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An Economic Lot Size Model with Shortages and Inflation

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Abstract: Inflation is a plague, which characterized the economy of many developing countries especially, in the current global economic meltdown. This effect influenced cases of both explainable and unexplainable shortages in production inventory. In this study, the management of inventory under twin limiting situations of inflation and shortages were considered. Also, presented was a model for determination of economic lot size, which includes the two conditions, time value of money and comparative study of corresponding cost implications using numerical example to illustrate the developed model.

Key words: Inflation rate, Inventory management, interest rate, production planning, shortage cost

INTRODUCTION

Inventory has been defined as idle resources that posses economic value (Monks, 1987). Usually, it is an important component of the investment portfolio of any productive system. Literature reveals that sometimes up to 60% of the annual production budget is spent on material and other inventories (Lucey, 1988; Datta, 1989). It cannot be overemphasized that better inventory management would invariably improve organizational profitability, reduce costs and lead to prudent use of scarce capital (Wemmerlov, 1982).

Apart from raw materials, other types of inventory include; in-process, components, supplies and finished goods inventory. The primary aim of inventory management is to determine how much resources or inputs are to be ordered and when to order so as to minimize production cost, while meeting the requisite requirements.

Due to ranging peculiarities of the production inventory no particular inventory model has general application to the entire variants inventory situations. Consequently, a variety of inventory models have emerged, which address specific inventory problems (Trigg and Pitts, 1962; Naddor, 1966; Lev *et al*., 1981; Silver, 1981; Harvey, 1987; Tersine, 1994).

The present global economic meltdown has placed much task on investment managers in the area monitoring the effects of inflation together with interest rate (return on capital) in relation to inventory problems. Usually, the problem is that of balancing the costs of less than adequate inventory (Under-stocking) and that of cost of more than adequate inventory (Over-stocking). The goal is to have adequate items at all times at minimal cost (Taha, 1982; Datta, 1989; Harris, 1990). Solution methods used to solving these problems are basically analytical techniques and the sophisticated application of mathematical programming. However, the mathematical complexity of the resulting models increases as we move away from the assumption of deterministic to probabilistic non-stationary demand (Taha, 1982; Lev and Soyster, 1979). Silver (1981) reviewed many classifications of the inventory problem, highlighting the limitations, while also advocating the bridging of the gap between theory and practice. Buzacott (1975) developed an inventory model with inflation factor included. He suggested obtaining the optimal order quantity by a process of iteration. However, closer examination reveals that the solution method is indeed an approximation; because of the assumptions inherent in his use of a quadratic approximation. Also, a critical review informed that while the main objective was achieved the solution obtained is also an approximation.

In this research, the development of an inventory model with shortage and inflation factors was described with the motive of providing a solution to a problem that is close to real life; as managers of inventory have to deal with problems with these twin features of shortages and inflation regularly.

Model development: Using the principles of the classical EOQ model, the following assumptions are made:

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- Demand rate is determinable and constant
- Supplies are delivered in batches
- Replacement is instantaneous on request
- Inflation rate is assumed constant over a period of time
- Unit purchase cost and other relevant costs are affected by inflation
- Shortages are allowed at a cost and over a given back-ordering time frame

The situation of a determinable demand rate in which shortages are allowed is illustrated in Fig. 1. The shortages could be backordered within the limit of the backlogged of the demand. The maximum inventory level is S and occur, when the inventory is replenished. The lot size is less than the order level as a result of the backorder.

The following notations are used in the model development:

C_0	=	Initial Purchasing cost/Unit
C_1	=	Set up cost
C_2	=	Holding cost/Unit/Unit time
C_3	=	Shortage cost /Unit/Unit time
$C_{(t)}$	=	Cost at time (t)
$C_{(0,L)}$	=	Cost over period (0, L)
D	=	Demand rate
MT	=	Megaton
R	=	Rate of return on inventory investment
K	=	Effective inflation rate
l	=	Number of orders
L	=	Planning Horizon
S	=	Maximum inventory level
Q	=	Lot size
I	=	Inventory cycle
t_1	=	Time interval before shortage
t_2	=	Shortage period
T	=	Ordering interval

TIC (L,T) Total Inventory cost over period (0, L) using ordering interval T.

Total inventory cost over the period (0, L), can be expressed as:

$$\text{TIC (L, T)} = \text{(Set-up costs)} + \text{(Shortage costs)} + \text{(Holding costs)} + \text{(Purchase costs)} \quad (1)$$

The expression for each cost component is derived as follow:

Ordering cost over the period (0, L): Considering that costs increase with time then express.

