. Proc. Math. Phys. Soc. Egypt, 53: 47-53.

Mathew, M. and R. Parimelazhagan, 2014. αm-closed sets in topological spaces. Int. J. Math. Anal., 8: 2325-2329.

Njastad, O., 1965. On some classes of nearly open sets. Pac. J. Math., 15: 961-970.

Singal, M.K. and A.R. Singal, 1968. Almost-continuous mappings. Yokohama Math. J., 16: 63-73.