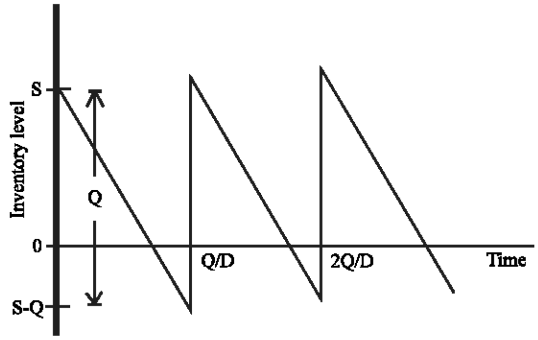

Fig. 1: Lot-size model with shortages allowed

$$C_1\,(T) = C_1\ell^{kt} \quad (2)$$

Assumption of batch supplies then:

$$L = lT$$

Over the period (0, L) then,

$$C_1\,(0, L) = C_1 + C_1\,(T) + C_1\,(2T) + \ldots + C_1\,((l-1)T) \quad (3)$$

Therefore,

$$C_1(0,L) = \sum_{n-0}^{1-1} C_1 \ell^{KTn} \quad (4)$$

$$C_1\,(0,L) = \left[C_1 + C_1\ell^{KT} + C_1\ell^{2KT} + \ldots + C_1\ell^{(l-1)KT}\right]$$

The geometric series can then be expressed as:

$$C_1(0,L) = C_1\left[\frac{e^{kL}-1}{e^{kT}-1}\right] \quad (5)$$

Holding costs over the period (0, L): In each order period, the holding cost can be expressed as:

$$\int_{w-0}^{T-t_2} DC_2(T, T+w)dw \quad (6)$$

over the period (0, L) we then have

$$C_2\,(0,L) - \sum_{n-1}^{1-1}\int_{w-0}^{T-t_2} DC_2\,(T, T+w)dw \quad (7)$$

but,

$$C_2\,(nT, nT+w) = RC_2\,(T)w$$

that is,

$$C_2\,(0,L) - \frac{DR(T-t_2)}{2}\sum_{n-0}^{1-1} C_2\,(T) \quad (8)$$

but,

$$C_2(T)=C_0+C_0e^{KT}+C_0e^{2KT}+.....+C_0e^{(l-1)KT}$$

Then express:

$$C_1(0,L)=\frac{C_0DR(T-t_2)^2}{2}\left[\frac{e^{KL}-1}{e^{KT}-1}\right] \quad (9)$$

Shortage costs over the period (0, L): Shortage cost per period of time according to Buzacott (1975) is given by:

$$\int_{n-0}^{t_2} DC_3(T)m\,dm \quad (10)$$

Thus, over the period (0, L) the shortage cost becomes:

$$C_3(0,L)=\sum_{n-0}^{l-1}\int_{m-T-t}^{T} DC_3(nT,nT+m)dm$$

But,

$$T\text{-}t = t_2$$

Hence,

$$C_1(0,L)=\frac{Dt_2^{\ 2}}{2}\left[\frac{e^{KL}-1}{e^{KT}-1}\right] \quad (11)$$

Purchasing costs over period (0, L): The initial purchase cost is DTC_0

But costs increase over time; such that:

$$C(t)=C_0e^{Kt} \quad (12)$$

Purchasing cost over the period (0, L) is then given by:

$$C_1(0,L)=DT(C_0+C_0(T)+C_0(2T)+C_0(l-1)T)$$

That is:

$$C_0(0,L)=\sum_{n-0}^{(l-1)}DTC_0e^{KTn} \quad (13)$$

This yield:

$$C_0(0,L)=DTC_0\left[\frac{e^{KL}-1}{e^{KT}-1}\right] \quad (14)$$

The total inventory cost over period (0, L), TIC (0, L), was obtained by adding all the component costs developed in Eq. 5, 9, 11 and 14.

$$TIC(0,L)-\left[\begin{matrix}C+\dfrac{C_0DR(T-t_2)^2}{2}\\ +C_3Dt_2^{\ 2}+C_0\,TD\end{matrix}\right]\left[\frac{e^{KL}-1}{e^{KT}-1}\right] \quad (15)$$

Given the objective of minimizing costs, the solution can be obtained by iteration (varying t-values to obtain minimum cost).

RESULTS AND DISCUSSION

In order to demonstrate the application of the proposed model, data for an imported item, in respect of a aluminum manufacturing company in Nigeria was obtained. Questionnaires, interviews and physical observation were utilized to collect the relevant data. Basic data for which the model was applied were stored in spreadsheet format and graphed using EXCEL 2007. Data used include: Time were varied with minimal interval (ΔT = 0.02 year) in order to locate point of inflection in the concave graph obtained.

Unit Cost (C_0)	=	N120,000/MT
Ordering Cost (C_1)	=	N100,000/MT
Stock out Cost (C_3)	=	N50,000/MT
Annual Demand (D)	=	250 MT
Effective inflation rate (k)	=	25%
Interest Rate (R)	=	40%
Time shortage is allowed (t_2)	=	1 month
Planning horizon (L)	=	1 year

Figure 2 shows, the Inventory cost profile. The minimum cost was obtained at the order interval 0.29 (3.45 orders). The associated Inventory Cost is N34,372,136.00. A comparison with the current Inventory cost of N35,664,453.00 shows a decrease of 3.76%. This is indicative of the benefits derivable from the use of the model by the organization.

Figure 3 shows that the inventory cost is quite sensitive to the shortage period allowed the longer the better. However, Fig. 4 shows that while inventory costs

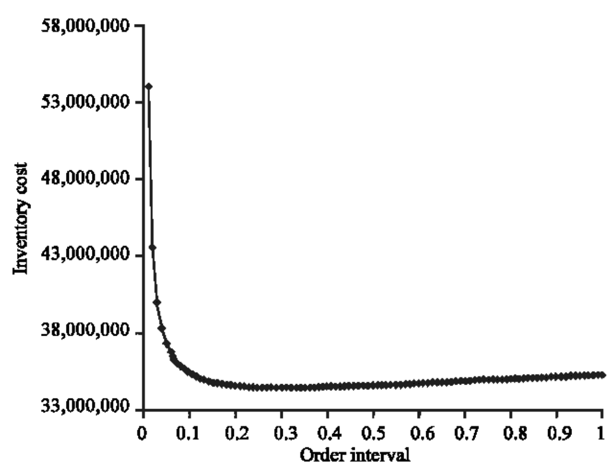

Fig. 2: Inventry cost profile

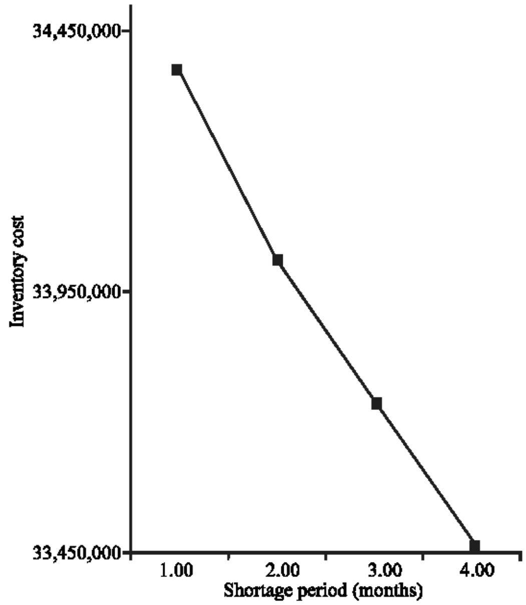

Fig. 3: Shortage period cost ptofile

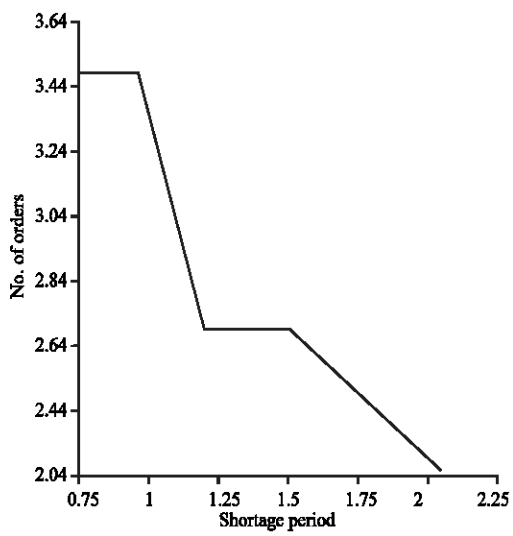

Fig. 4: Ordering policy profile

vary with shortage periods allowed; the ordering policy is not as sensitive. It is indeed a step function. This implies that while the shortage period may vary, the number of orders required in the planning horizon may not. For an example, the same ordering policy (3.45 orders/annum) is required for shortage periods of 0.75-0.96 months, while 2.56 orders/annum are required for shortage periods of 1.2-1.5 months and 2.06 orders/annum required for shortage period of 2.04 months. In real terms however, we an organization would probably prefer to let number of note that orders be an integer value. Given this, the optimal cost and number of orders can then be calculated by perturbation. For an example, given the solution of 3.45 orders/annum obtained in the case examined and compare the cost of 3 orders and 4 orders, respectively in the planning horizon. The policy with lesser cost is then chosen. In this case, 3 orders/year, which yields a savings of 3.75%.

CONCLUSION

We have reviewed the importance of inventory cost minimization with a view to increasing organizational profitability and liquidity. An inventory model with shortages and inflation factors included was derived and applied to a case example; in order to highlight the utility of the model.

The reliability of this model to produce optimal quantity Q* depends on the correctness of the inventory cost elements used. However the benefits derivable from the utilization of the model appear immense.

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